Macroeconomics and Term Structure of Interest Rates

Integrated Framework, Uncertainty and Forecasting

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1. Introduction

General motivation for this thesis lies in the realm of economic policy. Economics is a branch of social sciences. The domain of economic research lies in the methodological descriptions of the processes driving the economy - a main feature of human society. These methodological descriptions are not locked in into the sphere of academics and academic discussions. On the contrary: These descriptions serve as the foundation for rational economic decision making in the sphere of politics. Therefore these descriptions, with the economic policy decisions derived from them, govern in last instance the welfare and the wellbeing of people. With respect to the methodological descriptions in this context, there are two general questions: Where is a lack of methodological description of an economic policy issue of special importance? And secondly: How can this lack be fixed? For this thesis the first question is answered by referring to the important relationship between the macroeconomy and the term structure of interest rates. For the second question this thesis tries to find an answer or at least to show off a way in finding an adequate answer. Combining these two questions leads to the central question underlying this thesis: How can we adequately model the relationship between the macroeconomy and the term structure of interest rates in an integrated modeling framework? As outlined in the following of this introduction, this central question is related to further aspects like aspects regarding economic uncertainty, which this thesis also addresses.

In the moment of the beginning of this thesis the FED starts its third QE program. Until then, with the foregoing first QE and the related initialization of the purchases of US Treasuries announced by the Federal Open Market Committee (FOMC) in March 2009, the second QE in 2010 and Operation Twist in 2011, the volume of US bond purchases amounts to over one trillion Dollars. With these bond purchasing programs the FED starts to influence more directly the whole term structure of interest rates, with the intention to stabilize and stimulate the US economy. But what exactly is the working mechanism by which the monetary policy induced changes of the term structure of interest rates lead to the intended economic stimulations? What is the relationship between the term structure and the economy on which this mechanism builds on? From the perspective of this thesis, there is a lack in an integrated modeling of the relationship between the macroeconomy and the term structure of interest rates. Therefore we can repeat here the central question from above underlying this thesis: How can we adequately model the relationship between the macroeconomy and the term structure of interest rates in an integrated modeling framework?

The bond purchasing programs applied by the FED were supported in parallel by the FED's forward guidance strategy. This strategy aims to reduce the uncertainty about the future monetary policy path. Not in focusing on forward guidance strategies but in generally focusing on the phenomenon of uncertainty, this thesis takes up the aspect of uncertainty. In the literature uncertainty or volatility is viewed mainly as a phenomenon of financial markets, expressing the idiosyncratic fluctuations of asset prices. In this thesis the novelty in thinking about economic uncertainty is at first to switch attention from financial market uncertainties to macroeconomic uncertainties and to specify the various sources of economic uncertainty. Thereafter the challenging empirical task is the extraction of the time-varying uncertainty patterns from the specified sources of economic uncertainty. As the instrument of forward guidance with focus on mainly monetary policy issues suggests, the reasoning about economic uncertainty in a much broader sense could be an advantage for economic policy in general.

To methodologically capture the outlined aspects of this thesis we combine two exiting and promising strands of economic research: Macroeconomic modeling with large-scale dynamic stochastic general equilibrium (DSGE) models as the first strand and term structure of interest rate modeling as the second strand. By modeling the macroeconomy with larger DSGE models it becomes possible to endogenize a lot of different sector specific facets of the economy and their mutual interdependencies. Due to the encapsulated modeling implied by a DSGE model, every facet of the economy defines an own object of economic reasoning and modeling - with its own assumptions, derivations and implications. Due to its "divide and conquer" character this is a powerful tool for the economic researcher. The researcher can focus only on specific objects, can theoretically and empirically examine the phenomenal character of the specific objects and can compare them with economic reality. If necessary, the researcher can modify the assumptions and derivations, which define the configurations of the specific economic object under consideration, without modifying the DSGE model as a whole. Due to the encapsulated character, the researcher only reintegrates this modified object into the overall economic modeling framework of the DSGE model and can then investigate the interdependencies between this object and all other economic objects of the overall model. Such a kind of economic modeling is highly adaptive. If economic reality evolves in new and former unknown or unthinkable directions, the economic researcher only modifies the specific objects which are no longer compatible with changed economic reality and reintegrates these objects into the overall framework.

With the FED's bond purchasing programs economic reality evolved in new and former unknown directions. With its bond purchasing programs the FED starts to influence more directly the whole term structure of interest rates, which reflects the conditions of US Treasuries over the whole maturity spectrum and directly determines the borrowing conditions of the US government. Indirectly the term structure serves as the basis for fixing the credit conditions for private sector borrowers and lenders. The term structure of interest rates has a strong time-related focus. Loosely speaking it directly connects the present to the future. The term structure of interest rates presently determines the future cash flows economic agents face for their presently done middle- to long-term borrowing and lending activities in context to their present consumption and investment decisions and therefore shape their present expectations about the future. Going over from the micro- to the macro-perspective, these shaped present expectations reflect the expectations about the future path of economic development of the economy as a whole and its feedback mechanism on the economic state at the present.

This thesis is focused on the European Monetary Union (EMU). At the beginning of the work for this thesis, the ECB operated under the regime of conventional monetary policy measures. Large bond purchasing programs were not in the scope of monetary policy instruments of the ECB. Monetary policy in its conventional fashion only controls the very short end of the term structure. Changes in the short term rate directly controlled by the central bank leads indirectly via expectations and term premia, which determine the correlation structure between the interest rates of various maturities, to changes in middle and long term interest rates. The diffusion of monetary policy controlled changes of the short term rate through the correlation structure, implied by the term structure, into the middle and long term rates is time-delayed and less controlled, revealing the frictions of this diffusion. In this sense the bond purchasing programs executed by the FED or later by the ECB reduce such kind of frictions. This thesis does not take this more direct control of the term structure into account. Questions related to mechanisms and effects of these novel monetary policy measures define an own field of exiting and intense current economic research. Only the indirect mechanism implied by conventional monetary policy is part of the set of assumptions on which this thesis based on.

For this reason we select for our empirical investigations an observation horizon ranging between Q1/2005 and Q1/2014 - a phase where the Euro and the EMU institutions become more settled and before the ECB initializes its unconventional expanded asset purchase program (EAPP) in Q4/2014 and the public sector purchase program (PSPP) in Q1/2015 - but with critical events such as the starting of the sharp decline of U.S. housing prices in Q1/Q2 2007 and the FED's intervention by lowering its federal funds rate from 5.25% in September 2007 to 0.25% in December 2008. Further there is the bankruptcy of Lehman Brothers in September 2008, the spillover of the U.S. sub-prime and financial markets crisis to the EMU - becoming here a sovereign debt crisis and ECB's short term interest rates lowering interventions especially since Q4/2008. To pronounce this point again: Only conventional monetary policy measures are part of this thesis. But - and this is important for this thesis - the bond purchasing programs executed by the FED and later by the ECB have revealed the intention to strongly utilize the relationship between the term structure of interest rates and the macroeconomy for stimulating and stabilizing the economy. Finding an answer to the question for an adequate modeling of this relationship in an integrated modeling framework substantially shapes the concrete research agenda underlying this thesis in combining large-scale DSGE models and term structure of interest rate models to get such an integrated modeling framework. Therefore, beside the modeling of the economy by applying DSGE models, modeling of the term structure of interest rates defines the second exciting and promising strand of economic research mentioned above.

At the beginning of the work on this thesis the integration of the term structure of interest rates and economic issues related to the term structure of interest rates into the DSGE modeling framework were only at the beginning and were empirically focused on the US. But because of the outlined encapsulated characteristics of DSGE models, these kind of economic models are predestined for an integrated modeling of the macroeconomy on the one side and the term structure of interest rates on the other side. The term structure of interest rates becomes a further facet of the economy and defines therefore a further specific object of economic reasoning and modeling, with its own specific assumptions and derivations. This specific object can be integrated into the overall modeling framework to investigate the interdependencies between this newly integrated object and the other objects of the overall model.

With respect to the modeling of the term structure of interest rates there is a vast amount of research. For our purposes we have to focus on term structure models, which are - beside their technical character in mathematically describing the correlations and dynamics implied by the term structure - able for an integration into a broader macroeconomic modeling context. The most promising approach for this thesis is the arbitrage-free affine term structure modeling (ATSM) approach, where the dynamics and correlations of the term structure are driven by a small number of latent term structure factors. The dynamics of these factors are driven by a simple VAR model. In its latent factor formulations these pure term structure models tend to be captured in their own sphere: Latent factors are derived from the past dynamics of the term structure aiming to describe the future dynamics of the term structure. In this setting the latent factors serve more as a reduction in the complexity of the phenomenon than as an explanation of the phenomenon. But beside its latent or pure term structure form it becomes possible to integrate further observable macroeconomic factors into the equations of the VAR. For the bond pricing scheme implied by the ATSM the short term rate and its expected future path are used to determine the prices and therefore the interest rates implied by bonds of different maturities. In the ATSM approach the equation describing the short term rate is given. For our purposes we reformulate the short term rate equation in the fashion of a monetary policy Taylor-rule, such that the short term rate and its description becomes the connecting point between the interest rates - directly the short term interest rate and indirectly via the bond pricing scheme the whole term structure of interest rates - and the macroeconomy with monetary policy relevant measures like e.g. inflation or economic growth.

Chapter 2 outlines in detail how to integrate the term structure of interest rates into a larger DSGE model. In chapter 2 we select the Smets-Wouters 2007 DSGE model as the

DSGE model for our purposes. The Smets-Wouters model is very popular in the DSGE literature and implies in its fourteen structural equations important economic facets such as households and firms intertemporal consumption and investment decisions, firms price setting in markets under monopolistic competition, labor unions wage negotiations as well as monetary policy reactions facing price and wage stickiness. In the Smets-Wouters model monetary policy decisions follow a Taylor-rule like reaction function. As already mentioned the Taylor-rule is used by us to connect the closed-economy's micro- and macroeconomic processes implied by the Smets-Wouters DSGE model with the arbitrage-free pricing scheme implied by the ATSM. Here we reformulate the describing equation of the short term rate of the ATSM in the fashion of the monetary policy Taylor-rule of the Smets-Wouters DSGE model. In chapter 2 we separately estimate and analyze our term structure extended Smets-Wouters DSGE-ATSM for Germany, France and Italy - the three largest economies in the EMU.

Chapter 3 deviates from the country specific focus of the closed-economy Smets-Wouters model by focusing on the open-economy New Area Wide Model (NAWM) used by the ECB. The NAWM was developed for describing the EMU and its dependencies to the global economy as a whole, meaning that its focus lies on the interdependencies between EMU aggregates on the one side and the global economy on the other side. The NAWM endogenizes a lot of the closed-economy facets in the same way the Smets-Wouters DSGE faces these aspects. But because of its open-economy character, the NAWM is even larger in its modeling scope, reflected in an even larger number of equations describing the NAWM economy. In chapter 3 we use an alternative to the outlined ATSM for our term structure modeling. This has two reasons: The first lies in the complexity of the model. Keeping the integrated model tractable we integrate a term structure model into the NAWM, which is more parsimonious with respect to its parameters. The second reason is, that with the NAWM not a specific EMU country is modeled. Instead the NAWM models the EMU as a whole. This EMU wide approach should have an adequate equivalent on the term structure side. We find such an adequate equivalent in the parsimonious dynamic Nelson Siegel (DNS) model, where common EMU term structure factors are applied to the model. We are the first who apply such a common factor approach to the EMU for modeling and analyzing the common forces extracted from the country specific term structure developments. Further we are the first who integrate such a common factor term structure approach in a large-scale DSGE model, which is able to model the micro- and macroeconomic processes underlying the EMU's economic development as a whole.

As mentioned at the beginning of this introduction, a further aspect of this thesis is economic uncertainty. In the baseline DSGE models used in this thesis the dynamics of the economic state variables, describing the state of the economy, are disturbed by exogenously determined variables representing various sources of economic uncertainty. These exogenous shock variables follow Gaussian distributions, where the character of these Gaussians is static.

This means, that the distribution of possible disturbing events stays the same over the whole observation horizon. In every period of the observation horizon the economy is affected by randomly driven events realized from the same distribution. Economic uncertainty defined by the standard deviation (or synonymous the volatility) of the respective disturbance variable stays the same in all phases observed over the whole horizon. In Gaussian distributions the standard deviation determines the range of possible events centred around their mean. In the DSGE models used in this thesis the Gaussians have zero mean. Smaller standard deviations lead to Gaussians implying a narrower range of possible events with smaller values deviating from their zero mean. Larger standard deviations lead to Gaussians implying a broader range of possible events with larger deviations from their zero mean. For the economy this means in concrete: Larger standard deviations imply larger realized values of the shock variables leading to larger disturbing fluctuations in the economic development.

In chapters 4 and 5 we outline the methodological modeling and the empirical extraction of uncertainty patterns for single EMU economies like Germany, France and Italy in chapter 4 and for the EMU as a whole in chapter 5. The outlined approach in chapter 4 is novel in twofold: First it extends the Smets-Wouters DSGE model by integrating the arbitrage-free term structure of interest rates as outlined in chapter 2, where we focus only on this novel extension. The second, extending novelty is the modeling of time-varying volatilities in this combined DSGE and term structure approach. Here we methodologically introduce time-varying uncertainty patterns of both the macroeconomic as well as of the term structure determining variables. In chapter 5 we extend the scope of the sources of uncertainty in extending the NAWM of chapter 3 by integrating time-varying macroeconomic uncertainties. Therefore in chapter 5 we model, extract and analyze the volatility patterns induced by 21 sources of economic uncertainty over a time horizon ranging from the beginning of 1987 until the beginning of 2014 with the collapse of the Soviet Union, the first and the second Iraq war, the burst of the dot-com bubble, the introduction of the Euro, 09/11, the war in Afghanistan, the global financial crises and the upcoming European sovereign debt crisis - only to mention a few of the global events located in this horizon. With our extension of the NAWM by endogenizing time-varying volatilities, we are the first who reveal and discuss in this systematic manner the time-varying economic uncertainty patterns of a major economy over a larger time horizon.

To briefly summarize our empirical findings in chapter 2 we find that there are two phases where the macroeconomy and the term structure of interest rates are largely affected by macroeconomic shocks. The first phase starts immediately after the collapse of Lehman brothers in September 2008. For this phase the macroeconomy and the term structure are affected in similar ways. The second phase of larger shocks affecting the macroeconomy is between 2010 and 2011. For the term structure this second phase starts in Q3/2010 and dampens in the end of 2012 and therefore directly after Mario Draghi's London speech in July 2012 and the public discussions about more far-reaching ECB measures followed to this speech. Analyzing the shocks in detail we find that there are two kinds of shocks largely affecting the macroeconomic as well as the term structure developments. The first kind is related to risk premiums, reflecting the perception of and the aversion against economic risk in the eyes of the economic agents. The second kind is related to monetary policy issues. The last one reveals the crucial role the ECB plays for the economy in our observation sample. With respect to the weighting of macroeconomic phenomena in determining the term structure of interest rates, we find that issues related to government spending activities have a large influence on the term structure. This influence becomes stronger since 2009 and grows along the term structure with the interest rates' time to maturity, meaning that short term rates are less affected by government spending issues than middle to long term interest rates. In our observation sample this reflects the upcoming EMU's sovereign debt crisis and the high sensitivity of bond yields on issues related to government budget deficits and the sustainability of sovereign debt. We also reveal the imperative of the financial markets imposing the conditions for the governments' further spending activities and increasing the systemic pressure for stimulating self regulating as well as political forces for taking more sustainable future debt paths. With respect to the macroeconomic findings our empirical findings in chapter 3 are very similar to our findings in chapter 2, indicating the robustness of our findings. Here too risk-premiums and monetary policy shocks strongly affect the economy in both phases of high economic fluctuations. With respect to our common term structure factors we reveal the convergence pattern of the term structures of interest rates of various EMU countries. Whereas with the upcoming of the financial crisis the detailed analysis of the common EMU factors reveal the divergence of the country specific term structure developments - first in the levels and time-delayed in the slopes - expressing the differences between long and short term interest rates - of the term structure, indicating the current term structure heterogeneity of the EMU. Our empirical findings in chapter 4 are in common line with our findings in the foregoing chapters 2 and 3. In chapter 4 we reveal patterns of high macroeconomic volatilities in the two recession phases in our observation sample. With respect to the term structure of interest rate we also identify two phases of increased volatilities, whereas with focus on the magnitude of the volatility, the first phase is of even higher volatilities - reflecting the disruptive event of the collapse of Lehman brothers and its immediate consequences. With respect to monetary policy and the risk premiums in the EMU we find in our long-term empirical investigation of economic volatility, ranging between Q1/1987 to Q1/2014, that over this long-term horizon monetary policy issues are extremely uncertain since 2008. Looking at the EMU risk premiums we find that over the long horizon the introduction of the Euro as the common currency leads to a sharp reduction in the EMU risk premium. But since 2005 the uncertainty related to the risk premium is increasing, leading to sharp volatility peaks in the two recession phases observed in the last decade of our long-term horizon already identified in the foregoing chapters.

This thesis based on the (often forgotten) fact pronounce at the beginning of this introduction, that economic research is part of social science and is not part of natural science.

The mathematical formulations of quantitative economic models are subject to restrictions. As e.g. the Lucas critique makes clear: The economy is a highly adaptive system with a lot of feedback loops. There are no unchanging fundamental laws driving the economy, with the only task for the economic researcher to reveal these unchanging laws and to derive policy recommendation based on these revealed laws. Expressed in its very extreme: Every substantial economic insight can have direct feedback effects on the assumptions underlying these insights, changing the assumptions in such a kind, that the insights become invalid in the moment they become public. "This time is different" - this statement (taken as it is and not following the subtle intention of its authors) is in its validity not time-variant and should be regarded carefully in the context of quantitative economic modeling with respect to the suitability of its respective assumptions as well as of its application for policy recommendations. In the chapters 2 to 5 we have used larger quantitative modeling frameworks to analyze the status-quo characteristics of the modeled economy estimated over a specific past observation horizon. In every chapter we have implemented a vast amount of alternative macroeconomic and term structure of interest rate models to evaluate the robustness and the quality of our estimation results. In every chapter we have further tried to derive narratives based on the general mathematical structure of the model and its particular parameter estimates connecting the model equations to economic reality. These narratives should make the implications of the estimated models more plausible and more easy to verify or falsify. With focus on the term structure modeling, the mentioned restrictions become obvious in chapter 6. For chapter 6 we have implemented a larger number of term structure models, discussed in the literature. The implemented models cover the whole scope of past and current term structure of interest rate modeling discussed in the literature. In chapter 6 these models become objects of our forecasting experiments. In these experiments we split the underlying observation horizon into two parts: Over the first part we estimate the model parameters. Over the second part we do the out-of-sample forecasting, meaning that we use the estimated model implied dynamics and evolve the model variables, determining the interest rates of the term structure into the future. The future is represented by the out-of-sample part of our overall horizon not known in the first part of the horizon over which we estimate the model parameters. We evaluate the evolved forecasted interest rates by comparing the forecasts with the true interest rates observed over the second part of the overall horizon and evaluate the differences under different evaluation metrics. From a methodological point the summarizing result of these forecasting experiments is disenchanting: The simple drift-less random walk, where the currently observed value of a random variable is its future forecast, is hard or impossible to beat by the more sophisticated term structure of interest rate models discussed in the past and current literature. A further result of these experiments is, that the more structure these models assume, the poorer is its forecasting performance. Nevertheless it is important to have good economic theories that serve as the foundations for wide ranging economic decision making. These theories have restrictions in their applications e.g. in precise prediction making. Further these theories are no everlasting monoliths: Once formulated, lasting forever. On the contrary: Economic theories should have a strong dynamical character, meaning that they should be highly adaptive in rapidly endogenizing changed economic reality. In this line of thought this thesis was worked out.

2. EMU Term Structure Dynamics analyzed in a DSGE Model Setting

2.1 Introduction

The unconventional and far reaching monetary policy measures, especially the FED's quantitative easing and the ECB's outright monetary policy program as direct reactions of both central banks to the sharp recessions of their economies since 2007, reveal the deep interrelationship between the economy's real and financial sector. Central for the FED's and the ECB's stabilizing programs is the term structure of interest rates. Conventional measures of the central banks focus on setting the short term interest rate for controlling the economy's developments - especially the development of inflation. With respect to its maturity the central bank's short term interest rate is only one interest rate in a broad maturity spectrum. To be more concrete and taking into account the correlation structure between the different maturities of the term structure of interest rates, the short term interest rate directly influences only a small fraction of the term structure. The central bank's programs extend their direct influence on the whole term structure. It becomes possible to directly control the conditions of middle to long term bonds and credits, which are more conventional for financing the consumption and investment activities of households, firms and governments and so are more effective for stimulating the economy's development. These monetary policy measures require an understanding of the mentioned deep interrelationship between the economy's real and financial sector. For extending our understanding and our knowledge of this interrelationship, in this chapter we combine two promising strands of current economic research for getting an integrated modeling and analyzing approach of the real economy on the one side and the term structure of interest rates on the other side. The first strand is related to the macroeconomic modeling with middle- to large-scale DSGE models. These models create a broad modeling set up in which a wide variety of different macroeconomic facets can be integrated more or less easily. With the DSGE model proposed by Smets and Wouters [2003, 2007], which shows similarities to the model proposed by Christiano, Eichenbaum and Evans [2005], we use an established medium- to large-scale New-Keynesian DSGE model, which integrates in its fourteen structural equations important economic aspects such as households and firms intertemporal consumption and investment decisions, firms price setting in markets under monopolistic competition, labor unions wage negotiations as well as monetary poliyc reactions facing price and wage stickiness. Therefore this modeling framework enables in-depth analysis of a broad range of macroeconomic phenomena. The second strand is related to the arbitrage-free term structure of interest rates modeling. Here we focus on the class of arbitrage-free affine term structure models (ATSM) proposed by Duffie and Kan [1996], whereas Ang and Piazzesi [2003] were one of the first who extended the ATSM approach by endogenizing macroeconomic developments. More recently, integration of the term structure of interest rates into a larger macroeconomic modeling framework is addressed by Andreasen, Fernandez-Villaverde and Rubio-Ramirez [2018], De Greave, Emiris and Wouters [2009], Rudebusch and Swanson [2008, 2012], Beakert, Cho and Moreno [2010], van Binsenberg, Fernandez-Villaverde, Koijen and Rubio-Ramirez [2012] or Kliem and Meyer-Gohde [2017]. These works mainly focus on methodological aspects with empirical application to US data.

In this chapter we extend the literature in shifting our attention to the EMU - at a time national and EMU wide politics as well as the ECB as the monetary authority are largely confronted with the upcoming EMU's sovereign debt crisis. Therefore for our analysis we choose the horizon between Q1/2005 and Q1/2014 a phase where the Euro and the EMU institutions become settled and before ECB initializes its unconventional expanded asset purchase program (EAPP) in Q4/2014 and the public sector purchase program (PSPP) in Q1/2015 a separate field of current research. Estimation in such an outstanding economic environment is a challenging econometric task, so that we implemented and estimated for purposes of checking model quality and robustness a broad range of alternative macroeconomic and term structure of interest rate models. Here especially the fit to the observed data revealing the turbulences and disturbances of the crisis' impact on the EMU is crucial for us. Compared to the broad range of alternative macroeconomic and term structure models our modeling approach shows a remarkably good and robust fitting quality. Based on this good and credible data fit we find that the economies of Germany, France and Italy are largely exposed to macroeconomic shocks in the aftermath of the Lehman bankruptcy in September 2008. Shocks are mainly driven from sources related to risk premiums and disturbances affecting the monetary policy decisions in this critical phase. For all three countries there is a second phase of larger economic disturbances ranging for Germany and France between 2010 and 2011, whereas for Italy this second phase only starts in 2011 and lasts longer than in Germany and France on the Italian economy. Our model reveals large macroeconomic shocks on the Italian economy until the end of our data sample in Q1/2014. Interestingly the term structure of interest rates especially for Italy shows a very similar pattern as revealed for the macroeconomic shocks. There are large and fluctuating shocks also appearing in two phases. The first phase lies in the quarters immediately following the Lehman collapse. The second phase of larger shocks effecting the term structure starts in Q3/2010 and dampens in the end of 2012 and therefore directly after Mario Draghi's London speech in July 2012 and the public discussions about more far-reaching ECB measures followed to this speech. We further find that the decisive decrease in the short term rate in Q4/2008 and Q1/2009 in which the ECB's main refinancing operations rate fell by 225 basis points from 3.75% in the beginning of November 2008 to 1.50% in March 2009 is mainly effected by subjects concerning risk premiums. Beside this dominant factor the countries term structure of interest rates also have an impact of the decisions made in this crucial phase. This finding is in line with the estimation of our term structure extended version of the ECB's New Area Wide Model (NAWM) developed by Christoffel, Coenen and Warne [2008], a large-scale DSGE model of high practical relevance for monetary policy decision finding in the EMU outlined in chapter 3. We also find in chapter 3 that this crucial monetary policy decision was mainly driven by risk premiums demanded by foreign and EMU based investors. From our extended NAWM we further find, that the term structure also had an effect on this decision - strengthening the robustness of our findings in this paper.

Different to the classical reaction pattern of the monetary authority to shocks regarding its monetary policy decisions our model estimation reveals an interest rate decreasing response to shocks disturbing the decision finding process. This pattern reflects the "whatever it takes" monetary policy measures induced by the ECB and related institutions, credibly hedging the EMU against depressing exogenous influences. With respect to the countries term structure of interest rates we find that government spending activities have a large influence on the term structure of interest rates. This influence becomes stronger since 2009 and grows with the interest rates time to maturity, meaning that short term rates are less affected by government spending issues than middle to long term interest rates and reflecting the upcoming EMU's sovereign debt crisis and the high sensitivity of bond yields on issues related to government budget deficits and the sustainability of sovereign debt. We also reveal the imperative of the financial markets imposing the conditions for the governments' further spending activities and increasing the systemic pressure for stimulating self regulating as well as political forces for taking more sustainable future debt paths. With respect to Taylor-rule like monetary policy decision making we further find that especially Italian term structure issues, determining the Italian short, middle and long term financing conditions for the Italian government and the Italian economy as a whole play a dominant role in the rational of this kind of monetary policy rule since the second half of 2009.

This chapter is organized as follows: In section 2.2 we outline the fourteen structural log-linearized macroeconomic equations and there interpretations with respect to the processes and the functioning of the modeled economy. We briefly outline the model implied rational expectations building and the technical solution of the model. Further we outline the bond pricing scheme a rational and risk averse investor applies on bonds with different maturities in the ATSM framework. In the econometric part in section 2.3 we outline in short our Bayesian estimation procedure. Section 2.4 contains our empirical analysis and findings, starting with our macroeconomic analysis and continuing with our term structure related analysis and discussions. Due to the importance of the Taylor-rule like monetary policy reaction function with which we combine the macroeconomic developments with the dynamics of sovereign bond markets in section 2.4 we comparatively discuss different kinds of monetary policy reaction functions. Section 2.4 closes with a broad based checking of our model's quality and robustness. The last section 2.5 summarizes our conclusions.

2.2 Modeling Framework

2.2.1 DSGE log-linearized modeling equations

For modeling the economic processes and the diffusion of economic shocks we use the New-Keynesian medium- to large scale macroeconomic model proposed by Smets and Wouters [2007] (SW-2007 model) which is very similar to the model proposed by Christiano, Eichenbaum and Evans [2005] and Smets and Wouters [2003]. The SW-2007 model builds on optimal decisions related to firms in the intermediate and final goods market, households consumption decisions, wage and price related decisions made by firms and labor unions and on ECB's monetary policy decisions. These decision problems lead to 14 log-linearized equilibrium equations listed in brief in this section. Appendix A.4 outlines in more detail the optimization problems and its solutions.

Starting with the aggregated demand side, the economy's output y_t at time t is composed of:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \tag{2.1}$$

where the components are the aggregated consumption c_t and investment i_t, capital utilization z_t and the disturbance term representing economic shocks in exogenous spending ε_t^g. The first three components are weighted by their steady-state shares in output:

$$c_y = 1 - g_y - i_y \tag{2.2}$$

$$i_y = (\gamma - 1 + \delta)\, k_y \tag{2.3}$$

$$z_y = R^k k_y \tag{2.4}$$

where g_y is the steady-state exognous spending-to-output ratio, γ is the steady-state growth rate, δ is the depreciation rate of the economy's capital stock, k_y is the steady-state captital-to-output ratio and R^k is the steady-state rental rate of capital.

Consumption c_t follows:

$$\begin{aligned} c_t = {} & \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \frac{1}{1+\lambda/\gamma}\, \mathbb{E}_t\left[c_{t+1}\right] + \frac{w l_c\left(\sigma_c - 1\right)}{\sigma_c\left(1+\lambda/\gamma\right)}\left(l_t - \mathbb{E}_t\left[l_{t+1}\right]\right) \\ & - \frac{1-\lambda/\gamma}{\sigma_c\left(1+\lambda/\gamma\right)}\left(r_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \varepsilon_t^b \end{aligned} \tag{2.5}$$

so that consumption c_t at time t depends on the previous and expected furture consumption c_{t-1} and $\mathbb{E}_t\left[c_{t+1}\right]$, on the expected growth of working hours $l_t - \mathbb{E}_t\left[l_{t+1}\right]$, the nominal short term interest rate r_t, where the short term rate is adjusted by the expected price change between t and $t+1$ $\mathbb{E}_t\left[\pi_{t+1}\right] = \mathbb{E}_t\left[P_{t+1}/P_t - 1\right]$ and on the disturbance term ε_t^b related to the agent's risk taking behaviour. ε_t^b can be interpreted as a risk premium, stimulating the agent to substitute risk free deposits with more risky assets. σ_c stands for the elasticity of intertemporal substitution whereas λ models the agent's external habit formation. wl_c is the steady-state labor income-to-consumption ratio.

Investment activities are described by:

$$i_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} \mathbb{E}_t\left[i_{t+1}\right] + \frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi} q_t + \varepsilon_t^i \tag{2.6}$$

where β is the household's discount factor for discounting their future consumption and income streams. φ is the steady-state elasticity of the capital adjustment cost function S, described in more detail in Appendix A.4. q_t is the real value of the economy's capital stock and ε_t^i is a disturbance term representing technology shocks on the economy. The capital stock q_t follows the process:

$$q_t = \beta\left(1-\delta\right)\gamma^{-\sigma_c}\,\mathbb{E}_t\left[q_{t+1}\right] - r_t + \mathbb{E}_t\left[\pi_{t+1}\right] + \left(1 - \beta\left(1-\delta\right)\gamma^{-\sigma_c}\right)\mathbb{E}_t\left[r_{t+1}^k\right] - \varepsilon_t^b \tag{2.7}$$

Obviously the real value of the capital stock positively depends on its future value $\mathbb{E}_t\left[q_{t+1}\right]$ and the expected rental rate of capital $\mathbb{E}_t\left[r_{t+1}^k\right]$ and negatively on the current real short term interest rate and the risk premium ε_t^b.

The aggregated supply side of the economy is described by the aggregate log-linearized (Cobb-Douglas) two factor production function:

$$y_t = \Phi\left(\alpha k_t^s + \left(1-\alpha\right) l_t + \varepsilon_t^a\right) \tag{2.8}$$

where α and $(1-\alpha)$ are the share of capital and labor in the production process. Φ is a parameter including the share of fixed cost in production and the disturbance term ε_t^a represents productivity shocks in the economy's aggregated production process.

In the SW-2007 model the capital stock is divided into the overall capital stock k_t and the capital stock k_t^s, where the last one is denoted as capital service, which signals the amount of capital used in the production process. Current capital service k_t^s evolves according to:

$$k_t^s = k_{t-1} + z_t \tag{2.9}$$

so that capital service depends on the overall capital stock k_{t-1} of the previous period $t-1$ and the current degress of captital utilization z_t. The capital utilization z_t depends on the rental rate of capital r_t^k and is given by:

$$z_t = z_1 r_t^k \tag{2.10}$$

with $z_1 = (1-\psi)/\psi$ where $0 \leq \psi \leq 1$ reflects the elasticity of the adjustment costs for capital utilization. A higher ψ leads to higher costs for changing the utilization of the economy's capital stock, so that changes in the utilization from one period to the next are smaller than changes, where ψ lies close to 0.

Capital accumulation k_t follows the process:

$$k_t = \frac{(1-\delta)}{\gamma} k_{t-1} + \frac{(\gamma - 1 + \delta)}{\gamma} \left[i_t + \left(1 + \beta\gamma^{(1-\sigma_c)}\right) \gamma^2 \varphi \varepsilon_t^i \right] \tag{2.11}$$

where beside investments i_t the accumulation process is determined by technology shocks ε_t^i related to the investment process described in Equation 2.6.

Price setting under cost minimization by the firms follows the price setting scheme described by Calvo [1983] and is organized in a market under monopolistic competition, which is segmented into a fraction ξ_P of non-price-setting and $(1-\xi_P)$ of price-setting firms, with $0 \leq \xi_P \leq 1$ measures the degree of price-stickiness in the economy, where $\xi_P = 1$ stands for maximum price-stickiness, whereas $\xi_P = 0$ reflects full price flexibility. The non-price-setting firms act backward looking and index their current prices to prices of the past. The price setting-firms can force a price mark-up μ_t^P on their goods, which in equilibrium is equal to the difference between the marginal product of labor and the real wage for labor and is expressed by:

$$\mu_t^P = \alpha \left(k_t^s - l_t\right) + \varepsilon_t^a - w_t \tag{2.12}$$

In 2.12 the marginal product of labor depends positive on the current share of capital served in the production process and productivity shocks related to the economy's production and negative on the labor amount used in the economy's aggregate production process. w_t stands for the real wage.

Price indexation of non-price-setting firms and the mark-up of the price-setting firms are central drivers of the inflation process, where the mark-up in 2.12 positively relates the inflation rate to the amount of labor demanded in the production process, which leads to the (hybrid) forward-backward-looking New-Keynesian Phillips curve expression:

$$\begin{aligned} \pi_t = & \frac{\iota_P}{\left(1 + \beta\gamma^{(1-\sigma_c)}\iota_P\right)} \pi_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{\left(1 + \beta\gamma^{(1-\sigma_c)}\iota_P\right)} \mathbb{E}_t \left[\pi_{t+1}\right] \\ & - \frac{\left(1 - \beta\gamma^{(1-\sigma_c)}\xi_P\right)\left(1-\xi_P\right)}{\left(1 + \beta\gamma^{(1-\sigma_c)}\iota_P\right) \xi_P \left[\left(\Phi - 1\right)\varepsilon_P + 1\right]} \mu_t^P + \varepsilon_t^P \end{aligned} \tag{2.13}$$

Current inflation depends on the lagged inflation π_{t-1}, the expected future inflation $\mathbb{E}_t\left[\pi_{t+1}\right]$, the price mark-up μ_t^P and the price mark-up shock ε_t^P. ι_P is the degree of indexation to past inflation and determines how backward-looking the inflation process in 2.13 is. The constant ε_P represents the curvature of the goods market aggregator purposed by Kimball [1995].

Cost minimization of the firms implies for the rental rate of capital:

$$r_t^k = -(k_t - l_t) + w_t \tag{2.14}$$

where the rental rate depends negatively on the capital-to-labor ratio and positively on the real wage.

Similar to the price-setting in the goods market, the wage-setting in the SW-2007 model is done in a monopolistically competitive labor market, where a fraction of labor unions $(1-\xi_W)$ can negotiate a wage mark-up μ_t^W for their members, whereas the remaining fraction ξ_W acts only as wage takers, indexing their wages to wages of the past. The wage mark-up μ_t^W follows from:

$$\mu_t^W = w_t - \left[\sigma_l l_t + \frac{1}{(1-\lambda)}(c_t - \lambda c_{t-1})\right] \tag{2.15}$$

which is the difference between the real wage and the marginal rate of substitution between working and consuming. σ_l is the elasticity of labor supply with respect to the real wage. Combining wage maximizing unions with wage taking unions leads to the economy's wage equation:

$$\begin{aligned} w_t = {} & \frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)} w_{t-1} + \left[1 - \frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\right]\left(\mathbb{E}_t\left[w_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right]\right) - \frac{\left(1+\beta\gamma^{(1-\sigma_c)}\iota_W\right)}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\pi_t \\ & + \frac{\iota_W}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\pi_{t-1} - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_W\right)(1-\xi_W)}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\xi_W\left[(\lambda_W - 1)\varepsilon_W + 1\right]}\mu_t^W + \varepsilon_t^W \end{aligned} \tag{2.16}$$

where the current wage w_t depends on the lagged wage and inflation w_{t-1} and π_{t-1}, on the expected future wage and inflation $\mathbb{E}_t[w_{t+1}]$ and $\mathbb{E}_t[\pi_{t+1}]$, on the wage mark-up μ_t^W and the disturbance term ε_t^W representing wage mark-up shocks. Similar to the inflation process in 2.13 ξ_W can be interpreted as a measure of wage stickiness and ι_W determines the degree of backward-looking in the wage process in 2.16. $(\lambda_W - 1)$ is hte steady-state wage mark-up and ε_W defines the curvature of the Kimball labor market aggregator.

Monetary policy decisions by the ECB based on a Taylor-rule like reaction function:

$$\begin{aligned} r_t = {} & \rho r_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_y(y_t - \tilde{y}_t)\right] + r_{\Delta y}\left[(y_t - \tilde{y}_t) - (y_{t-1} - \tilde{y}_{t-1})\right] \\ & + \omega_l f_{l,t} + \omega_s f_{s,t} + \omega_c f_{c,t} + \varepsilon_t^r \end{aligned} \tag{2.17}$$

where the ECB adjusts the short term rate r_t in response to current inflation π_t and the current and lagged output gap, measured as the difference between actual y_t and potential output $\tilde{y}_t$. Potential output $\tilde{y}_t$ is determined by equations 2.1 - 2.17 under full price and wage flexibility, with $\xi_P = \xi_W = 0$ and in the absence of the price and wage mark-up disturbances ε_t^P and ε_t^W. The resulting equation system under full price and wage flexibility is outlined in Appendix A.5.1.

The reaction function in 2.17 is extended by three latent term structure factors $\mathbf{f}_t^T = [f_{l,t}, f_{s,t}, f_{c,t}]$, which determine level, slope and curvature of the interest rates. The coefficients ω_l, ω_s and ω_c in 2.17 determine the weighting of the term structure factors in ECB's policy decision findings. $\mathbf{f}_t$ evolves according to the following VAR[1] process:

$$\boldsymbol{f}_t = \mathbf{A}\tilde{\boldsymbol{f}}_{t-1} + \boldsymbol{\epsilon}_t^f \tag{2.18}$$

with $\tilde{\boldsymbol{f}}_t^T = [y_t, \pi_t, \boldsymbol{f}_t]$. $\mathbf{A} = [\mathbf{A}_{l,m}, \mathbf{A}_{ll}]$ is a 3×5 matrix which contains the 3×2 matrix $\mathbf{A}_{l,m}$ and the 3×3 lower triangular matrix. $\mathbf{A}_{l,l}$. $\mathbf{A}_{l,m}$ describes the interdependencies between the economy's output and inflation on the term structure factors, whereas $\mathbf{A}_{ll}$ models the interdependencies between the three latent term sturcture factors. The process of the term structure factors in 2.18 is disturbed by $\left(\boldsymbol{\epsilon}_t^f\right)^T = \left[\epsilon_t^l, \epsilon_t^s, \epsilon_t^c\right]$, the shocks on the term structure's level, slope and curvature factor respectively. With $\boldsymbol{\varepsilon}_t^f \sim N\left(\mathbf{0}, \mathbf{I}_{3\times3}\right)$ the identically and independent distributed (i.i.d.) Gaussian term structure disturbances are uncorrelated. The VAR[1] process in 2.18 is used by the agents for the endogenous arbitrage-free bond pricing according to the macro-finance arbitrage-free affine term structure model (MF-ATSM) component discussed in Section 2.2.3.

To close our brief description of our used DSGE model, the disturbance terms representing shocks from productivity, financial risk premiums, exogenous spending, technology, monetary policy as well as of price and wage mark-ups on the economy are described by the following serially correlated stochastic processes:

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \sigma_a \epsilon_t^a \tag{2.19}$$

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \sigma_b \epsilon_t^b \tag{2.20}$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \epsilon_t^g \tag{2.21}$$

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \sigma_i \epsilon_t^i \tag{2.22}$$

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \sigma_r \epsilon_t^r \tag{2.23}$$

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \sigma_p \epsilon_t^p \tag{2.24}$$

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \sigma_w \epsilon_t^w \tag{2.25}$$

where the i.i.d. random variables $\epsilon_t^a, \epsilon_t^b, \epsilon_t^g, \epsilon_t^i, \epsilon_t^r, \epsilon_t^p, \epsilon_t^w$ in 2.19 - 2.25 are standard normal distributed.

2.2.2 Rational expectations building

2.2.2.1 Canonical rational expectations form

As described in Herbst and Schorfheide [2016] or Dejong and Dave [2011] to determine the agent's expectations in a first step the log-linearized equations for the 14 endogenous macroeconomic variables of the SW-2007 DSGE model under sticky and flexible price and wage setting combined with the term structure factors $\boldsymbol{f}_t$ are transferred into the canonical linear rational expectations form:

$$\boldsymbol{\Gamma}_0 \boldsymbol{s}_t = \boldsymbol{\Gamma}_1 \boldsymbol{s}_{t-1} + \boldsymbol{\Psi}\boldsymbol{\epsilon}_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t \tag{2.26}$$

where

$$\begin{aligned}\boldsymbol{s}_t^T = [&y_t, c_t, i_t, q_t, k_t^s, z_t, k_t, \mu_t^P, \pi_t, r_t^k, \mu_t^w, w_t, r_t, l_t, \varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^g, \varepsilon_t^i, \varepsilon_t^r, \varepsilon_t^p, \varepsilon_t^w, \mathbb{E}_t\left[c_{t+1}\right], \mathbb{E}_t\left[i_{t+1}\right],\\ &\mathbb{E}_t\left[l_{t+1}\right], \mathbb{E}_t\left[\pi_{t+1}\right], \mathbb{E}_t\left[q_{t+1}\right], \mathbb{E}_t\left[r_{t+1}^k\right], \mathbb{E}_t\left[w_{t+1}\right], \tilde{y}_t, \tilde{c}_t, \tilde{i}_t, \tilde{q}_t, \tilde{k}_t^s, \tilde{z}_t, \tilde{k}_t, \tilde{r}_t^k, \tilde{w}_t, \tilde{r}_t, \tilde{l}_t,\\ &\mathbb{E}_t\left[\tilde{c}_{t+1}\right], \mathbb{E}_t\left[\tilde{i}_{t+1}\right], \mathbb{E}_t\left[\tilde{l}_{t+1}\right], \mathbb{E}_t\left[\tilde{q}_{t+1}\right], \mathbb{E}_t\left[\tilde{r}_{t+1}^k\right], y_{t-1}, c_{t-1}, i_{t-1}, w_{t-1}, \tilde{y}_{t-1}, f_{l,t}, f_{s,t}, f_{c,t},\\ &\tilde{f}_{l,t}, \tilde{f}_{s,t}, \tilde{f}_{c,t}]\end{aligned}$$

defines the 55×1 state vector.

$$\boldsymbol{\epsilon}_t^T = \left[\epsilon_t^a, \epsilon_t^b, \epsilon_t^g, \epsilon_t^i, \epsilon_t^r, \epsilon_t^p, \epsilon_t^w, \epsilon_t^l, \epsilon_t^s, \epsilon_t^c\right]$$

is the 10×1 vector of stochastic innovations and

$$\begin{aligned}\boldsymbol{\eta}_t^T = [&\pi_t - \mathbb{E}_{t-1}\left[\pi_t\right], c_t - \mathbb{E}_{t-1}\left[c_t\right], l_t - \mathbb{E}_{t-1}\left[l_t\right], q_t - \mathbb{E}_{t-1}\left[q_t\right], r_t^k - \mathbb{E}_{t-1}\left[r_t^k\right], i_t - \mathbb{E}_{t-1}\left[i_t\right],\\ &w_t - \mathbb{E}_{t-1}\left[w_t\right], \tilde{c}_t - \mathbb{E}_{t-1}\left[\tilde{c}_t\right], \tilde{l}_t - \mathbb{E}_{t-1}\left[\tilde{l}_t\right], \tilde{q}_t - \mathbb{E}_{t-1}\left[\tilde{q}_t\right], \tilde{r}_t^k - \mathbb{E}_{t-1}\left[\tilde{r}_t^k\right], \tilde{i}_t - \mathbb{E}_{t-1}\left[\tilde{i}_t\right]]\end{aligned}$$

is the 12×1 vector of expectation errors. $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_1$ are 55×55 matrices. $\boldsymbol{\Psi}$ and $\boldsymbol{\Pi}$ are 55×10 and 55×12 matrices respectively, relating the vectors of innovations and expectation errors $\boldsymbol{\epsilon}_t$ and $\boldsymbol{\eta}_t$ to the dynamics of the state variables $\boldsymbol{s}_t$. In Appendix A.5.2 we outline in detail the row-wise specification of the matrices $\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}$ and $\boldsymbol{\Pi}$.

2.2.2.2 Sim's solution algorithm

In this work we apply the solution algorithm proposed by Sims [2002] to the canonical linear rational expectations model expressed in 2.26. For the implementation of the algorithm the following steps are necessary:

In the first step the matrices $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_1$ are decomposed by applying the QZ-factorization:

$$\boldsymbol{\Gamma}_0 = \mathbf{Q}^T \boldsymbol{\Lambda} \mathbf{Z}^T \tag{2.27}$$

$$\boldsymbol{\Gamma}_1 = \mathbf{Q}^T \boldsymbol{\Omega} \mathbf{Z}^T \tag{2.28}$$

where $(\mathbf{Q}, \mathbf{Z})$ are unitary matrices with $\mathbf{Q}^T\mathbf{Q} = \mathbf{Z}^T\mathbf{Z} = \mathbf{I}$ and $(\mathbf{\Lambda}, \mathbf{\Omega})$ are upper triangular matrices.

The next step organizes the factorized matrices $\mathbf{Q}, \mathbf{Z}, \mathbf{\Lambda}, \mathbf{\Omega}$ according to an increasing ordering of the set of (absolute) generalized eigenvalues $|\omega_{i,i}/\lambda_{i,i}|$ of $(\mathbf{\Gamma}_0, \mathbf{\Gamma}_1)$ defined as the ratios of the diagonal elements of $(\mathbf{\Lambda}\mathbf{\Omega})$. Ordering of the generalized eigenvalues is from left to right, such that the largest absolute eigenvalue appears at the lower right. Defining:

$$\boldsymbol{z}_t = \mathbf{Z}^T \boldsymbol{s}_t \tag{2.29}$$

and premultiplying 2.29 by $\mathbf{Q}$ the canonical linear rational expectations model in 2.26 can be written as:

$$\boldsymbol{z}_t = \mathbf{\Omega}\boldsymbol{z}_{t-1} + \mathbf{Q}\boldsymbol{\mu} + \mathbf{Q}\mathbf{\Psi}\boldsymbol{\epsilon}_t + \mathbf{Q}\mathbf{\Pi}\boldsymbol{\eta}_t \tag{2.30}$$

As in Blanchard and Kahn [1980] the system is separated into an explosive and a nonexplosive block where the explosive block is defined by the (absolute) generalized eigenvalues larger than one and is located in the lower equations of the system:

$$\begin{bmatrix} \mathbf{\Lambda}_{1,1} & \mathbf{\Lambda}_{1,2} \\ \mathbf{0} & \mathbf{\Lambda}_{2,2} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_{1,t} \\ \boldsymbol{z}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathbf{\Omega}_{1,1} & \mathbf{\Omega}_{1,2} \\ \mathbf{0} & \mathbf{\Omega}_{2,2} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_{1,t-1} \\ \boldsymbol{z}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} [\boldsymbol{\mu} + \mathbf{\Psi}\boldsymbol{\epsilon}_t + \mathbf{\Pi}\boldsymbol{\eta}_t] \tag{2.31}$$

where $z_{1,t}$ is the $n_s \times 1$ vector of stable and $z_{2,t}$ the $n_e \times 1$ vector of explosive state variables.

In the next step the explosive block in 2.31 is solved. Writing:

$$\mathbf{\Lambda}_{2,2}\boldsymbol{z}_{2,t} = \mathbf{\Omega}_{2,2}\boldsymbol{z}_{2,t-1} + \boldsymbol{w}_{2,t} \tag{2.32}$$

with $w_{2,t} = \mathbf{Q}_2 [\boldsymbol{\mu} + \mathbf{\Psi}\boldsymbol{\epsilon}_t + \mathbf{\Pi}\boldsymbol{\eta}_t]$. Iterating from t to $t+1$ we get from 2.32:

$$\boldsymbol{z}_{2,t} = \mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\boldsymbol{z}_{2,t+1} + \mathbf{\Omega}_{2,2}^{-1}\boldsymbol{w}_{2,t+1} \tag{2.33}$$

Further iteration yields to:

$$\begin{aligned} \boldsymbol{z}_{2,t} &= \left(\mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\right)^2 \boldsymbol{z}_{2,t+2} - \mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\Omega_{2,2}^{-1}\boldsymbol{w}_{2,t+2} - \mathbf{\Omega}_{2,2}^{-1}\boldsymbol{w}_{2,t+1} \\ &\vdots \qquad\qquad (2.34) \\ \boldsymbol{z}_{2,t} &= \left(\mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\right)^n \boldsymbol{z}_{2,t+n} - \sum_{i=1}^{n} \left(\mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\right)^{i-1} \Omega_{2,2}^{-1}\boldsymbol{w}_{2,t+i} \qquad (2.35) \end{aligned}$$

Due to the explosive behaviour expressed by $|\omega_{j,j}/\lambda_{j,j}| > 1$ with $j = 1, 2, ..., n_c$, such that $\lim_{n\to\infty} \left(\mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\right)^n \boldsymbol{z}_{2,t+n} = 0$ for $n \to \infty$ expression 2.33 becomes:

$$\begin{aligned} \boldsymbol{z}_{2,t} &= -\sum_{i=1}^{n} \left(\mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\right)^{i-1} \mathbf{\Omega}_{2,2}^{-1}\boldsymbol{w}_{2,t+i} \\ &= -\sum_{i=1}^{n} \left(\mathbf{\Omega}_{2,2}^{-1}\mathbf{\Lambda}_{2,2}\right)^{i-1} \mathbf{\Omega}_{2,2}^{-1}\mathbf{Q}_2 \left[\boldsymbol{\mu} + \mathbf{\Psi}\boldsymbol{\epsilon}_{t+i} + \mathbf{\Pi}\boldsymbol{\eta}_{t+i}\right] \end{aligned} \tag{2.36}$$

where the structural and expectation errors $\boldsymbol{\epsilon}_{t+i}$ and $\boldsymbol{\eta}_{t+i}$ in 2.36 are future values with $\mathbb{E}_t\left[\boldsymbol{\epsilon}_{t+i}\right] = \mathbb{E}_t\left[\boldsymbol{\eta}_{t+i}\right] = \mathbf{0}$ for $i > 0$. The explosive state variables $\boldsymbol{z}_{2,t}$ in 2.36 can be expressed as:

$$\boldsymbol{z}_{2,t} = -\sum_{i=1}^{n}\left(\boldsymbol{\Omega}_{2,2}^{-1}\boldsymbol{\Lambda}_{2,2}\right)^{i-1}\boldsymbol{\Omega}_{2,2}^{-1}\mathbf{Q}_2\boldsymbol{\mu} \tag{2.37}$$

With $\sum_{i=1}^{n}\left(\boldsymbol{\Omega}_{2,2}^{-1}\boldsymbol{\Lambda}_{2,2}\right)^{i-1} = -\left(\mathbf{I} - \boldsymbol{\Omega}_{2,2}^{-1}\boldsymbol{\Lambda}_{2,2}\right)^{-1} = \left(\boldsymbol{\Lambda}_{2,2}^{-1}\boldsymbol{\Omega}_{2,2} - \mathbf{I}\right)$ in 2.37 yields to the solution:

$$\boldsymbol{z}_{2,t} = \left(\boldsymbol{\Lambda}_{2,2} - \boldsymbol{\Omega}_{2,2}\right)^{-1}\mathbf{Q}_2\boldsymbol{\mu} \tag{2.38}$$

for the explosive variables $\boldsymbol{z}_{2,t}$.

Conditional to the solution $\boldsymbol{z}_{2,t}$ in 2.38 the last step is the solution for the stable variables $\boldsymbol{z}_{1,t}$ in 2.31, which eliminates the influence of the expectational errors. To solve for $\boldsymbol{z}_{1,t}$ the relation:

$$\mathbf{Q}_1\boldsymbol{\Pi} = \boldsymbol{\Phi}\mathbf{Q}_2\boldsymbol{\Pi} \iff [\mathbf{I}, -\boldsymbol{\Phi}]\begin{bmatrix}\mathbf{Q}_1\\ \mathbf{Q}_2\end{bmatrix}\boldsymbol{\Pi} = \mathbf{0} \tag{2.39}$$

is exploited, where the existence of some matrix $\boldsymbol{\Phi}$ is the necessary and sufficient condition for the unique existence of a solution. Premultiplying 2.31 by $[\mathbf{I}, -\boldsymbol{\Phi}]$ yields the stable upper block for $\boldsymbol{z}_{1,t}$ to:

$$\boldsymbol{\Lambda}_{1,1}\boldsymbol{z}_{1,t} = -\left(\boldsymbol{\Lambda}_{1,2} - \boldsymbol{\Phi}\boldsymbol{\Lambda}_{2,2}\right)\boldsymbol{z}_{2,t} + \boldsymbol{\Omega}_{1,1}\boldsymbol{z}_{1,t-1} + \left(\boldsymbol{\Omega}_{1,1} - \boldsymbol{\Phi}\boldsymbol{\Omega}_{2,2}\right)\boldsymbol{z}_{2,t-1} + \left(\mathbf{Q}_1 - \boldsymbol{\Phi}\mathbf{Q}_2\right)\left(\boldsymbol{\mu} + \boldsymbol{\Psi}\right) \tag{2.40}$$

Premultiplying 2.40 by $\boldsymbol{\Lambda}_{1,1}^{-1}$:

$$\begin{aligned}\boldsymbol{z}_{1,t} = &-\boldsymbol{\Lambda}_{1,1}^{-1}\left(\boldsymbol{\Lambda}_{1,2} - \boldsymbol{\Phi}\boldsymbol{\Lambda}_{2,2}\right)\boldsymbol{z}_{2,t} + \boldsymbol{\Lambda}_{1,1}^{-1}\boldsymbol{\Omega}_{1,1}\boldsymbol{z}_{1,t-1}\\ &+ \boldsymbol{\Lambda}_{1,1}^{-1}\left(\boldsymbol{\Omega}_{1,1} - \boldsymbol{\Phi}\boldsymbol{\Omega}_{2,2}\right)\boldsymbol{z}_{2,t-1} + \boldsymbol{\Lambda}_{1,1}^{-1}\left(\mathbf{Q}_1 - \boldsymbol{\Phi}\mathbf{Q}_2\right)\left(\boldsymbol{\mu} + \boldsymbol{\Psi}\right)\end{aligned} \tag{2.41}$$

With the solution $\boldsymbol{z}_{2,t} = \left(\boldsymbol{\Lambda}_{2,2} - \boldsymbol{\Omega}_{2,2}\right)^{-1}\mathbf{Q}_2\boldsymbol{\mu}$ from 2.38 we get:

$$\begin{aligned}\boldsymbol{z}_{1,t} &= -\boldsymbol{\Lambda}_{1,1}^{-1}\left(\boldsymbol{\Lambda}_{1,2} - \boldsymbol{\Phi}\boldsymbol{\Lambda}_{2,2}\right)\left(\boldsymbol{\Lambda}_{2,2} - \boldsymbol{\Omega}_{2,2}\right)^{-1}\mathbf{Q}_2\boldsymbol{\mu} + \boldsymbol{\Lambda}_{1,1}^{-1}\boldsymbol{\Omega}_{1,1}\boldsymbol{z}_{1,t-1}\\ &\quad + \boldsymbol{\Lambda}_{1,1}^{-1}\left(\boldsymbol{\Omega}_{1,1} - \boldsymbol{\Phi}\boldsymbol{\Omega}_{2,2}\right)\boldsymbol{z}_{2,t-1} + \boldsymbol{\Lambda}_{1,1}^{-1}\left(\mathbf{Q}_1 - \boldsymbol{\Phi}\mathbf{Q}_2\right)\boldsymbol{\mu} + \boldsymbol{\Lambda}_{1,1}^{-1}\left(\mathbf{Q}_1 - \boldsymbol{\Phi}\mathbf{Q}_2\right)\boldsymbol{\Psi}\\ &= \left[\boldsymbol{\Lambda}_{1,1}^{-1}, \boldsymbol{\Lambda}_{1,1}^{-1}\left(\boldsymbol{\Lambda}_{1,2} - \boldsymbol{\Phi}\boldsymbol{\Lambda}_{2,2}\right)\right]\begin{bmatrix}\left(\mathbf{Q}_1 - \boldsymbol{\Phi}\mathbf{Q}_2\right)\\ \left(\boldsymbol{\Omega}_{2,2} - \boldsymbol{\Lambda}_{2,2}\right)^{-1}\mathbf{Q}_2\end{bmatrix}\boldsymbol{\mu}\\ &\quad + \boldsymbol{\Lambda}_{1,1}^{-1}\left[\boldsymbol{\Omega}_{1,1}, \left(\boldsymbol{\Omega}_{1,2} - \boldsymbol{\Phi}\boldsymbol{\Omega}_{2,2}\right)\right]\begin{bmatrix}\boldsymbol{z}_{1,t-1}\\ \boldsymbol{z}_{2,t-1}\end{bmatrix} + \boldsymbol{\Lambda}_{1,1}^{-1}\left(\mathbf{Q}_1 - \boldsymbol{\Phi}\mathbf{Q}_2\right)\boldsymbol{\Psi}\end{aligned} \tag{2.42}$$

Combining both solutions $[\boldsymbol{z}_{1,t}, \boldsymbol{z}_{2,t}]$ from 2.42 and 2.38 and premultiplying by $\mathbf{Z}$ we get the conventional state space form:

$$\mathrm{s}_t = \boldsymbol{\theta}_c + \boldsymbol{\Theta}_0\boldsymbol{s}_{t-1} + \boldsymbol{\Theta}_1\boldsymbol{\epsilon}_t \tag{2.43}$$

with:

$$\mathbf{H} = \mathbf{Z}\begin{bmatrix} \mathbf{\Lambda}_{1,1}^{-1}, & -\mathbf{\Lambda}_{1,1}^{-1}\left(\mathbf{\Lambda}_{1,2} - \mathbf{\Phi}\mathbf{\Lambda}_{2,2}\right) \\ \mathbf{0}, & \mathbf{I} \end{bmatrix} \qquad \boldsymbol{\theta}_c = \begin{bmatrix} \left(\mathbf{Q}_1 - \mathbf{\Phi}\mathbf{Q}_2\right) \\ \left(\mathbf{\Omega}_{2,2} - \mathbf{\Lambda}_{2,2}\right)^{-1}\mathbf{Q}_2 \end{bmatrix}\boldsymbol{\mu}$$

$$\mathbf{\Theta}_0 = \mathbf{Z}_{*,[1:n_c]}\mathbf{\Lambda}_{1,1}^{-1}\left[\mathbf{\Omega}_{1,1}, \left(\mathbf{\Omega}_{1,2} - \mathbf{\Phi}\mathbf{\Omega}_{2,2}\right)\right]\mathbf{Z} \qquad \mathbf{\Theta}_1 = \begin{bmatrix} \left(\mathbf{Q}_1 - \mathbf{\Phi}\mathbf{Q}_2\right) \\ \mathbf{0} \end{bmatrix}\mathbf{\Psi} \tag{2.44}$$

where the matrix $\mathbf{Z}_{*,[1:n_c]}$ contains the n_s first columns of the matrix $\mathbf{Z}$.

2.2.3 Term structure of interest rates under rational expectations

To find the price $P(t,T)$ of a government bond at time t which matures in $t < T$ the risk neutral and the risk averse investors use the DSGE rational expectation solution in 2.43 and 2.44 for regarding possible future states of the economy in its bond pricing scheme. The pricing scheme proposed by Ang and Piazzesi [2003] and discussed in detail in Hamilton and Wu [2012, 2014] is an arbitrage-free pricing approach, based on an affine term structure of interest rates model (ATSM) for which it is possible to combine macroeconomic and term structure related phenomena. Using this kind of term structure modeling, we can endogenize the arbitrage-free pricing scheme into our macroeconomic DSGE framework. Our model becomes now the combined SW-DSGE-ATSM. The pricing scheme used by the risk neutral investor is:

$$\begin{aligned} P^{RN}\left(t, T, \boldsymbol{s}_t\right) &= exp\left(-r_t\right)\mathbb{E}_t^{RN}\left[P\left(t+1, T-1, \boldsymbol{s}_{t+1}\right)\right] \\ &= exp\left(-r_t\right)\int f\left(\boldsymbol{s}_{t+1}|\boldsymbol{s}_t, \mathbf{\Theta}\right)P\left(t+1, T-1, \boldsymbol{s}_{t+1}\right)\boldsymbol{ds}_{t+1} \end{aligned} \tag{2.45}$$

with $\mathbf{\Theta} = [\theta_c, \mathbf{\Theta}_0, \mathbf{\Theta}_1]$ from 2.43. $f\left(\boldsymbol{s}_{t+1}|\boldsymbol{s}_t, \mathbf{\Theta}\right)$ in 2.45 denoutes the Gaussian PDF for observing the economy in the state $\boldsymbol{s}_{t+1}$ at time $t+1$:

$$f\left(\boldsymbol{s}_{t+1}|\boldsymbol{s}_t, \mathbf{\Theta}\right) = \frac{1}{\sqrt{(2\pi)^M\left|\mathbf{\Theta}_1\mathbf{\Theta}_1^T\right|}} exp\left(-\frac{1}{2}\left(\boldsymbol{s}_{t+1} - \boldsymbol{\theta}_c - \mathbf{\Theta}_0\boldsymbol{s}_t\right)^T\left(\mathbf{\Theta}_1\mathbf{\Theta}_1^T\right)^{-1}\left(\boldsymbol{s}_{t+1} - \boldsymbol{\theta}_c - \mathbf{\Theta}_0\boldsymbol{s}_t\right)\right) \tag{2.46}$$

where $|\mathbf{\Theta}_1\mathbf{\Theta}_1^T|$ denotes the determinant of the covariance $\mathbf{\Theta}_1\mathbf{\Theta}_1^T$. The pricing scheme in 2.45 implies that for the risk neutral investor the current bond price is the discounted expected one step ahead price $P\left(t+1, T-1, \boldsymbol{s}_{t+1}\right)$. The risk neutral investor calculates the expectation value only by the use of the PDF expressed in 2.46. Distinguished from the risk neutral investor the risk averse investor uses in addition to the PDF in 2.46 the stochastic bond pricing kernel M_t for calculating expectations about the bond price in $t+1$. So the risk averse investor's pricing scheme is:

$$\begin{aligned} P^{RA}\left(t, T, \boldsymbol{s}_t\right) &= \mathbb{E}_t^{RA}\left[P\left(t+1, T-1, \boldsymbol{s}_{t+1}\right)\right] \\ &= \int M_{t+1} f\left(\boldsymbol{s}_{t+1}|\boldsymbol{s}_t, \mathbf{\Theta}\right)P\left(t+1, T-1, \boldsymbol{s}_{t+1}\right)\boldsymbol{ds}_{t+1} \end{aligned} \tag{2.47}$$

where the bond pricing kernel which expresses the investors risk aversion is defined here analogue to Duffie and Kan [1996] as:

$$M_{t+1} = exp\left(-r_t - \frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t^T\boldsymbol{\epsilon}_{t+1}\right) \tag{2.48}$$

According to Duffee [2002] and Dai and Singleton [2002] the 10×1 vector $\boldsymbol{\lambda}_t$ contains the time varying market prices of risk related to the sources of uncertainty $\boldsymbol{\epsilon}_t^T = \left[\epsilon_t^a, \epsilon_t^b, \epsilon_t^g, \epsilon_t^i, \epsilon_t^r, \epsilon_t^p, \epsilon_t^w, \epsilon_t^l, \epsilon_t^s, \epsilon_t^c\right]$ of the state space dynamics in 2.43, which directly influence the term structure and are described by the affine equation:

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1\boldsymbol{s}_t \tag{2.49}$$

with the 10×1 constant vector $\boldsymbol{\lambda}_0$ and the 10×55 block matrix $\boldsymbol{\lambda}_1$ which expresses the influence of the state variables $\boldsymbol{s}_t$ on the market prices of risk $\boldsymbol{\lambda}_t$ at time t. $\boldsymbol{\lambda}_1$ is specified as:

$$\boldsymbol{\lambda}_1 = \begin{bmatrix} \mathbf{0}_{7\times14} & \boldsymbol{\lambda}_{m,m} & \mathbf{0}_{7\times28} & \mathbf{0}_{7\times3} & \mathbf{0}_{7\times3} \\ \mathbf{0}_{3\times14} & \mathbf{0}_{3\times7} & \mathbf{0}_{3\times28} & \boldsymbol{\lambda}_{l,l} & \mathbf{0}_{3\times3} \end{bmatrix} \tag{2.50}$$

where $\boldsymbol{\lambda}_{m,m}$ and $\boldsymbol{\lambda}_{l,l}$ are 7×7 and 3×3 matrices respectively. From the risk averse investor's pricing scheme in 2.47 the rational expectation VAR is modified in its risk averse form:

$$\boldsymbol{s}_{t+1} = \boldsymbol{\theta}_c^{RA} + \boldsymbol{\Theta}_0^{RA}\boldsymbol{s}_t + \boldsymbol{\Theta}_1\boldsymbol{\epsilon}_t \tag{2.51}$$

with $\boldsymbol{\theta}_c^{RA} = \boldsymbol{\theta}_c - \boldsymbol{\Theta}_1\boldsymbol{\lambda}_0$, $\boldsymbol{\Theta}_0^{RA} = \boldsymbol{\Theta}_0 - \boldsymbol{\Theta}_1\boldsymbol{\lambda}_1$ and $\boldsymbol{\epsilon}_t \sim N\left(\mathbf{0}, \mathbf{I}_{10\times10}\right)$.

As derived in Appendix A.5.3 the bond price $P(t, T, \boldsymbol{s}_t)$ is defined by the exponential affine function:

$$P(t, T, \boldsymbol{s}_t) = exp\left(A_\tau + \mathbf{B}_\tau^T\boldsymbol{s}_t\right) \tag{2.52}$$

where the constant A_τ and the $N \times 1$ coefficient vector $\mathbf{B}_\tau$ depends on the bond's time to maturity $\tau = T - t$. For the risk averse investor A_τ and $\mathbf{B}_\tau$ are defined by the following system of difference equations:

$$A_\tau^{RA} = A_{\tau-1}^{RA} + \left(\mathbf{B}_{\tau-1}^{RA}\right)^T\left(\boldsymbol{\theta}_c - \boldsymbol{\Theta}_1\boldsymbol{\lambda}_0\right) + \frac{1}{2}\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\boldsymbol{\Theta}_1\mathbf{B}_{\tau-1}^{RA} \tag{2.53}$$

$$\left(\mathbf{B}_\tau^{RA}\right)^T = \left(\mathbf{B}_{\tau-1}^{RA}\right)^T\left(\boldsymbol{\Theta}_0 - \boldsymbol{\Theta}_1\boldsymbol{\lambda}_1\right) - \boldsymbol{\delta}^T \tag{2.54}$$

whereas the risk neutral investors who do not adjust their expectations about the future bond prices use the following bond pricing system:

$$A_\tau^{RN} = A_{\tau-1}^{RN} + \left(\mathbf{B}_{\tau-1}^{RN}\right)^T\boldsymbol{\theta}_c + \frac{1}{2}\left(\mathbf{B}_{\tau-1}^{RN}\right)^T\boldsymbol{\Theta}_1\mathbf{B}_{\tau-1}^{RN} \tag{2.55}$$

$$\left(\mathbf{B}_\tau^{RN}\right)^T = \left(\mathbf{B}_{\tau-1}^{RN}\right)^T\boldsymbol{\Theta}_0 - \boldsymbol{\delta}^T \tag{2.56}$$

The recursive systems in 2.53 and 2.54 and 2.55 and 2.56 are initiated with $A_1^{RA} = A_1^{RN} = 0$ and $\mathbf{B}_1^{RA} = \mathbf{B}_1^{RN} = -\boldsymbol{\delta}_r$ respectively, where $\boldsymbol{\delta}_r^T = \left[\mathbf{0}_{(r-1)\times 1}, 1, \mathbf{0}_{(N-r)\times 1}\right]$ is an $N \times 1$ vector, indicating the position r of the short rate r_t in the N-dimensional state vector $\boldsymbol{s}_t$. With:

$$y(t,T) = -ln\left(P(t,T)\right)/\tau \tag{2.57}$$

the maturity dependent zero-coupon rates of the risk averse and risk neutral investors are:

$$y^{RA}(t,T) = a_\tau^{RA} + \left(\boldsymbol{b}_\tau^{RA}\right)^T \boldsymbol{s}_t \tag{2.58}$$

$$y^{RN}(t,T) = a_\tau^{RN} + \left(\boldsymbol{b}_\tau^{RN}\right)^T \boldsymbol{s}_t \tag{2.59}$$

where $a_\tau^{RA} = -A_\tau^{RA}/\tau$ and $\boldsymbol{b}_\tau^{RA} = -\mathbf{B}_\tau^{RA}/\tau$ and $a_\tau^{RN} = -A_\tau^{RN}/\tau$ and $\boldsymbol{b}_\tau^{RN} = -\mathbf{B}_\tau^{RN}/\tau$ respectively.

2.3 Model estimation

2.3.1 State space form of the SW-DSGE-ATSM

For estimating our SW-DSGE-ATSM as a combination of the SW-2007 DSGE and the MF-ATSM we formulate the model in state space form. The measurement equation of the system is specified as follows:

$$\boldsymbol{y}_t = \boldsymbol{c} + \mathbf{M}\boldsymbol{s}_t + \boldsymbol{\vartheta}_t \tag{2.60}$$

where the 17×1 vector:

$$\begin{aligned}\boldsymbol{y}_t^T = [&ln(\Delta GDP_t), ln(\Delta CONS_t), ln(\Delta INV_t), ln(\Delta WAGE_t), ln(LABOR_t), ln(INF_t), ECB_t,\\ &y(t,12), y(t,24), ..., y(t,120)]\end{aligned}$$

contains the measurements GDP, consumption, investment, wage, labor (measured in working hours), GDP deflator based inflation, the monetary policy rate set by the ECB (approximated by the EONIA swap rate) and the zero-coupon rates with time to maturities $\tau = 12, 24, ..., 120$ month. We extract the zero-coupon rates from government bonds for Germany, France and Italy by applying the parametric Nelson-Siegel-Svensson (NSS) approach proposed by Nelson and Siegel [1987] and Svensson [1995]. Bond data and the NSS approach are listed and outlined in Appendix A.2 and A.3. Details of the used macroeconomic data are outlined in Appendix A.1. The constant vector $\boldsymbol{c}$ is specified as:

$$\boldsymbol{c}^T = \left[\bar{y}, \bar{y}, \bar{y}, \bar{y}, \bar{l}, \bar{\pi}, \bar{r}, a_{12}^{RA}, a_{24}^{RA}, ..., a_{120}^{RA}\right]$$

$\bar{y}, \bar{l}, \bar{\pi}$ and $\bar{r}$ are the quarterly trend growth rate, the quarterly working hours, the quarterly inflation and nominal short term rate in the steady-state of the Smets-Wouters economy. M is the 17×55 coefficient matrix of the SW-DSGE-ATSM's measurement equation which is specified in Appendix A.5.4. $\boldsymbol{\vartheta}_t \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}\right)$ is the Gaussian measurement error with diagonal covariance $\boldsymbol{\Sigma}$. The system's transition equation for the dynamics of the state variables is given by 2.43.

2.3.2 Mixed MH-MCMC procedure

The SW-DSGE-ATSM developed in this chapter has 126 parameters, collected in the vector $\boldsymbol{\theta}$. $\boldsymbol{\theta}^T\left[\boldsymbol{\theta}_{SW}, \boldsymbol{\theta}_{ATSM}\right]$ s seperated into the 33×1 vector:

$$\begin{aligned}\boldsymbol{\theta}_{SW}^T = [&\bar{\gamma}, \alpha, \lambda, \sigma_c, \beta, \varphi, \psi, \iota_p, \Phi, \xi_p, \iota_w, \xi_w, \sigma_l, \rho, r_\pi, r_y, r_{\Delta y}, \rho_g, \rho_b, \rho_i, \rho_a, \rho_p, \rho_w,\\ &\rho_r, \sigma_g, \sigma_b, \sigma_i, \sigma_a, \sigma, \sigma_w, \sigma_r, \bar{\pi}, \bar{l}]\end{aligned}$$

of the SW-2007 DSGE structural model parameters and the 93 vector:

$$\boldsymbol{\theta}_{ATSM}^T = \left[\omega_l, \omega_s, \omega_c, \boldsymbol{\lambda}_0, vec\left[\boldsymbol{\lambda}_{1,mm}\right], vec\left[\boldsymbol{\lambda}_{1,ll}\right], vec\left[\boldsymbol{\Psi}_{lm}\right], vech\left[\boldsymbol{\Psi}_{ll}\right], \sigma_{12M}, \sigma_{24M}, ..., \sigma_{120M}\right]$$

containing the ATSM parameters. For estimating the parameters of the SW-DSGE-ATSM we use a block Gibbs sampler where the sampler alternates between the SW-2007 DSGE and the ATSM parameter blocks. Sampling is done by drawing randomly selected blocks of parameters from $\boldsymbol{\theta}_{SW}$ and $\boldsymbol{\theta}_{ATSM}$ with three differently specified types of the Metropolis-Hastings (MH) algorithm. In concrete the Markov-Chain-Monte-Carlo (MCMC) procedure applied in this chapter works as follows:

In every iteration $i = 1, 2, ..., N$ of the MCMC procedure both parameter vectors $\boldsymbol{\theta}_{SW}$ and $\boldsymbol{\theta}_{ATSM}$ are partitioned into N_b blocks $\boldsymbol{\theta}_{SW}^T = [\boldsymbol{\theta}_{1,SW}, \boldsymbol{\theta}_{2,SW}, ..., \boldsymbol{\theta}_{N_b,SW}]$ and $\boldsymbol{\theta}_{ATSM}^T = [\boldsymbol{\theta}_{1,ATSM}, \boldsymbol{\theta}_{2,ATSM}, ..., \boldsymbol{\theta}_{N_b,ATSM}]$. As in Chibb and Ramamurthy [2010] the grouping of the $b = 1, 2, ..., N_b$ blocks of parameters is randomly, which means that in every iteration the parameter composition of every single block is different. After partitioning $\boldsymbol{\theta}_{SW}$ and $\boldsymbol{\theta}_{ATSM}$ the parameter blocks $b = 1, 2, ..., N_b$ are drawn with:

$$\boldsymbol{\vartheta}_b \sim q\left(*|\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\theta}_b^{(i-1)}, \boldsymbol{\theta}_{>b}^{(i-1)}\right) \tag{2.61}$$

with acceptance probability:

$$\alpha = max\left[\frac{p\left(\left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\vartheta}_b, \boldsymbol{\theta}_{>b}^{(i-1)}\right]|\mathbf{Y}\right) q\left(\boldsymbol{\theta}_b^{(i)}|\left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\vartheta}_b, \boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)}{p\left(\left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\theta}_b^{(i-1)}, \boldsymbol{\theta}_{>b}^{(i-1)}\right]|\mathbf{Y}\right) q\left(\boldsymbol{\vartheta}_b|\left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\theta}_b^{(i-1)}, \boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)}, 1\right] \tag{2.62}$$

the b-th parameter block is updated $\boldsymbol{\theta}_b^{(i)} = \boldsymbol{\vartheta}_b$, whereas with $1-\alpha$ updating is renounced such that $\boldsymbol{\theta}_b^{(i)} = \boldsymbol{\theta}_b^{(i-1)}$ To keep notation simple 2.61 and 2.62 holds for$\boldsymbol{\theta}_{SW}$ and $\boldsymbol{\theta}_{ATSM}$ respectively. p in 2.62 is the systems posterior distribution, which is recursively computed by applying the Kalman filter. The computation of p is outlined in short in Appendix A.5.5. q is the MH's proposal distribution. To increase the mixing of the sampling procedure we use two various specifications of the proposal q . The specifications follow the random block (RB-) Random-Walk MH (RW-MH) and the Newton-MH algorithm. Last one is proposed by Qi and Minka [2002]. Switching between these two algorithms is generated randomly, whereby due to the speed of executing the algorithms most of the sampling in this mixed MH-MCMC procedure is done by the random block RW-MH algorithm. Appendix A.6 outlines the two algorithms in more detail. The random block (RB-) RW-MH needs a covariance matrix as input. This covariance matrix is the inverse Hessian of the numerically maximized posterior distribution determined in a pre-estimation step. Pre-estimation is done by using a hybridized genetic Nelder-Mead algorithm. Due to its innovative character in econometrics we outline the steps of this optimization algorithm in short in Appendix A.7.1.

2.4 Empirical results

2.4.1 EMU parameter estimates

For the estimation of the DSGE component in our term structure extended SW-DSGE-ATSM we use the prior distributions proposed by Smets and Wouters [2007]. All priors we used for our estimations are listed in detail in Appendix A.7.2. In Appendix A.9 we list the estimated parameters of our SW-DSGE-ATSM, as well as the estimated parameters for the SW-DSGE and the alternative small-scale New-Keynesian DSGE proposed by Beakert, Cho and Moreno (BCM) [2010] at the mode of the respective posterior distributions. In Appendix A.9 we further show the histograms of the estimated parameters from the MCMC procedure for our SW-DSGE-ATSM. For our SW-DSGE-ATSM we use 150000 MCMC iterations, with the first 50000 being discarded as burn-in draws. Most of the parameter estimates deviate from their respective prior mean, indicating a large informative content implied by our macroeconomic modeling. Our estimates of the model's structural parameters for the three EMU countries are in line with the estimation results of Smets and Wouters [2007]. In Appendix A.8 we outline in short the alternative small-scale New-Keynesian DSGE proposed by Beakert, Cho and Moreno.

2.4.2 Macroeconomic shock transition in the EMU

2.4.2.1 Structural macroeconomic shock variables

In Figure 2.1 we show the SW-DSGE-ATSM implied innovations of the structural macroeconomic DSGE variables $\hat{\boldsymbol{\epsilon}}_t^T = \left[\hat{\epsilon}_t^a, \hat{\epsilon}_t^b, \hat{\epsilon}_t^g, \hat{\epsilon}_t^i, \hat{\epsilon}_t^r, \hat{\epsilon}_t^p, \hat{\epsilon}_t^w\right]$ for the three EMU countries Germany, France and Italy between Q2/2005 and Q1/2014. For all three countries Figure 2.1 shows that innovations of the technology related shock component followed by innovations in the productivity and risk premium related components show the largest fluctuations in their dynamics. Interestingly Germany and France show a very similar pattern in their technology related fluctuations. For both countries there is a peak in Q2/2008. Italy differs from this pattern in showing this peak in the second half of the first recession phase. All three countries have in common the larger risk premium related shock between Q3/2008 and Q1/2009. In this period the ECB lowers its short term rate for main refinancing operations by 275 basis points from 4.25% in September 2008 to 1.50% in March 2009. There is a second phase of larger fluctuations in the structural shock components.This phase nearly runs in parallel for

Germany and France between the second half of 2010 and the end of 2012. For Italy this phase of larger macroeconomic fluctuations starts with a delay in Q3/2011 and lasts until the end of our time horizon in Q1/2014.

2.4.2.2 Decomposing EMU's macroeconomic variables

For understanding the effects driving the macroeconomic variables in Figure 2.2 we plot the historical decomposition of structural shocks affecting the macroeconomic development of the EMU economies of Germany, France and Italy. In Figure 2.2 we focus on the macroeconomic state variables GDP, inflation and ECB's controlled short term rate. From the countries' GDP decompositions it becomes clear that in the phase between Q2/2008 and Q2/2009 the impact of monetary policy induced shocks and shocks related to risk premiums on GDP largely increased. For Germany and France in its beginning monetary policy shocks stimulate the countries output growth. With the financial crisis spilling over to Europe discussions and decisions surrounding ECB's monetary policy negatively effects German and French GDP. In Italy this effect is weaker than in Germany and France. For the Italian economy in the phase since Q2/2008 term structure issues become more important, reflecting the negative effects of the first yield factor on the Italian GDP between Q2/2008 and Q2/2009. There is also a strong negative term structure effect in Q1/2009 on the German real economy. Obviously in Italy there are stronger uncertainties related to monetary policy decisions between Q3/2011 and Q3/2012. In all three EMU countries the largest risk premium effects on GDP are located in Q4/2008 and Q1/2009 directly in the phase after the bankruptcy of Lehman brothers in September 2008. Our findings concerning the increased real economy impact of shocks induced by disturbances in the monetary policy decision finding process as well as of shocks related to risk premiums in the critical phase surrounding the collapse of Lehman Brothers is consistent to our findings outlined in 3, where we analyze the EMU's common macroeconomic and term structure developments in the period ranging between the upcoming of the international financial crisis in the U.S. and the initialization of the ECB induced expanded asset purchase program (EAPP) in Q4/2014 and the public sector purchase program (PSPP) in Q1/2015. Looking at the nominal short term rates reveals a similar pattern to our analysis concerning the development of the whole EMU outlined in the following chapter 3. ECB's monetary policy decisions made in the crucial phase Q2/2008 to Q2/2009 are mainly effected by shocks related to risk premiums. For Germany and France Figure 2.2 shows that the decisive decrease in the short term rate in Q4/2008 and Q1/2009 in which the ECB's main refinancing operations rate fell by 225 basis points from 3.75% to 1.50% is also affected by shocks concerning the countries term structure of interest rates. Term structure effects are also revealed in the historical decomposition of the price development. Especially in Germany and France term structure effects spill over into the firm's price setting decisions. A further interesting aspect are the negative price mark-up shocks affecting the price developments in the phase of economic prosperity until 2007.

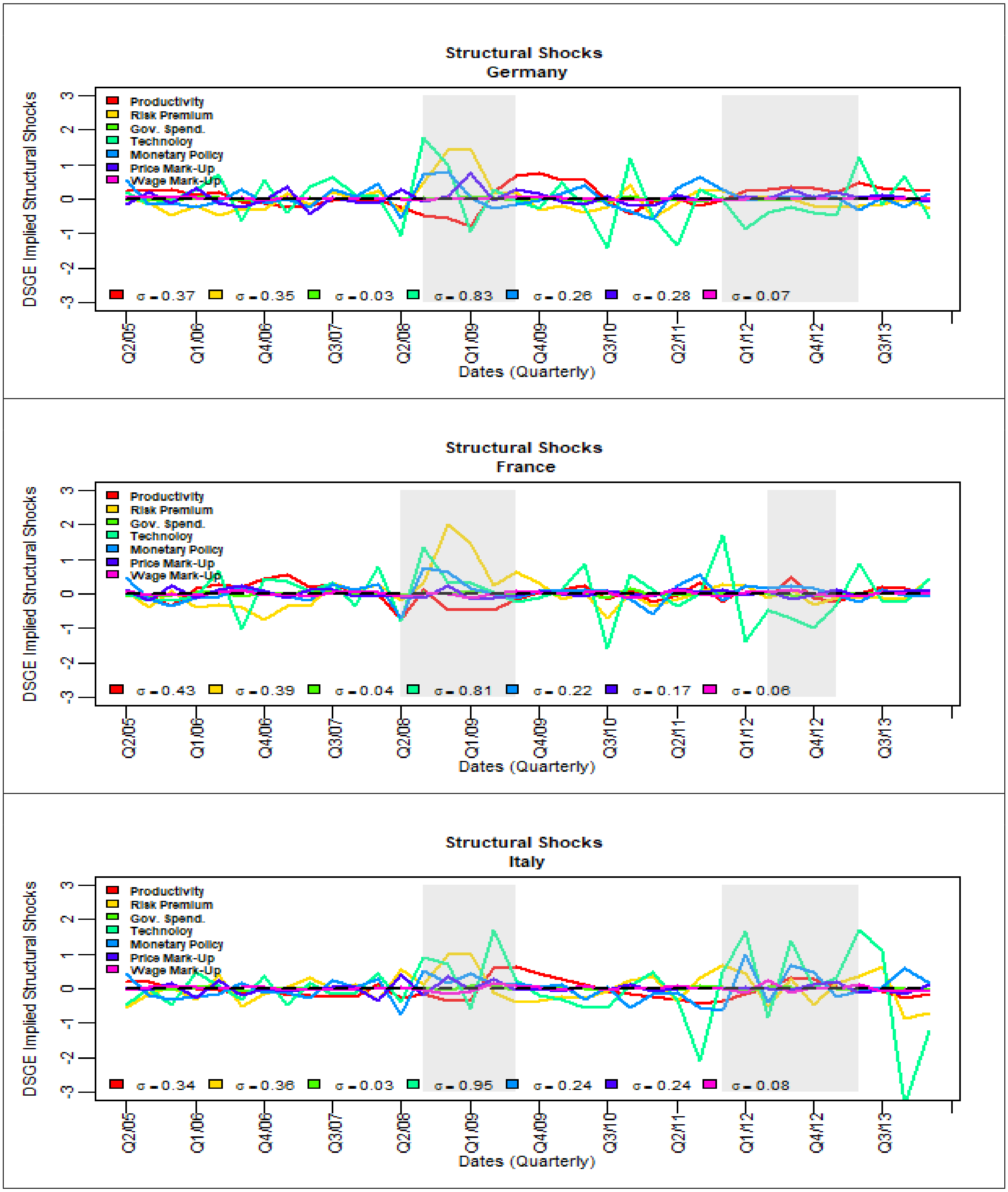


Table 2.1: SW-DSGE-ATSM implied innovation component of the structural shock variables $\hat{\boldsymbol{\epsilon}}_t^T = \left[\hat{\epsilon}_t^a, \hat{\epsilon}_t^b, \hat{\epsilon}_t^g, \hat{\epsilon}_t^i, \hat{\epsilon}_t^r, \hat{\epsilon}_t^p, \hat{\epsilon}_t^w\right]$ for Germany, France and Italy (with the constant structural shock volatilities in the legend at bottom left) at the mode of the model's posterior

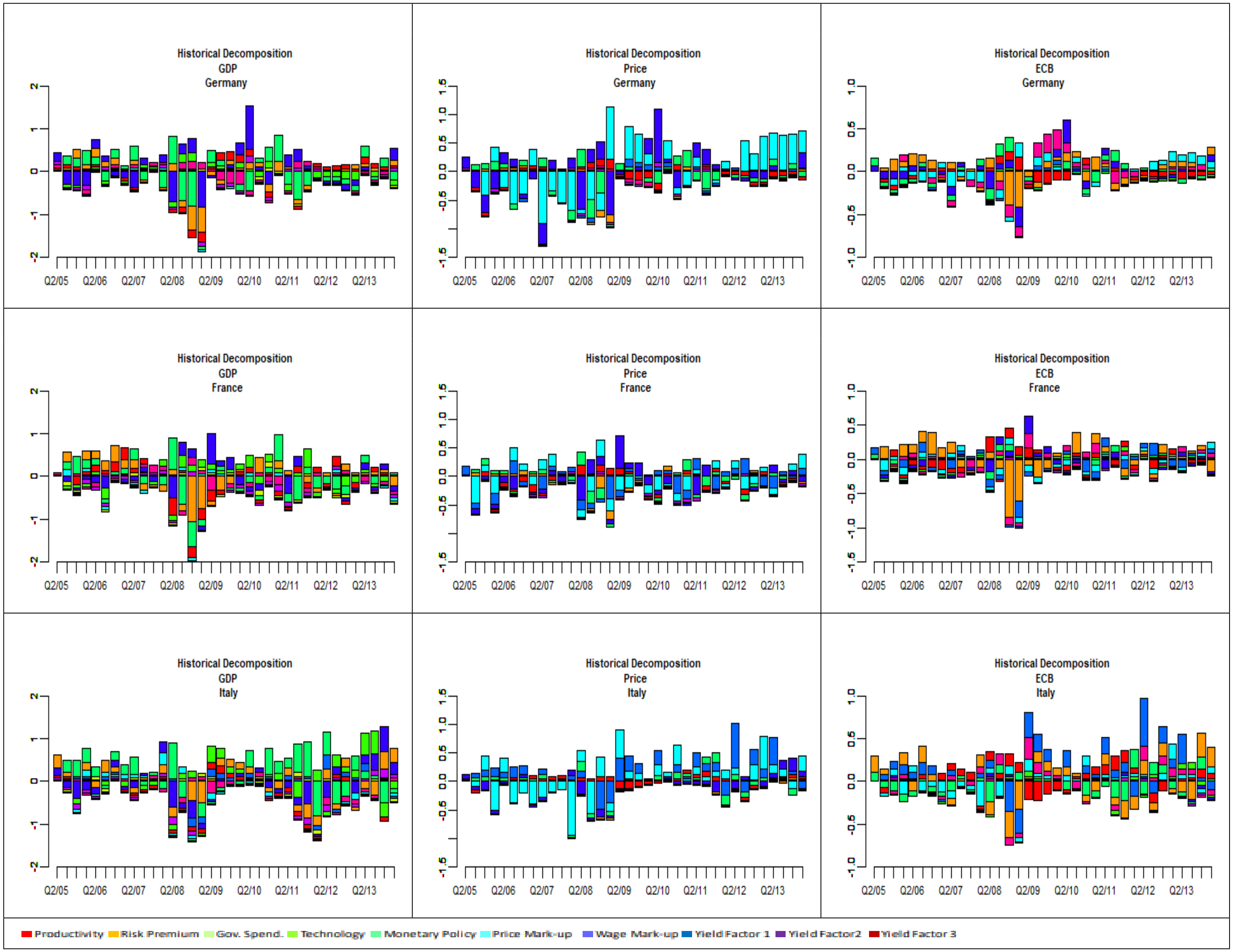


Table 2.2: Historical decomposition of the SW-DSGE-ATSM implied macroeconomic variables GDP, Inflation and ECB's monetary policy rate for Germany, France and Italy at the mode of the model's posterior

2.4.2.3 Macroeconomic shock processing

In Figure 2.3 we plot the responses of GDP and the economy's nominal short term rate on one standard deviation shocks coming from the 10 structural shocks $\boldsymbol{\varepsilon}_t^T = \left[\varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^g, \varepsilon_t^i, \varepsilon_t^r, \varepsilon_t^p, \varepsilon_t^w, \varepsilon_t^l, \varepsilon_t^s, \varepsilon_t^c,\right]$ for Germany and Italy. As outlined above $\varepsilon_t^l, \varepsilon_t^s, \varepsilon_t^c$ are the three latent term structure factors determining level, slope and curvature of the term structure of interest rates. The impulse-response (IR) functions for the French economy are shown in Figure A.19 in Appendix A.9.7. The response patterns of GDP and the short term rate are mainly in line with common wisdom regarding shock transmission through the economy's processes. The main driving force in the movement of GDP is the economy's productivity component, increasing the GDP in the short term and revealing an increasing impact in the long run. Beside the economy's productivity component the first yield factor drives the development of GDP. The first yield factor has its largest impact on GDP in the middle term, whereas the broader confidence interval around the GDP's response signals here an increased uncertainty. Depressing on GDP are shocks coming from increased price and wage mark-ups as well as shocks induced by monetary policy decisions. The first two shocks depress the economy's development mainly in the short-term, whereas shocks from the ECB have their largest impact also in the short term but in total affect the economy more persistently. The response patterns of GDP for France and Italy are very similar to Germany. Obviously from Figure 2.3 it follows that the Italian economy reacts more sensitive especially on price and wage mark-ups. In their impact these shocks are much larger than observed for Germany and France. Beside its strength the response of Italy's GDP on wage mark-up shocks lasts longer on the development of the Italian GDP than the according responses observed for Germany and France. As for the first term structure factor the broader confidence interval signals an increased uncertainty about GDP's long term response on wage mark-up shocks. The nominal short term rates are mainly driven by productivity shocks and shocks coming from the first factor of the term structure. Both components reveal a positive reaction pattern, whereas the first yield factor has its largest impact in the short term. A productivity shock affects the short term interest rate more persistently. Price and wage mark-up increases reveal an ECB short term counter reaction inducing a more restrictive monetary policy reaction on an increased inflationary pressure on the economy. Interestingly monetary policy only reacts in the short term on this increased pressure. In the middle to long term there is no effect in response to price mark-up shocks and a negative long term ECB response on wage mark-up shocks. Taking the contrary perspective: The responses of inflation rates on increases in the ECB's controlled short term rate (not shown here) reveal persistent inflation lowering effects especially in the short to middle term for all three EMU countries. Interestingly the response of the ECB's controlled short term rate to shocks disturbing monetary policy decisions show an interest rate decreasing pattern. The initializing positive one standard deviation shock is observable only in the ECB's first quarter response. After that the ECB induces a counter reaction to this interest increasing shock. This response pattern is in line with the term structure of interest rate factor loadings discussed in 2.4.3.1. Here the structural monetary policy shocks negatively load on the yields over the whole maturity spectrum, reflecting the "what ever it takes" policy measures induced by the ECB, hedging the EMU against depressing exogenous influences. ECB also sharply reacts on increases in the risk premiums investors demand, implying higher financing costs and lead to a decline in GDP and investment activities (last one not shown here). The ECB reaction on increased risk premiums in decreasing the monetary policy rate is larger than the depressing risk premium effect on the economy's GDP.

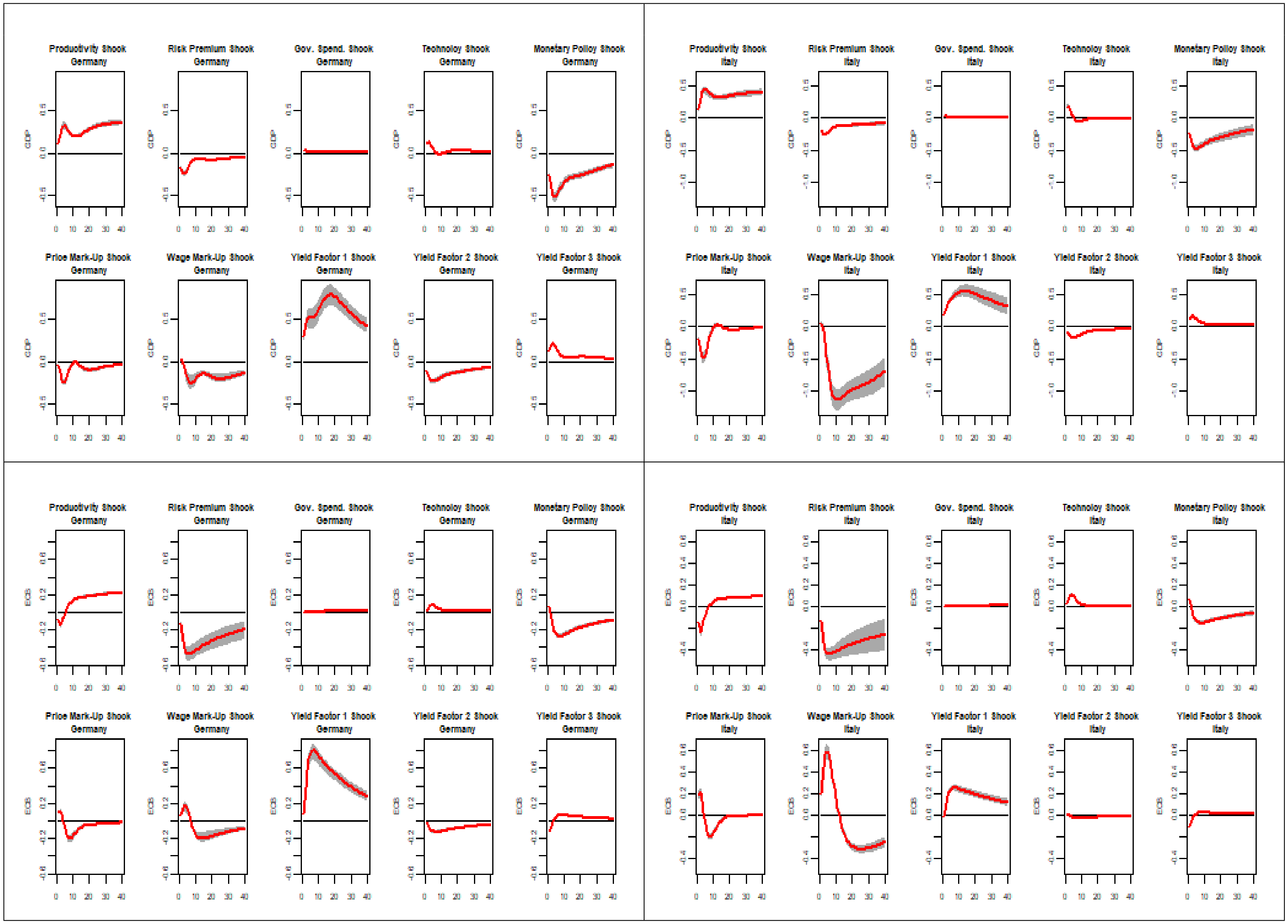


Table 2.3: SW-DSGE-ATSM implied responses of GDP and the nominal short term rate to a one standard deviation shock coming from the 10 structural shock variables of our SW-DSGE-ATSM for Germany and Italy. For generating the IR's we take the mean of 1000 draws from the models posteriors. The shaded areas are the [10% , 90%] confidence intervals.

2.4.3 Term structure shock transition in the EMU

2.4.3.1 Macroeconomic and term structure factor weights

In Figure 2.4 we plot the macroeconomic term structure weights $\boldsymbol{b}_\tau^{RA}$ of the yields with time to maturity $\tau = 12, 24, ..., 120$ respectively. Interestingly from Figure 2.4 it follows that structural shocks related to government spending have dominant effects on middle and long term interest rates reflecting the EMU's sovereign debt crisis and the high sensitivity of bond yields on issues relating government budget deficits and the sustainability of sovereign debt in our time horizon ranging between Q1/2005 and Q1/2014. Figure 2.4 further points out that structural shocks related to monetary policy decisions have a persistent influence over the whole term structure's maturity spectrum. Only for short term and long term yields the term structure effects of monetary policy shocks are weaker. The yield loadings for monetary policy shocks are negative, reflecting the "what ever it takes" policy measures induced by the ECB for stabilizing the EMU with the sharp decreases in the ECB's deposit and marginal lending facility rates as well as in the main refinancing operations rate since Q4/2008. These monetary policy environment manifests itself in the negative sign of the monetary policy shock loadings. Additional to these two factors the yield loadings reveal a high (positive) sensitivity especially in the term structure's short to middle term maturity segment on structural wage mark-up shocks driving the inflation rate via the (hybrid) forward-backward-looking New Keynesian Phillips curve in 2.13. With focus on the three latent term structure factors only the first latent factor loads stronger on the term structure.

2.4.3.2 Decomposing EMU's term structure of interest rates

To get an understanding of the real economy spillover effects into the government bond markets in Figure 2.5 we plot the decomposition of the short and long term 12 and 120 month government bond yields for Germany, France and Italy. Similar to the decomposition of the real economy variables in section 2.4.2.2 the historical yield decomposition reveals that the countries' short and long term interest rates are largely effected by structural shocks between Q2/2008 and Q1/2009. Beside this first phase there is a second phase of larger impacts of shocks. For Germany and France this second phase starts in Q3/2010 and ends in Q4/2011, whereas for Italy this phase lasts until Q4/2012. The magnitude of the shocks affecting the long and short term interest rates in the first phase are similar between Germany and France. In this phase the dominant shocks affecting the German and French term structure are induced by monetary policy decisions or are related to risk premiums. Both shocks lower the countries' level of interest rates, where the monetary policy shocks reflect the mentioned sharp (perhaps surprising) decrease in the ECB's controlled short term policy rates. In Germany and France risk premium shocks have a decreasing effect on both the short and the long term interest rates indicating the save haven status of German and French government bonds in this critical phase. Interestingly since the beginning of 2009 shocks related to government spending issues become more dominant in the development of the long term interest rates of all three countries, indicating the changing characteristic of the EMU's economic crisis in becoming a sovereign debt crisis. In the second phase of larger shock impacts on the EMU countries' term structures especially Italy is affected by highly fluctuating shocks. This phase ends in the end of 2012 and therefore with Mario Draghi's London speech in July 2012 and the discussions about more far-reaching ECB measures followed to this speech.

2.4.3.3 Term structure shock processing

In Figure 2.6 we plot the term structure responses to one standard deviation shocks from the 10 structural shock variables of our SW-DSGE-ATSM for Germany and Italy. Obviously similar to the short term rate's reaction patterns shown in Figure 2.3 the term structure of interest rates of both countries react on shocks disturbing ECB's monetary policy decisions with downward shifts affecting the whole maturity spectrum of the term structure, reflecting the efforts by the ECB in stabilizing the EMU. Interestingly different to the short term rate reaction the German and Italian term structure immediately decreases, where for both countries the middle term maturity segment shows the strongest reaction whereas the short end of the term structure shows the weakest response. Shocks disturbing ECB's policy decision process effect the slope of the countries term structure, leading in its further development to a flattening of the yield curve. With a look at the responses of the three term structure factors from Figure 2.6 these factors become more tangible. The first and the third factor affect the term structure in flattening the yield curve, whereas the second factor leads to a steepening of the term structure. Figure 2.3 has revealed, that in response to a flattening of the term structure expressed in shocks coming from the first and third yield factor the trajectory of ECB's short term interest rate shows a middle to long term increase. This finding is in line to the response pattern revealed in our estimation of a term structure extended version of the ECB's NAWM in the next chapter 3. Here we also find an upward directed trajectory of the short term monetary policy rate induced by a flattening of the term structure of interest rates in the EMU. With respect to risk premium shocks the short term maturities of the term structure of interest rates react analogue to the short term rate in Figure 2.3, where the largest impact of the risk premium shock lies in the middle term. This response is also in line with our findings from the estimation of the term structure extended NAWM in chapter 3 - further strengthening the robustness of our findings. In its total effect a risk premium shock leads to a steepening of the term structure of interest rates. Similar effects come from price mark-up shocks. Here the short end shows the same reaction path as the short term rate in Figure 2.3 and in total the shock leads to a steepening of the term structure, reflecting the inflation premium investors demand for holding long term maturity bonds.

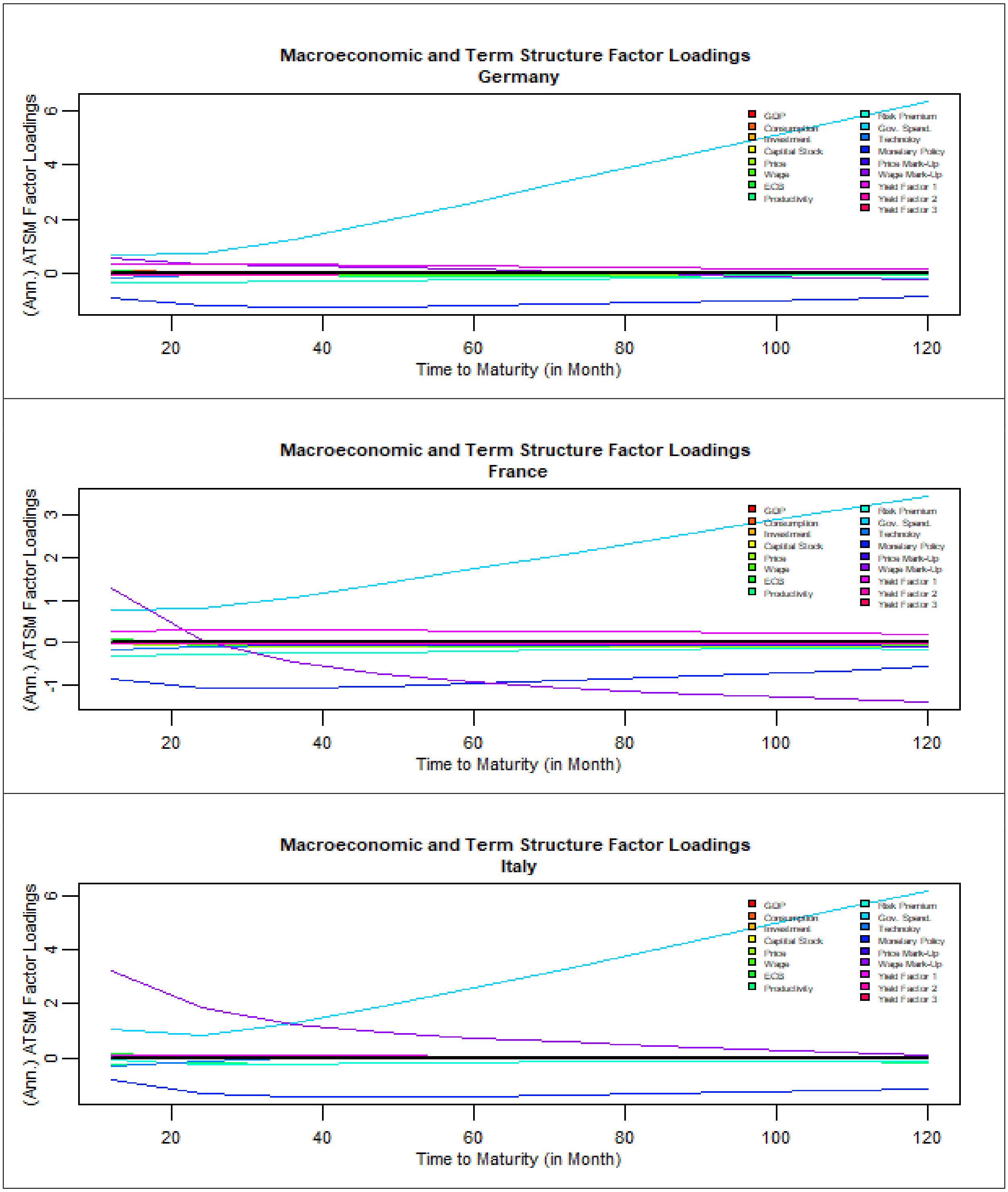


Table 2.4: (Annualized) macroeconomic and latent term structure of interest rate factor weights $\boldsymbol{b}_{\tau}^{RA}$ implied by the ATSM component of our SW-DSGE-ATSM evaluated at the mode of the models posterior for Germany, France and Italy

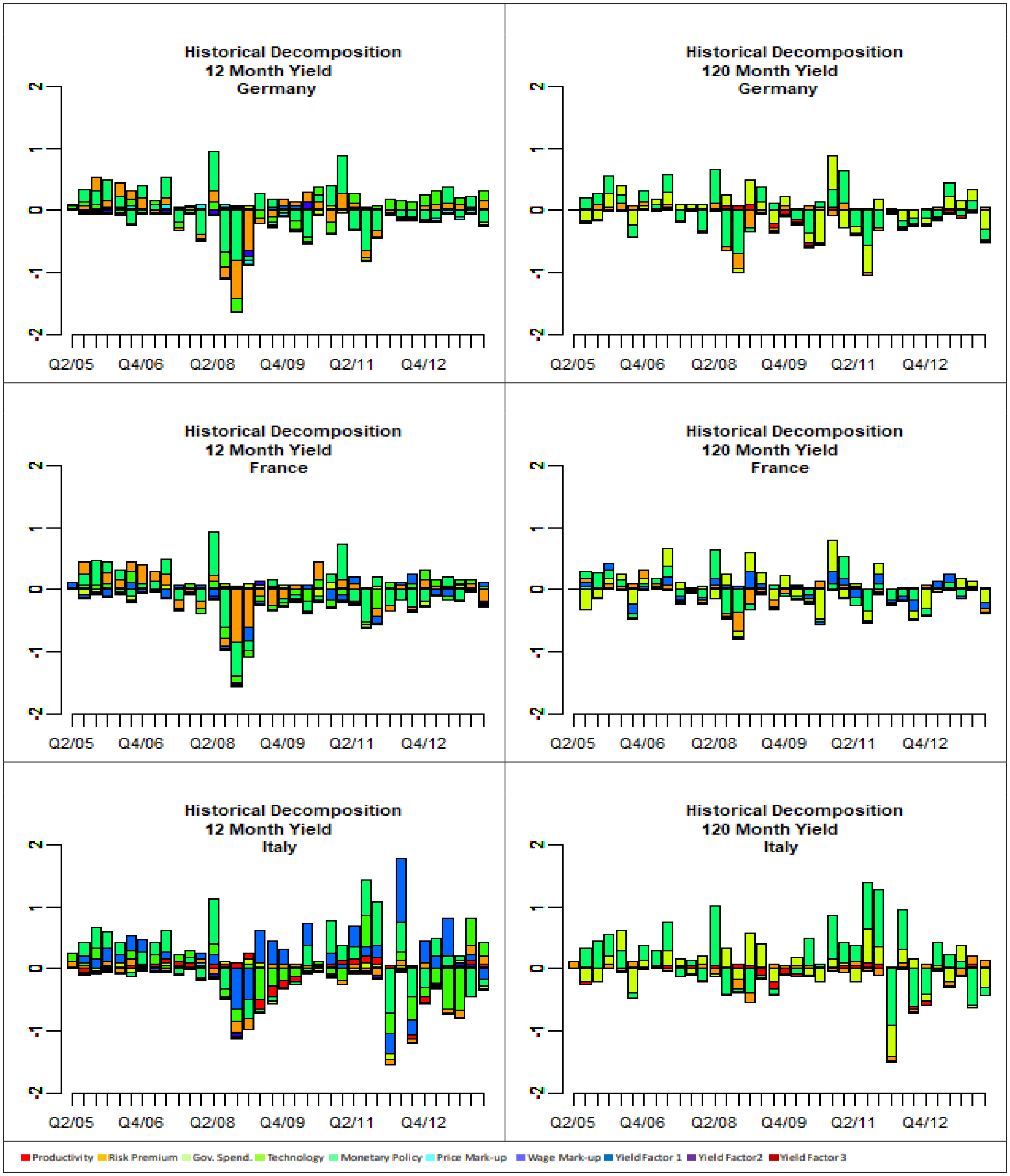


Table 2.5: Historical decomposition of the SW-DSGE-ATSM implied 12 and 120 month short and long term interest rates between Q1/2005 and Q1/2014 for Germany, France and Italy evaluated at the mode of the models posterior

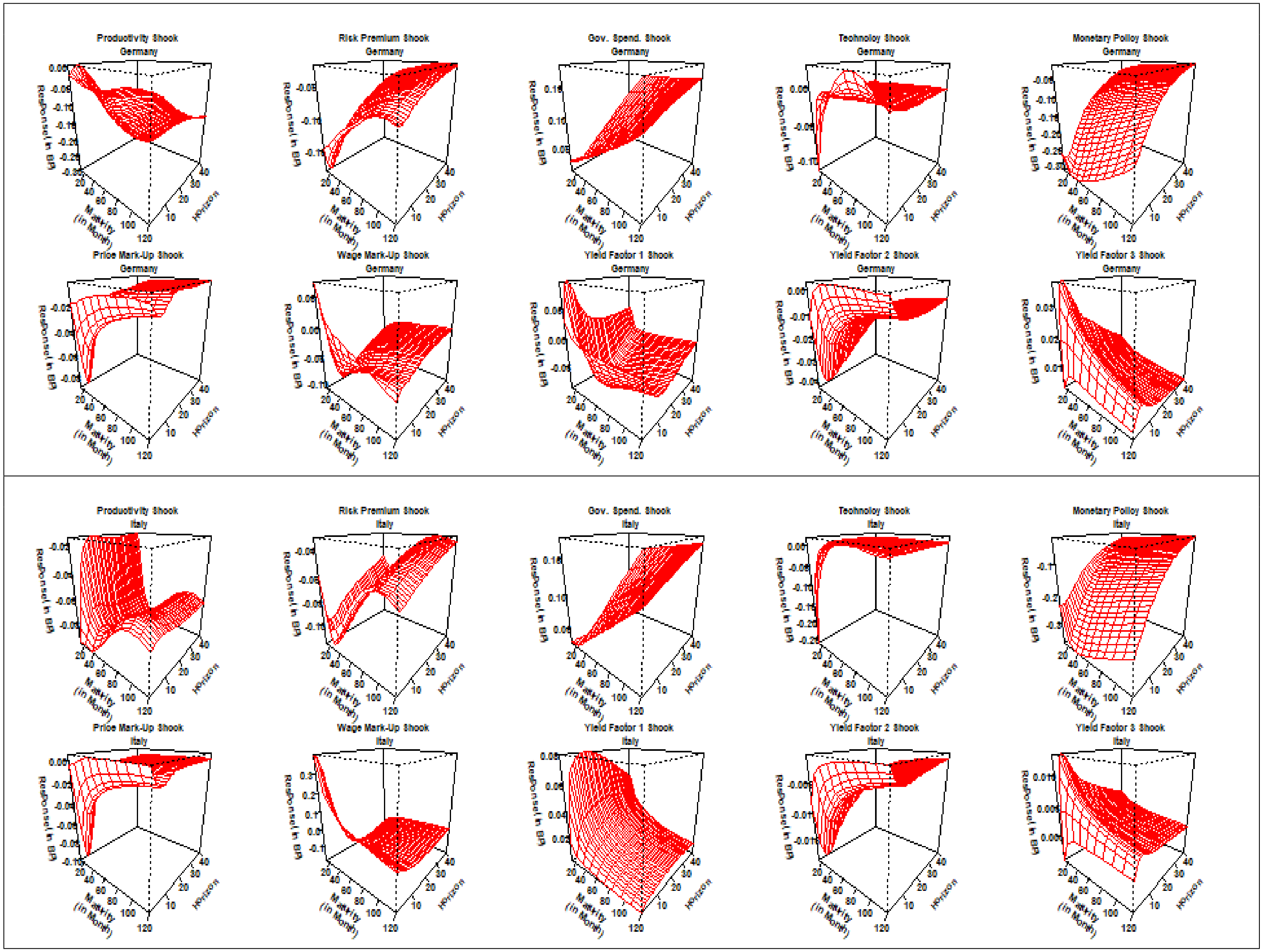


Table 2.6: Term structure responses to one standard deviation shocks coming from the 10 structural shock variables of our SW-DSGEATSM for Germany and Italy. For generating the IR's we take the mean of 1000 draws from the models posteriors.

As we have seen in the previous sections, there is a large impact of government spending issues on the term structure, expressed in a (nearly) linear increasing yield loading with respect to government spending shocks. Especially the long term interest rates react very sensitive on issues related to government spending. Not surprisingly in Figure 2.6 we see a linear increasing response pattern over the maturity spectrum in reaction to a one standard deviation shock coming from government spending. The economic environment reflected in our data between Q1/2005 and Q1/2014 is dominated by the upcoming EMU's sovereign debt crisis. Political decisions - national and EMU wide - as well as the ECB as the monetary authority were driven by subjects related to government spending and government deficits. Therefore our SW-DSGE-ATSM signals that shocks related to government spending have larger effects and last very persistently on EMU's term structure of interest rates imposing the conditions for the governments further spending activities and reflecting the increased pressure induced by the financial markets for stimulating self regulating as well as political forces for taking more sustainable future debt paths.

2.4.4 Monetary policy reaction function

In combining the SW-DSGE model with the ATSM the monetary policy reaction function expressed in 2.17 is a central cornerstone. In 2.17 we have extended the Taylor-rule like reaction function of the SW-DSGE model in which the monetary authority adjusts the short term policy rate with respect to the two components inflation and output gap. Beside these two conventional components our extension integrates a third component. This component reflects the increased influence of issues related to the term structure of interest rates on the process of monetary policy decision finding. For validating our extension we discuss our variant in comparison with the estimation results of three alternative variants of the Taylor-rule. These three alternatives include the standard Taylor-rule with partial adjustment:

$$r_t = c + \rho r_{t-1} + (1-\rho)\left[\psi_\pi \pi_t + \psi_y \left(y_t - \tilde{y}_t\right)\right] + \epsilon_t^r \tag{2.63}$$

where π_t is the annual (log) inflation rate calculated from the GDP implied price deflator and $\tilde{y}_t$ is the potential (trend) GDP computed by using the Hodrick-Prescott filter. Both y_t and $\tilde{y}_t$ are in logs and the disturbance term is Gaussian $\epsilon_t^r \sim N\left(0, \sigma_r^2\right)$. The second variant is the monetary policy reaction function proposed by Smets and Wouter [2007]:

$$r_t = \rho r_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_y \left(y_t - \tilde{y}_t\right)\right] + r_{\Delta y}\left[\left(y_t - \tilde{y}_t\right) - \left(y_{t-1} - \tilde{y}_{t-1}\right)\right] + \rho_r \varepsilon_{t-1}^r + \sigma_r \epsilon_t^r \tag{2.64}$$

similar to 2.17 combined with the monetary shock process in 2.23 except to the influence of the latent term structure factors. 2.64 expresses a backward looking type of Taylor rule considered among others by Eichenbaum and Evans [1995], Christiano, Eichenbaum and Evans [1996] and Clarida, Gali and Gertler [1998], where beside the current macroeconomic variables the monetary policy rule also includes lagged variables. With ε_t^r and ε_{t-1}^r the modified Taylor rule in 2.64 also includes serially correlated policy shocks as discussed in Ang, Dong and Piazzesi [2007]. The third variant we use for our validation is the monetary policy rule used in the small-scale New-Keynesian DSGE proposed by Beakert, Cho and Moreno (BCM) [2010], which is defined as:

$$r_t = \alpha_{MP} + \rho r_{t-1} + (1-\rho)\left[\beta\left(\mathbb{E}_t\left[\pi_{t+1} - \tilde{\pi}_t\right]\right) + \gamma\left(y_t - \tilde{y}_t\right)\right] + \epsilon_t^r \tag{2.65}$$

Distinct to the standard Taylor-rule and the backward looking Taylor-rule with serially correlated policy shocks implied by the SW-DSGE and the SW-DSGE-ATSM, the BCM integrates expectations about future inflation and an inflation target $\tilde{\pi}_t$ in its implied monetary policy rule. The BCM New-Keynesian DSGE's modified policy rule is in line with the forward looking Taylor-rule discussed in Clarida and Gertler [1997] and Clarida, Gali and Gertler [2000], where expectations about future inflation and output are integrated into the policy rule. In Appendix A.8 the BCM New-Keynesian economy is outlined in more detail. Table 2.7 lists the parameter estimates of the SW-DSGE-ATSM and the three variants for Germany, France and Italy. For all three countries the SW-DSGE-ATSM, SW-DSGE and the standard Taylor rule have large and significant coefficients ρ indicating interest rate smoothing. For all three countries the coefficients r_π of our SW-DSGE-ATSM are significantly larger than one reflecting a positive long run response to inflation consistent with the Taylor principle. For the standard Taylor rule, this is only true for France. For Germany and Italy the inflation related coefficient r_π has a negative sign and therefore lead to the counter-intuitive interpretation of decreasing interest rates in times of higher inflation rates. The SW-DSGE's inflation coefficient r_π for France and Italy is very large reflecting a strong response to increasing inflation rates. Our estimates of r_y and $r_{\Delta y}$ as well as our estimates of the GDP related coefficient ψ_y implied by the standard Taylor rule are in line with estimates of backward looking FED Taylor rules done by Ang, Dong and Piazzesi [2008] for the U.S. Looking at our estimates of the coefficient ρ_r determining the strength of serial correlation of the policy shocks the SW-DSGE-ATSM reveals a high autocorrelation of the monetary policy shocks for all three countries. Different are the estimates for the SW-DSGE. Here we find high persistence of the policy shock only for Germany. Compared to the standard Taylor-rule and the SW-DSGE implied backward-looking monetary policy rule the SW-DSGE-ATSM has the highest volatility parameters. The forward looking Taylor-rule implied by the BCM New-Keynesian DSGE shows also very high volatilities for its estimated residuals. There are low estimated coefficients ρ reflecting no pronounced interest rate smoothing. As the SW-DSGE the BCM model has significant negative coefficients γ indicating an immediate restrictive monetary policy response to (positive) output gaps. The significant estimates of the BCM model's β coefficient reflecting the impact of inflation's one-period ahead expectation are of similar magnitudes as in the estimates done by Ang, Dong and Piazzesi [2008] of the one-quarter ahead forward looking FED Taylor rule proposed by Clarida and Gertler [1997] and Clarida, Gali and Gertler [2000]. Except for the BCM New-Keynesian DSGE the R^2 of all Taylor rule estimates are close to or larger than 0.9 . All three monetary policy rules imply from their respective macroeconomic state variables a large content of predictable variation in their short term interest rates r_t. In Table 1 the backward looking Taylor rule of the SW-DSGE has the highest R^2 for all three EMU countries.

In Figure 2.8 we plot the decomposition of the SW-DSGE-ATSM implied monetary policy reaction function with respect to the EMU countries Germany, France and Italy evaluated at the mode of the models posteriors. We further plot the time-varying percentage proportions of the monetary policy rules components. In Germany and France beside the steady-state value of the short-term interest rate r_t (the Taylor-rule in the state-space model's transition equation is expressed in (log) deviations from the state variables steady-state values) r_t is mainly driven by the Taylor-rule implied lagged short rate and the monetary policy shock component. Interestingly in 2009 the term structure of interest rate factors internalized in the SW-DSGE-ATSM's monetary policy rule become more dominant. Obviously for Italy this phenomenon is very pronounced. Until the beginning of 2009 the Italian short term interest rate is dominated by interest rate smoothing and monetary policy shocks respectively. In the first half of 2009 there is a change in the components driving the Italian short term rate. The three term structure factors become the dominant driving forces with interruption in late 2011 and early 2012 when monetary policy shocks become dominant again.

2.4.5 Goodness of fit of the SW-DSGE-ATSM

To evaluate the quality of our SW-DSGE-ATSM in describing the observed macroeconomic and term-structure data we implemented a large set of various established macroeconomic and term-structure models. The goodness of fit for the observed macroeconomic variables is evaluated by the implementation of the SW-DSGE model proposed by Smets and Wouters [2007], by a conventional VAR process where we restrict the number of lags due to our small observation horizon to one and by the implementation of the small-scale New-Keynesian DSGE with integrated term-structure modeling proposed by Beakert, Cho and Moreno (BCM) [2010]. For our comparison the BCM-DSGE model has the restriction that it only describes the macroeconomic variables GDP, Inflation and the ECB's monetary policy rate. For Germany, France and Italy Table 2.10 lists the RMSE of our SW-DSGE-ATSM compared to the three alternative macroeconomic model implementations. More details about the implementation and Bayesian estimation of the small scale New-Keynesian BCM-DSGE model are outlined in Appendix A.8.

Calculating for each model the mean over the RMSEs of all seven macroeconomic variables listed in Table 2.10 shows for Germany an average RMSE for our SW-DSGE-ATSM of 49 basis points (BP) comparable to the 47 BP mean RMSE of the SW-DSGE. Both errors are significantly lower than the 65 BP mean RMSE implied by the VAR[1]. The model estimations for France show on average a 32 BP RMSE for our SW-DSGE-ATSM, a 22 BP mean error implied by the SW-DSGE and a 37 BP mean RMSE of the VAR[1] over all of the seven macroeconomic variables. For Italy the quality of the in-sample-fit is similar between the three models SW-DSGE-ATSM, SW-DSGE and VAR[1]. Our SWDSGE-ATSM implies on average a 61 BP RMSE, SW-DSGE and VAR[1] imply mean RMSEs of 59 BP and 60 BP respectively. Compared to these three models, the BCM-DSGE for all three countries shows a RMSE of the short-term-rate, which is similar to the three other models, whereas for GDP the BCM-DSGE shows a poorer in-sample-fit as the aforementioned three models. In Figure 2.9 we plot the macroeconomic variables implied by the four alternative models compared to the observed data. From Figure 2.9 it becomes clear that the BCM-DSGE also shows a poor in-sample-fit for the inflation variable which is not directly reflected by the BCM-DSGE's RMSE of inflation.

SW-DSGE-ATSM

	ρ	r_π	r_y	$r_{\Delta y}$	$\delta_{f,1}$	$\delta_{f,2}$	$\delta_{f,3}$	ρ_r	σ_r	R^2
Germany	0.942	3.881	0.524	0.612	-0.202	0.087	-0.198	0.916	0.263	0.889
	(0.018)	(0.007)	(0.014)	(0.005)	(0.001)	(0.005)	(0.001)	(0.008)	(0.013)	
France	0.854	3.916	0.364	0.627	-0.200	0.069	-0.201	0.932	0.220	0.959
	(0.009)	(0.003)	(0.016)	(0.005)	(0.001)	(0.006)	(0.001)	(0.004)	(0.010)	
Italy	0.879	3.903	0.242	0.597	-0.174	0.069	-0.191	0.936	0.238	0.929
	(0.016)	(0.006)	(0.030)	(0.011)	(0.001)	(0.003)	(0.001)	(0.005)	(0.016)	

SW-DSGE

	ρ	r_π	r_y	$r_{\Delta y}$	ρ_r	σ_r	R^2
Germany	0.826	-0.925	-0.169	0.077	0.940	0.151	0.991
	(0.029)	(0.567)	(0.047)	(0.021)	(0.007)	(0.139)	
France	0.752	13.950	-0.629	0.143	0.419	0.144	0.980
	(0.104)	(8.210)	(0.930)	(0.416)	(0.081)	(0.126)	
Italy	0.810	5.769	0.181	0.270	0.306	0.030	0.980
	(0.073)	(1.425)	(0.138)	(0.080)	(0.272)	(0.067)	

Standard Taylor-rule with partial adjustment

	c	ρ	ψ_π	ψ_y	r	R^2
Germany	0.307	0.878	-4.565	1.040	0.058	0.960
	(0.107)	(0.038)	(2.129)	(0.298)	(0.021)	
France	-0.011	0.840	3.838	1.223	0.073	0.967
	(0.092)	(0.037)	(1.393)	(0.337)	(0.017)	
Italy	0.192	0.862	-0.611	1.458	0.077	0.966
	(0.079)	(0.038)	(0.695)	(0.351)	(0.018)	

BCM New Keynesian DSGE

	α_{MP}	ρ	β	r_γ	σ_r	R^2
Germany	0.320	0.188	1.000	-0.345	1.51	0.150
	(0.015)	(0.008)	(0.001)	(0.015)	(0.036)	
France	0.360	0.188	1.000	-0.329	1.484	0.476
	(0.010)	(0.003)	(0.001)	(0.008)	(0.006)	
Italy	0.401	0.188	1.000	-0.429	1.541	0.485
	(0.029)	(0.004)	(0.005)	(0.020)	(0.018)	

Table 2.7: Estimation results of the monetary policy reaction functions of the SW-DSGE-ATSM model and three alternative variants for Germany, France and Italy

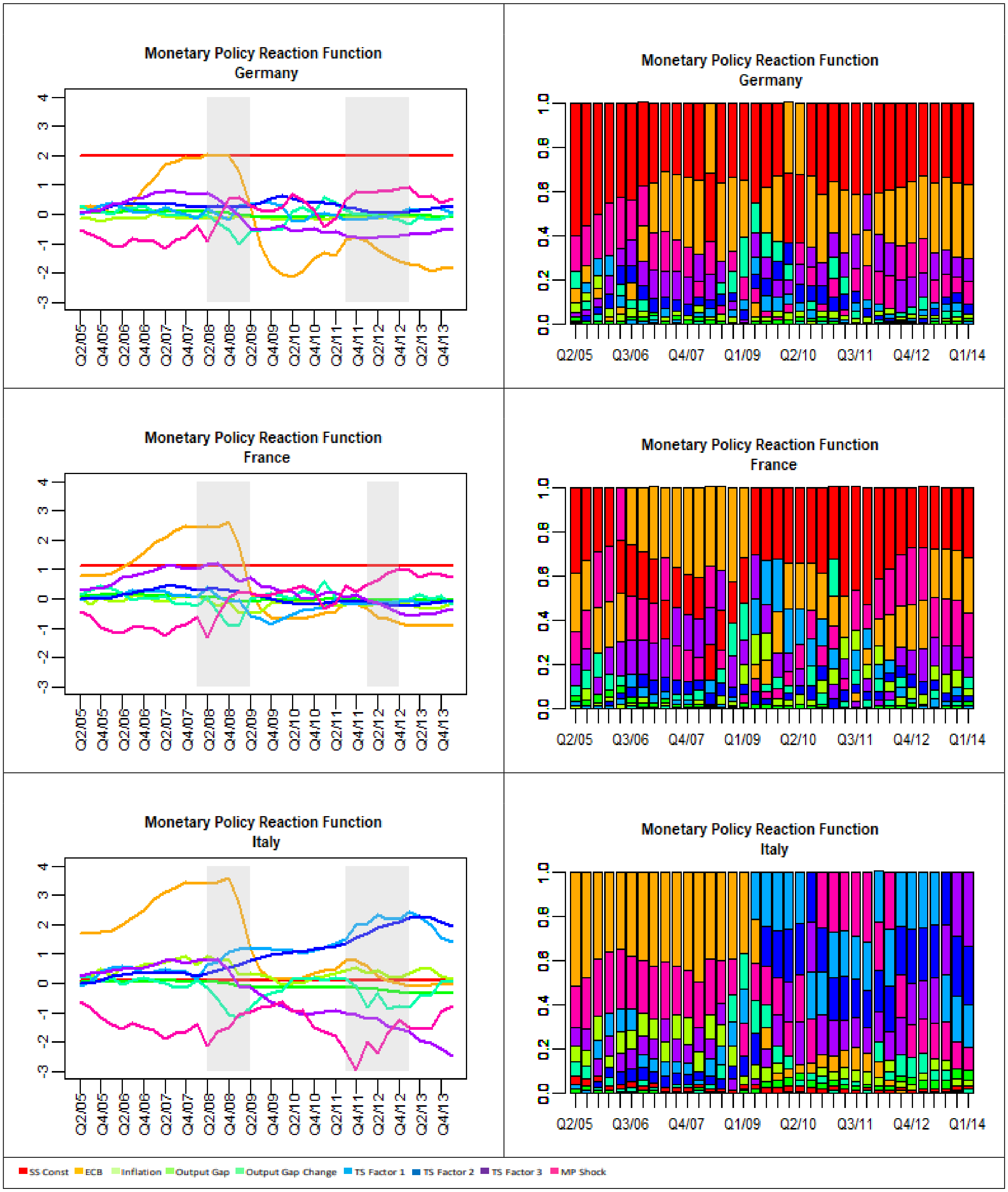


Table 2.8: Decomposition of the SW-DSGE-ATSM implied monetary policy reaction function with respect to the EMU countries Germany, France and Italy at the mode of the models posteriors

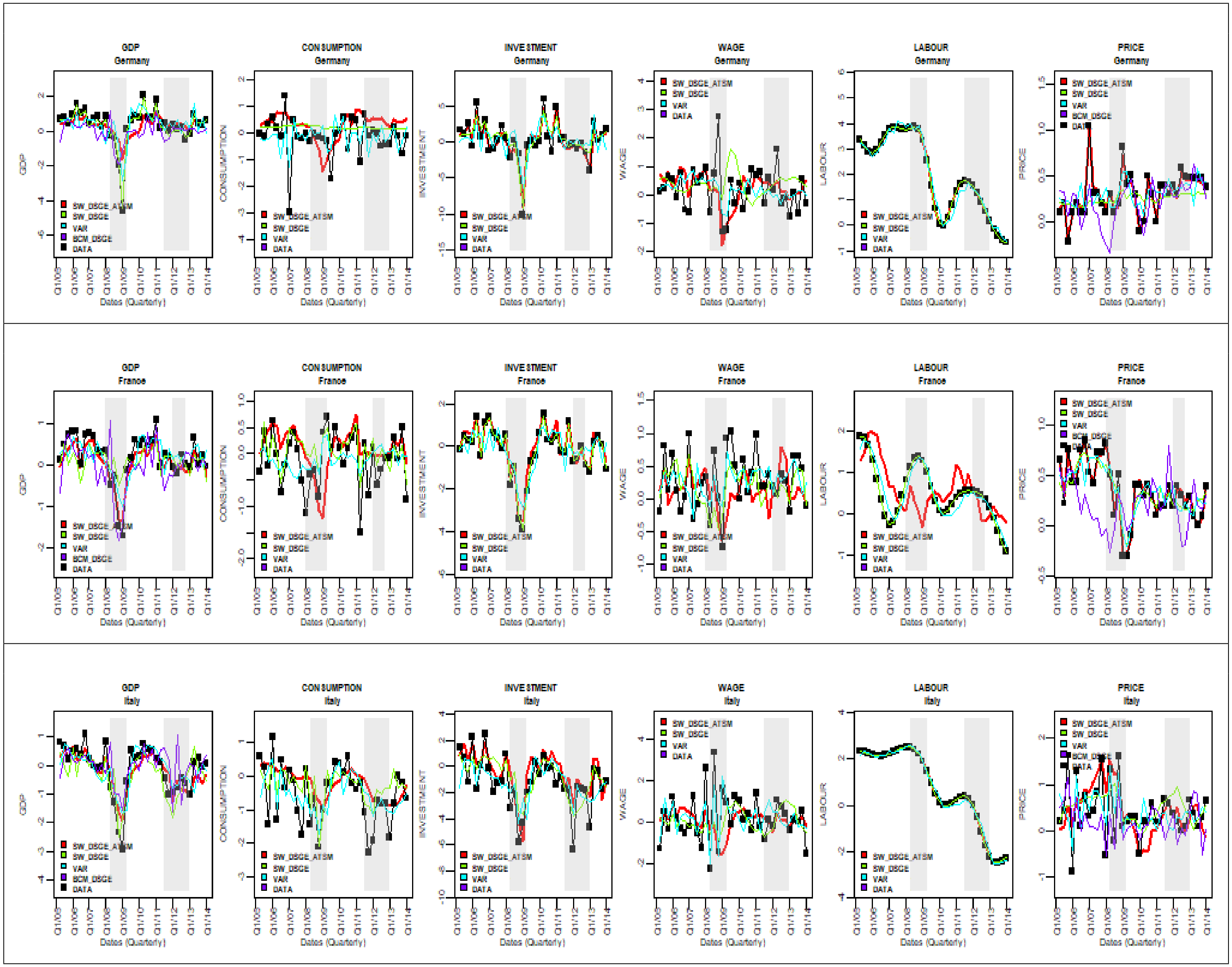


Table 2.9: Model implied macroeconomic variables for Germany, France and Italy from four alternative macroeconomic model implementations compared to our SW-DSGE-ATSM calculated at the mode of the model's posteriors

Macroeconomic Variables : Germany

	$ln(\Delta GDP_t)$	$ln(\Delta CONS_t)$	$ln(\Delta INV_t)$	$ln(\Delta WAGE_t)$	$ln(LABOUR_t)$	$ln(INF_t)$	ECB_t
SW-DSGE-ATSM	72.984	90.345	75.283	71.135	0.682	3.647	26.386
SW-DSGE	16.459	79.675	118.235	90.947	0.000	21.980	4.718
BCM-DSGE	102.529	$--$	$--$	$--$	$--$	26.296	20.465
VAR[1]	72.255	56.273	188.079	69.714	24.379	20.404	25.697

Macroeconomic Variables : France

	$ln(\Delta GDP_t)$	$ln(\Delta CONS_t)$	$ln(\Delta INV_t)$	$ln(\Delta WAGE_t)$	$ln(LABOUR_t)$	$ln(INF_t)$	ECB_t
SW-DSGE-ATSM	22.078	55.873	27.320	44.008	67.113	3.431	2.393
SW-DSGE	44.006	37.610	7.570	31.824	0.000	15.042	15.376
BCM-DSGE	69.353	$--$	$--$	$--$	$--$	38.733	16.145
VAR[1]	39.626	47.521	72.131	35.489	20.377	17.956	24.106

Macroeconomic Variables : Italy

	$ln(\Delta GDP_t)$	$ln(\Delta CONS_t)$	$ln(\Delta INV_t)$	$ln(\Delta WAGE_t)$	$ln(LABOUR_t)$	$ln(INF_t)$	ECB_t
SW-DSGE-ATSM	42.479	80.498	122.674	119.455	0.520	53.377	4.795
SW-DSGE	54.637	60.137	128.777	108.971	0.000	57.015	1.746
BCM-DSGE	84.918	$--$	$--$	$--$	$--$	54.763	1.618
VAR[1]	56.918	65.511	140.265	76.569	14.921	40.715	26.347

Table 2.10: RMSE of the SW-DSGE-ATSM and alternative macroeconomic model implementations for the three EMU countries Germany, France and Italy

Figure 2.11 separately shows the observed and the model implied short-term interest rate linked to our three macroeconomic DSGE models by different specifications of the Taylor rule. Figure 2.11 reveals that all four macroeconomic models imply good in-sample-fits for the short term interest rate. For evaluating the goodness of fit of the term structure of interest rates, we have implemented the short rate model originally proposed by Vasicek [1977] in a more general three-factor version as outlined in Boulder [2001] and Brigo and Mercurio [2007], the latent and macro-finance ATSM proposed by Ang and Piazzesi [2003], the independent and correlated dynamic Nelson Siegel (DNS) models developed in Diebold and Li [2006], the independent and correlated arbitrage-free DNS by Christensen, Diebold and Rudebusch [2011], the macro-finance DNS proposed by Diebold, Rudebusch and Aruoba [2006] and the above mentioned BCM small-scale New-Keynesian DSGE with integrated term-structure modeling as model alternatives. Table 2.12 lists the RMSYE for the maturities $\tau = 12, 24, 36, ..., 120$ month of our SW-DSGE-ATSM compared to the nine alternative term structure models for Germany, France and Italy. From Table 2.12 it becomes clear that for all three countries our SW-DSGE-ATSM shows the best in-sample-fit compared to the nine alternative models. Only the RMSYE for the 12 month yields implied by our SW-DSGE-ATSM seems to be systematically larger than the RMSYE implied by the latent ATSM as the term structure model with the second best in-sample-fit in the set of our implemented models. The BCM-DSGE as the second model which implies a DSGE modeling component for the macroeconomic variables also shows a good in-sample-fit for maturities ranging from 24 month to 96 month. But especially for the 12 month maturity at the short end and for

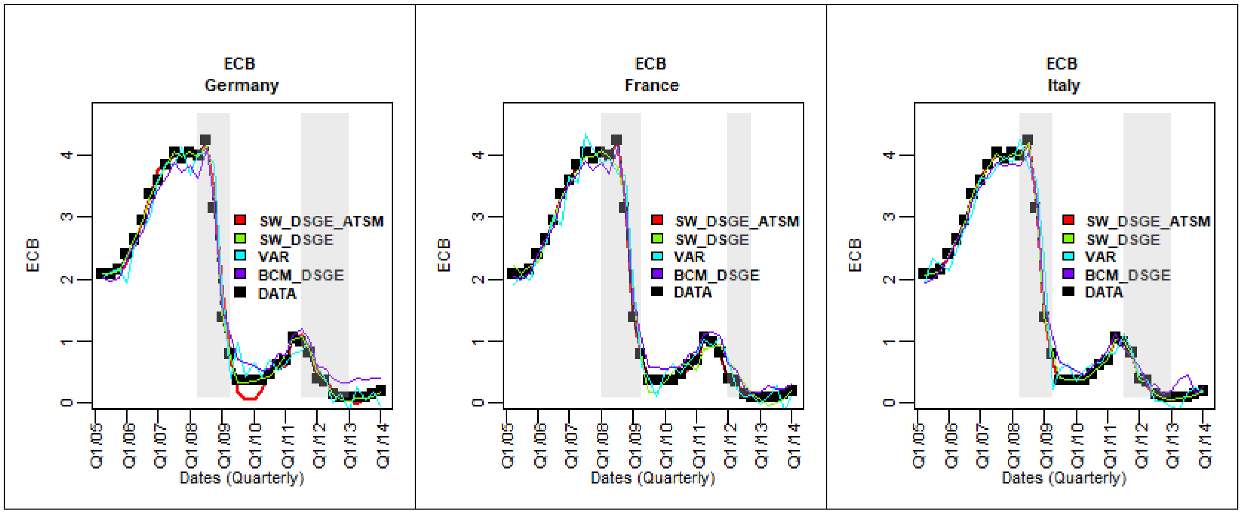


Table 2.11: Model implied ECB's monetary policy rate from four alternative macroeconomic model implementations with data from Germany, France and Italy compared to our SW-DSGE-ATSM calculated at the mode of the model's posteriors

the maturities 108 and 120 month at the long end, the BCM-DSGE implies larger RMSYE's. For all three countries the three-factor Vasicek-model shows the poorest in-sample-fit. Figure 2.13 shows the yields with maturities 12, 60 and 120 month implied by the nine term-structure models compared to the observed yields of Germany, France and Italy. Obviously the latent ATSM and our SW-DSGE-ATSM are very close to the observed data and show the best in-sample-fit, whereas the Vasicek-model for Italy shows a very poor fit.

Term Structure of Interest Rates: Germany

	$y(12M)$	$y(24M)$	$y(36M)$	$y(48M)$	$y(60M)$	$y(72M)$	$y(84M)$	$y(96M)$	$y(108M)$	$y(120M)$
SW-DSGE-ATSM	6.142	2.979	1.570	0.517	0.565	0.513	0.521	0.834	1.024	2.197
MF-ATSM	0.020	13.548	0.009	17.245	25.975	27.067	23.148	16.414	8.378	0.102
Latent-ATSM	0.000	4.854	0.000	4.680	7.030	7.348	6.290	4.447	2.255	0.000
Ind. AFDNS	37.771	23.302	8.198	7.957	11.548	12.327	10.713	7.927	6.721	10.073
Corr. AFDNS	29.958	19.993	11.661	5.584	0.038	4.595	8.542	12.932	18.905	26.976
Ind. DNS	36.346	23.543	11.730	5.238	1.882	0.000	1.765	3.398	3.586	0.141
Corr. DNS	18.111	5.843	6.106	5.642	4.113	2.133	0.000	2.160	4.296	6.373
MF-DNS	23.216	7.715	1.649	1.561	1.727	1.690	1.380	0.958	1.314	2.499
Vasicek	91.264	47.510	14.613	14.962	11.728	8.513	7.797	8.714	10.684	14.849
BCM-DSGE	30.910	3.917	0.964	1.122	2.558	5.009	5.743	7.533	16.602	32.172

Term Structure of Interest Rates: France

	$y(12M)$	$y(24M)$	$y(36M)$	$y(48M)$	$y(60M)$	$y(72M)$	$y(84M)$	$y(96M)$	$y(108M)$	$y(120M)$
SW-DSGE-ATSM	5.035	2.581	1.074	0.667	0.385	0.486	0.475	0.514	0.761	0.333
MF-ATSM	0.047	5.327	0.071	10.116	18.236	21.283	19.434	14.271	7.410	0.033
Latent-ATSM	0.000	0.874	0.000	2.064	2.998	3.125	2.919	2.276	1.218	0.000
Ind. AFDNS	34.525	22.304	9.524	6.549	6.039	4.692	2.844	3.291	6.494	10.383
Corr. AFDNS	56.256	33.461	40.288	27.440	17.122	9.802	2.794	4.661	11.942	18.839
Ind. DNS	17.760	10.903	3.320	0.001	1.256	1.403	0.902	0.000	1.150	2.453
Corr. DNS	42.557	29.091	13.686	6.679	2.922	0.011	2.772	5.487	8.137	10.740
MF-DNS	29.549	17.884	5.331	1.233	1.420	1.458	1.417	1.267	0.812	0.795
Vasicek	77.959	41.650	19.734	10.331	8.905	8.631	8.030	8.084	9.498	12.178
BCM-DSGE	33.144	3.599	0.713	0.985	1.577	3.172	4.608	6.270	14.011	31.439

Term Structure of Interest Rates: Italy

	$y(12M)$	$y(24M)$	$y(36M)$	$y(48M)$	$y(60M)$	$y(72M)$	$y(84M)$	$y(96M)$	$y(108M)$	$y(120M)$
SW-DSGE-ATSM	5.091	4.310	0.003	1.770	1.726	1.022	0.377	0.639	0.924	2.462
MF-ATSM	0.022	12.016	0.065	27.825	42.737	43.962	37.357	26.769	13.918	0.110
Latent-ATSM	0.000	6.744	0.000	7.203	10.015	9.724	7.816	5.245	2.546	0.000
Ind. AFDNS	45.641	24.955	10.896	11.908	14.130	13.196	10.459	9.420	13.802	21.896
Corr. AFDNS	68.193	39.474	18.601	6.369	0.123	2.212	2.015	1.151	2.653	5.311
Ind. DNS	67.777	51.201	27.364	11.155	3.522	1.666	0.006	2.278	3.560	2.045
Corr. DNS	40.857	30.495	17.097	6.008	0.000	2.133	1.828	0.000	2.850	6.451
MF-DNS	34.966	21.759	10.197	1.530	3.081	3.762	2.493	0.491	2.707	5.733
Vasicek	49.843	36.081	30.450	24.352	20.497	18.028	16.672	16.359	17.180	19.850
BCM-DSGE	36.149	5.417	0.457	1.662	2.642	5.750	7.420	9.305	18.858	37.171

Table 2.12: RMSYE of the SW-DSGE-ATSM and alternative term structure of interest rates model implementations (in BP) for the three EMU countries Germany, France and Italy

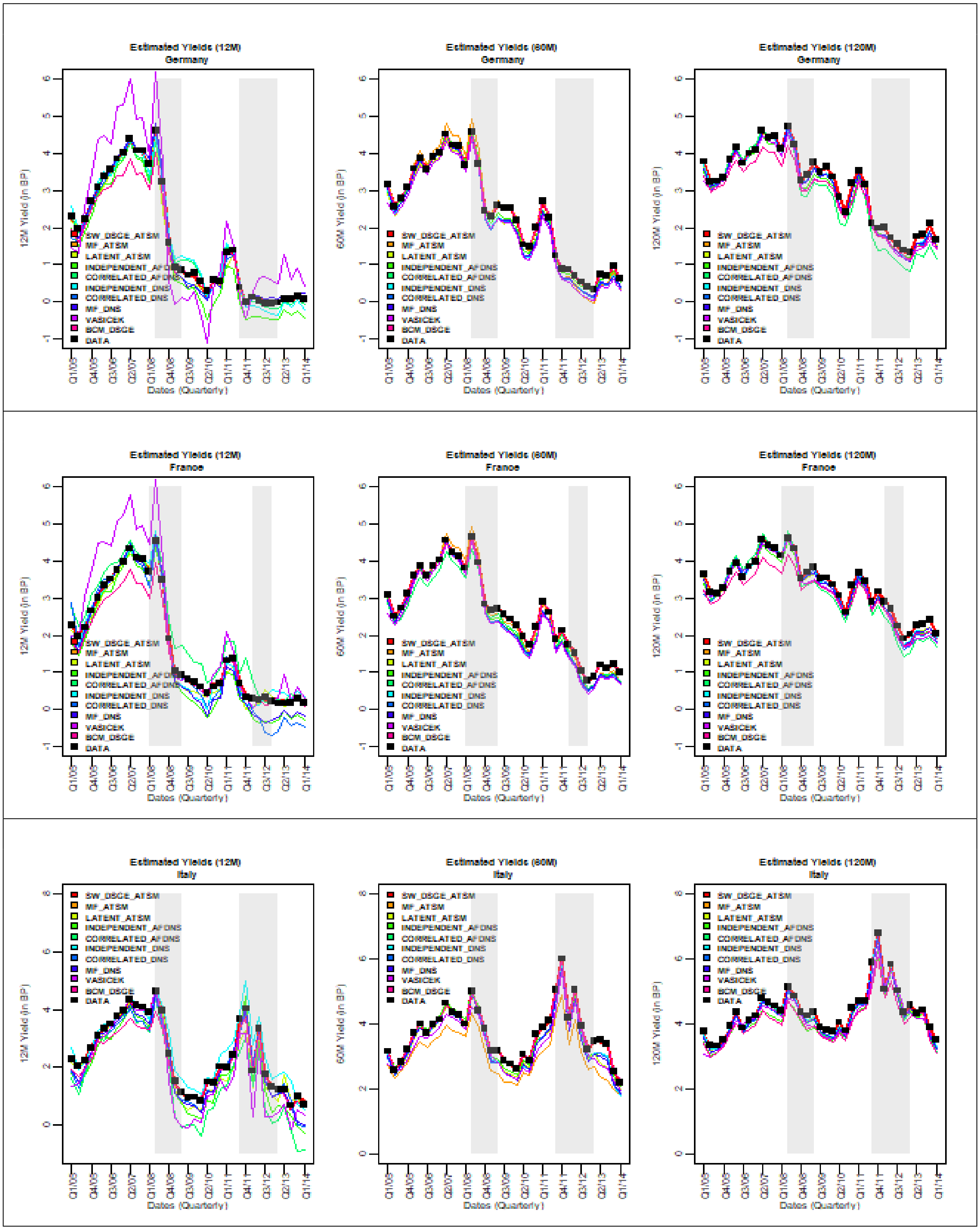


Table 2.13: Model implied yields of nine estimated term structure models compared to our SW-DSGE-ATSM for the three EMU countries Germany, France and Italy

2.5 Conclusion

In this chapter we combine the macroeconomic processes and their sectoral interrelationships implied by the Smets-Wouters economy with an ATSM induced recursive pricing scheme of rational and risk averse investors describing the developments in the sovereign bond markets for revealing the specific patterns of relationship between the real economy on the one hand and the financial sector in form of the crucial sovereign bond markets on the other hand. We do this in the specific time horizon ranging between the time where the Euro and its institutions become more settled and the moment the ECB initializes one of the largest monetary policy interventions in the history of modern western market economies. Our findings show how the decisive decision of the ECB in decreasing its main refinancing operations rate by 225 basis points from 3.75% in the beginning of November 2008 to 1.50% in March 2009 is mainly effected by subjects concerning risk premiums demanded by EMU investors. With focus on the Italian economy we further reveal that issues concerning term structure of interest rates and therefore the short, middle and long term financing conditions of the Italian government and the Italian economy as a whole become more dominant in the rational of Taylor-rule like decision supporting rules since the Lehman bankruptcy and the upcoming EMU's sovereign debt crisis in 2008 and 2009. For all three EMU countries regarded in this chapter, we find that sovereign bond markets become more sensitive on subjects related to government deficits and the sustainability of the governments overall debt. Here we find that the sensitivity is increasing with the government bond's time to maturity. Especially the middle to long term debt instruments react very sensitive on topics related to government spending activities. We further find that this reaction patterns become especially clear since the second half of 2009 revealing a form of an imperative of the financial markets (Krippner [2012]) imposing the conditions for the governments further spending activities and increasing the systemic pressure for stimulating self regulating as well as political forces for taking more sustainable future spending and debt paths. With respect to the negative sign of the monetary policy reaction to shocks disturbing monetary policy decisions and its induced measures we reveal a"whatever it takes" reaction pattern in credibly hedging the EMU against depressing exogenous influences. Regarding our model's complexity and the inherent modeling risk to which our findings of this chapter are exposed we implemented a larger number of both macroeconomic as well as term structure of interest rate models. The comparison to these alternative models points out the goodness-of-fit our modeling framework shows to both - the macroeconomic as well as the observed bond data - in this crucial phase of the EMU.

3. Common macroeconomic and term structure of interest rate dynamics in the EMU

3.1 Introduction

The term structure of interest rates is a central component in the well functioning of advanced and highly diversified economies. The term structure of interest rates determines the conditions under which economic decisions are made by households, firms and governments. From this point of view an understanding of the dynamics of the term structure and its impact on the macroeconomic development becomes crucial. With focus on the European Monetary Union (EMU) we should not narrow our view in describing and understanding only country specific phenomena. Here of special importance is an understanding of the relations and mechanisms determining the economic development of the EMU as a whole. How can country specific phenomena concerning the term structure of interest rates and the EMU's macroeconomic development be integrated in an overall modeling framework generating a deeper understanding of the EMU's underlying economic structure? In this chapter this crucial question is central for us. To find an answer we can access to a vast amount of work already done in the two strands of economic research related to this chapter. The first strand concerning the term structure of interest rates is separated into three parts, where the lines between these parts can become fuzzy. The first part implies the so called short rate models mainly driven by the early work done by Vasicek [1977] and Cox, Ingersoll and Ross [1985]. In these models the arbitrage-free term structure of interest rates in economic equilibrium is derived only by the short term rate. Connected with this first part of term structure related research are the more generally defined affine term structure models (ATSM) proposed by Duffie and Kan [1996] also concerning the short rate models. Here Ang and Piazzesi [2003] made an influential contribution in integrating macroeconomic factors into the term structure of interest rates modeling. Focal point in their modeling is a Taylor-rule like monetary policy decision rule combining (non-observable) latent term structure factors like the term structure's level and slope with observable macroeconomic factors, providing a bicausal description of the relations between the term structure of interest rates and the macroeconomic development. The third part of term structure research builds on the parsimonious and time-invariant Nelson-Siegel (NS) approach originally proposed by Nelson and Siegel [1987] and reformulated in a time-varying three factor fashion by Diebold and Li [2006] designated in the following as dynamical NS (DNS) model. On a hint by Filipovic [1999] in a more recent work Christensen, Diebold and Rudebusch [2011] reformulate the DNS in accordance to the ATSM defined by Duffie and Kan [1996] such that the line between the DNS and ATSM approaches becomes fuzzy too. In this chapter we use the DNS by Diebold and Li [2006] in its more reduced two factor form introduced and discussed in Diebold, Piazzesi and Rudebusch [2005] and Diebold, Li and Yue [2008]. Our motivation for embedding the DNS in our line of thought is twofold: From a methodological point of view the reduced DNS captures important phenomenological features of the ATSM by Ang and Piazzesi [2003], namely the interpretation of the (latent) term structure factors as level and slope factors. Keeping the modeling of a larger number of EMU countries in mind, from a more practical perspective - without loss of economic insights - the parsimonious character of the DNS makes the model's solving and computational tasks more tractable. Based on the country specific interest rate dynamics in this chapter the common Euro area term structure factors are extracted following the global yield curve approach outlined in Diebold, Li and Yue [2008]. In modeling the EMU wide aggregated macroeconomic development we use the large-scale open economy New Area-Wide Model (NAWM) proposed by Christoffel, Coenen and Warne [2008]. As outlined by the ECB [2016] the NAWM is used by the ECB for their EMU wide economic policy analysis and their macroeconomic staff projections and is therefore of high practical relevance for the monetary policy decisions made by the ECB. The NAWM is in direct line with the large-scale open economy models GEM (Global Economy Model by the IMF, cf. Bayoumi, Laxton and Pesenti [2004]) and the Federal Reserve Board's SIGMA model (cf. Erceg, Guerrieri and Gust [2006]). The EMU wide modeling characteristics and the practical importance of the NAWM for EMU concerning policy decisions are the reasons why we use the NAWM as our macroeconomic modeling framework. The NAWM implies the modeling of EMU related intertemporal decision problems of households and firms in a second generation New-Keynesian formulation close to the decision problems formulated by Smets and Wouters [2003, 2007] or Christiano, Eichenbaum and Evans [2005] with monopolistic competition in the intermediate and final goods sectors as well as the non-neutrality of money through price and wage stickiness where monetary policy measures are implemented by a Taylor rule like monetary policy decision rule. With respect to the causal effectiveness of the EMU's common term structure of interest rates, in this chapter we choose a unidirectional integration of the EMU's term structure into the macroeconomic NAWM framework. In concrete this means that the common EMU term structure directly effects the macroeconomic state variables of the NAWM, whereas the macroeconomic state variables do not have a direct effect on the EMU's term structure of interest rates. Integration of the term structure into a larger macroeconomic modeling framework is addressed by Andreasen, Fernandez-Villaverde and Rubio-Ramirez [2018], De Greave, Emiris and Wouters [2009], Rudebusch and Swanson [2008, 2012], Beakert, Cho and Moreno [2010], van Binsenberg, Fernandez-Villaverde, Koijen and RubioRamirez [2012] or Kliem and Meyer-Gohde [2017].

Our modeling approach outlined here differs to these works in that we are the first who use a modeling framework, where we combine a large-scale open-economy macroeconomic DSGE model with a multi-country term structure of interest rate model component in revealing the relations and mechanisms building the economic structure of the EMU - beside the U.S. and China the world's major economy - as a whole. With respect to the common factor term structure modeling, we are the first who apply such a common factor approach to the EMU for modeling and analyzing the common forces underlying the country specific term structure developments.

Focusing on the time horizon ranging from Q1/2005 to Q1/2014 - the phase where the Euro and the EMU institutions become more settled and before ECB started its expanded asset purchase program (EAPP) in Q4/2014 and the public sector purchase program (PSPP) in Q1/2015 - we find that especially in the month around the collapse of Lehman Brother's in September 2008 monetary policy unfolded shocks to the EMU strongly effecting the development of EMU's macroeconomic aggregates such as GDP, consumption, investment, exports and imports. The monetary policy decisions by the ECB are itself effected largely by shocks concerning the risk-premiums paid to EMU's investors. Looking at the decisive decision of the sharp decrease in the short term interest rate in Q4/2008 and Q1/2009, where the ECB's main refinancing operations rate fell by 225 basis points from 3.75% in the beginning of November 2008 to 1.50% in March 2009 reveals, that this decision is also effected by shocks induced by a changing slope of the EMU's common term structure of interest rates. Related to the common EMU term structure factors we further find, that the EMU's common level factor has no significant effect on the EMU's macroeconomic development, whereas EMU's macroeconomic responses to common slope factor shocks are of the same magnitude as conventional short term interest rate shocks induced by monetary policy decisions.

The following sections of this chapter are organized as follows: In part 3.2 we briefly outline the multi-country term structure modeling for revealing the country specific interest rate dynamics as well as the common EMU factors driven by a heterogeneous subset of EMU countries. We further outline our main empirical findings concerning the country specific and common EMU wide term structure dynamics in part 3.3. Part 3.4 describes the economic structure of the NAWM. Central here becomes the Taylor rule as the connecting point for us combining the area wide term structure of interest rates block with the large-scale open economy macroeconomic modeling block. Here we extend the NAWM's monetary policy rule by integrating EMU's common level and slope factors. Combined with the ECB controlled short term interest rate the extended monetary policy rule takes into account the whole maturity spectrum of the EMU's interest rates with respect to its level as well as to its steepness, where the last one reflects the interest rates differential between long and short term maturities. As a political consequence our extension endogenizes EMU's term structure development into the EMU's monetary policy decision framework. In part 3.5 we outline and discuss in detail our empirical findings of our integrated EMU wide economic modeling. The chapter closes with a summarizing conclusion in part 3.6.

3.2 EMU term structure of interest rates modeling framework

3.2.1 Country specific term structure modeling

According to Diebold, Li and Yue [2008] for every single country $i = 1, 2, ..., N$ the dynamics of the zero-coupon rates are based on the dynamical Nelson-Siegel (DNS) model proposed by Diebold and Li [2006]. To keep the model and its estimation simple we use for the country specific term structure of interest rate modeling the reduced two factor DNS applied by Diebold, Li and Yue [2008], methodologically discussed in Diebold, Piazzesi and Rudebusch [2005]. The reduced two factor DNS is defined by:

$$y_i(t,\tau) = l_{i,t} + s_{i,t}\left(\frac{1 - exp\,(-\lambda\tau)}{\lambda\tau}\right) \tag{3.1}$$

where $y_i(t,\tau)$ is the zero coupon rate of country $i = 1, 2, ..., N$ with time to maturity $\tau \geq 0$. As in Diebold and Li [2006] we keep the decay parameter λ in 3.1 constant with $\lambda = 0.0609$. The two latent term structure factors $l_{i,t}$ and $s_{i,t}$ collected in a 2×1 vector $\boldsymbol{f}_{i,t}^T = [l_{i,t}, s_{i,t}]$ are interpreted as the i-th country's term structure level and slope factor. $\boldsymbol{f}_{i,t}$ follows:

$$\boldsymbol{f}_{i,t} = \boldsymbol{\Gamma}_i \boldsymbol{f}_{i,t-1} + \boldsymbol{v}_{i,t} \quad \boldsymbol{v}_{i,t} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_{v,i}\right) \tag{3.2}$$

where we specify the country specific DNS in line to the independent DNS outlined in Diebold and Li [2006], and define $\boldsymbol{\Gamma}_i$ and $\boldsymbol{\Sigma}_{v,i}$ in 3.2 as diagonal, with $diag(\boldsymbol{\Gamma}_i) = \left[\gamma_i^l, \gamma_i^s\right]$ and $diag(\boldsymbol{\Sigma}_{v,i}) = \left[\sigma_{v,i}^l, \sigma_{v,i}^s\right]$.

3.2.2 EMU's common term structure of interest rates

Based on the country specific term structure factors $l_{i,t}$ and $s_{i,t}$, with countries $i = 1, 2, ..., N$ we can extract a (latent) common term structure of interest rates for the EMU. Similar to the country specific dynamics the dynamics of this EMU wide term structure is also described by a reduced two factor DNS:

$$Y^{EMU}(t,\tau) = L_t + S_t\left(\frac{1 - exp\,(-\lambda\tau)}{\lambda\tau}\right) \tag{3.3}$$

where $Y^{EMU}(t,\tau)$ is the extracted common EMU zero rate at time t with time to maturity $\tau \geq 0$. L_t and S_t are the EMU's common level and slope factors determined at time t . The dynamics of the latent EMU term structure factors L_t and S_t are modeled by two independent AR[1] processes:

$$\mathbf{F}_t = \mathbf{\Phi}\mathbf{F}_{t-1} + \boldsymbol{\epsilon}_t \tag{3.4}$$

where $\mathbf{F}_t^T = [L_t, S_t]$ and $\boldsymbol{\epsilon}_t^T = [\epsilon_t^l, \epsilon_t^s]$ with $\epsilon_t \sim N(\mathbf{0}, \mathbf{\Sigma}_\epsilon)$ contain the extracted latent EMU level and slope factors and their disturbance factors. The VAR's matrices $\mathbf{\Phi}$ and $\mathbf{\Sigma}_\epsilon$ are diagonal.For $\mathbf{\Sigma}_\epsilon$ we set$\mathbf{\Sigma}_\epsilon = \mathbf{I}_{2\times 2}$ with $\mathbf{I}_{2\times 2}$ as the 2×2 identity matrix. The relation between the EMU's common level and slope factor and the country specific factors is determined by:

$$\boldsymbol{f}_{i,t} = \boldsymbol{\alpha}_i + \mathbf{B}_i\mathbf{F}_t + \boldsymbol{\varepsilon}_{i,t} \tag{3.5}$$

where $\boldsymbol{\alpha}_i^T = \left[\alpha_i^l, \alpha_i^s\right]$ is the vector of constants and $\mathbf{B}_i$ is diagonal, with $diag(\mathbf{B}_i) = \left[\beta_i^l, \beta_i^s\right]$. The MA[1] error term $\boldsymbol{\varepsilon}_{i,t}^T = \left[\varepsilon_{i,t}^l, \varepsilon_{i,t}^s\right]$ follows:

$$\boldsymbol{\varepsilon}_{i,t} = \mathbf{\Psi}_i\boldsymbol{\varepsilon}_{i,t-1} + \boldsymbol{u}_{i,t} \tag{3.6}$$

with $\boldsymbol{u}_{i,t} \sim N(\mathbf{0}, \mathbf{\Sigma}_{u,i})$, where we define the dynamics of $\boldsymbol{\varepsilon}_{i,t}$ as two independent AR[1] processes defining $\mathbf{\Psi}_i$ and $\mathbf{\Sigma}_{u,i}$ as diagonal with $diag(\mathbf{\Psi}_i) = \left[\psi_i^l, \psi_i^s\right]$ and $diag(\mathbf{\Sigma}_{u,i}) = \left[\left(\sigma_{u,i}^l\right)^2, \left(\sigma_{u,i}^s\right)^2\right]$.

3.2.3 Extracting country specific and common EMU yield factors

Following Diebold, Li and Yue [2008] estimation of the common EMU term structure of interest rates is done in two steps. In the first step the country specific term structure factors $l_{i,t}$ and $s_{i,t}$ for $t = 1, 2, ..., T$ and $i = 1, 2, ..., N$ are estimated by MLE. For applying MLE we formulate the following state space system:

$$\boldsymbol{y}_{i,t} = \mathbf{A}\boldsymbol{f}_{i,t} + \boldsymbol{\vartheta}_{i,t} \quad \boldsymbol{\vartheta}_{i,t} \sim N(\mathbf{0}, \mathbf{\Sigma}_{\vartheta,i}) \tag{3.7}$$

$$\boldsymbol{f}_{i,t} = \mathbf{\Gamma}_i\boldsymbol{f}_{i,t-1} + \boldsymbol{v}_{i,t} \quad \boldsymbol{v}_{i,t} \sim N(\mathbf{0}, \mathbf{\Sigma}_{v,i}) \tag{3.8}$$

with the system's transition equation 3.8 similar to 3.2, where $\boldsymbol{y}_{i,t}^T = [y_i(t, 12), y_i(t, 24), ..., y_i(t, 120)]$ is the i-th country's $M \times 1$ vector of yields $y_i(t, \tau)$ observed at time t with the $M = 10$ time to maturities (in month) $\tau = 12, 24, ..., 120$. $\mathbf{A} = [\mathbf{e}, \mathbf{a}]$ is a $M \times 2$ matrix, where $\boldsymbol{e}$ is the $M \times 1$ unit vector and a is a $M \times 1$ vector with the j-th element defined as $a_j = \left[\frac{1-exp(-\lambda\tau_j)}{\lambda\tau_j}\right]$ for all $j = 1, 2, ..., M$. As mentioned above $\boldsymbol{f}_{i,t}^T = [l_{i,t}, s_{i,t}]$ is the vector of the i-th country's level and slope factor. $\mathbf{\Sigma}_{\vartheta,i}$ is diagonal and contains the squared measurement errors. Country specific estimation of the 14 parameters $\boldsymbol{\theta}^T \left[\sigma_{\vartheta,i,12}, \sigma_{\vartheta,i,24}, ..., \sigma_{\vartheta,i,120}, \gamma_i^l, \gamma_i^s, \sigma_{v,i}^l, \sigma_{v,i}^s\right]$ and the extraction of the factors $\boldsymbol{f}_t$ is done by maximizing the Kalman filter's likelihood with respect to the state space system in 3.7 and 3.8. In the second step the estimation of the $8N+2$ parameters $\boldsymbol{\theta}_{EMU}^T = \left[\phi^l, \phi^s, \alpha_1^l, \alpha_1^s, \beta_1^l, \beta_1^s, \psi_1^l, \psi_1^s, \sigma_{1,u}^l, \sigma_{1,u}^s, ..., \alpha_N^l, \alpha_N^s, \beta_N^l, \beta_N^s, \psi_N^l, \psi_N^s, \sigma_{N,u}^l, \sigma_{N,u}^s\right]$ and the extraction of the (latent) EMU factors $\mathbf{F}_t$ is done. Here we use a Gibbs sampler embeded in a Markov-Chain-Monte-Carlo (MCMC) procedure which we outline in more detail in Appendix B.2.

3.3 Empirical implications of the EMU term structure framework

3.3.1 Country specific term structure of interest rates

For extracting the common EMU yield curve we focus on the five EMU countries Germany, France, Netherlands, Italy and Spain. Based on government bond data of these five countries between 03/2005 and 02/2014 in a data preparing step previous to our further estimations we estimate the countries (zero-coupon) term structure of interest rates by applying the Nelson-Siegel-Svensson approach proposed by Svensson [1994, 1995]. Data details related the to the bond data and the data preparing step are given in Appendix A.2 and A.3. In Figure 3.1 we plot the (zero-coupon) term structure of interest rates for Germany, France, Italy and Spain. Based on the countries zero-coupon rates we estimate the reduced independent DNS getting the country specific DNS' latent level and slope factors for the five EMU countries. In Figure 3.2 we plot the country specific latent term structure factors of the reduced independent DNS. The estimated parameters of the DNS are listed in Appendix B.6. Until the beginning of the EMU recession phase in Q1/2008 all five countries show a slightly increasing level factor. The level factors of all five countries evolve more or less parallel to each other. Only Spain shows a larger (nearly) constant spread to the four other countries. With the beginning of the recession phase there is a diverging behavior in the dynamics of the country specific level factors, which accelerates in 2010. The divergence separates the group of five EMU countries into two subgroups consisting of Germany, France and the Netherlands and of Italy and Spain. For the first group the level factors show a declining pattern, whereas the second group show a large increase in their level factors reflecting their increased credit spreads. Interestingly in the recession phase beginning in Q3/2011 both groups in itself show a diverging behavior with larger spreads between their level factors. These spreads narrow after the end of the recession phase in Q1/2013. The country specific slope factors in Figure 3.2 show an increasing pattern with slope factors near zero, reflecting the flat EMU countries term structure of interest rates in 2008. As for the level factors, the slope factors evolve in this phase parallel to each other. And as for the level factors the Spanish slope factor implies a spread to the factors of the other four EMU countries. With the sharp decrease of the ECB's main refinancing operations rate from 3.25% to 1.00% between Q4/2008 and Q2/2009, the slope factors for all five countries sharply decrease, reflecting a steepening of the countries term structures. In the phase between the two recession phases of our data sample the slope factors increase but become more diverged since the beginning of 2011. The group of EMU countries separates similar to the level factors into the two subgroups of Germany, France and the Netherlands and Italy and Spain. In the beginning of 2012 the divergence of the countries slope factors accelerates, where compared to the first subgroup the slope factors of Italy and Spain sharply decreased. Similar to the level factors there is the interesting phenomena that the slope factors of both subgroup show in their respective group a diverging behavior.

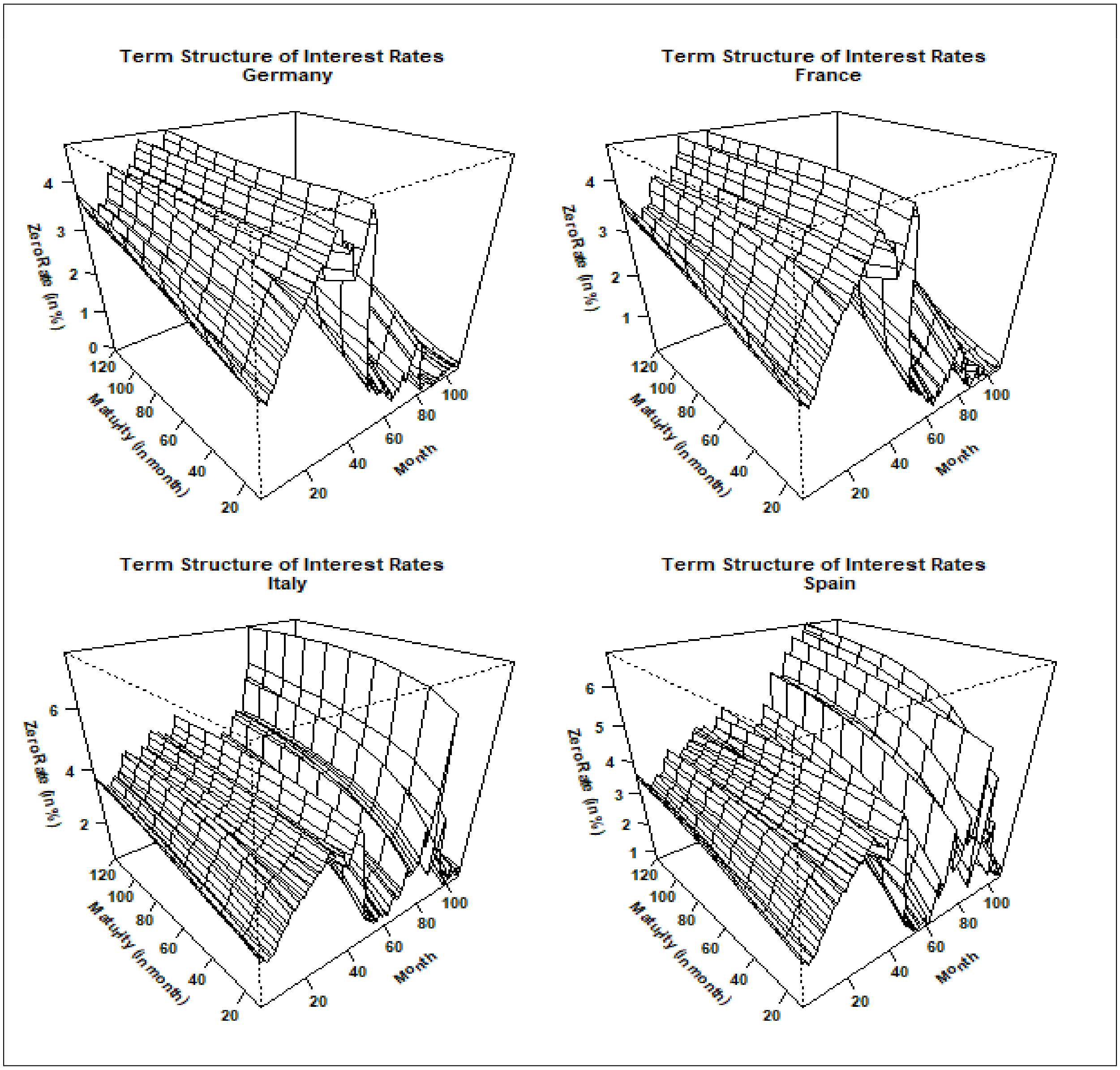


Table 3.1: Term structure of interest rates for Germany, France, Italy and Spain between 03/2005 and 02/2014

3.3.2 EMU's common term structure factors

Figure 3.3 shows the posterior mean of the common EMU term structure factors. Around the posterior means we lay the factor's 90% confidence intervals. For both factors these intervals are very narrow reflecting a high accuracy of the extracted factors. Obviously in their dynamics both factors strongly reflect the country specific patterns of the subgroup consisting Germany, France and the Netherlands. As for this subgroup the level factor

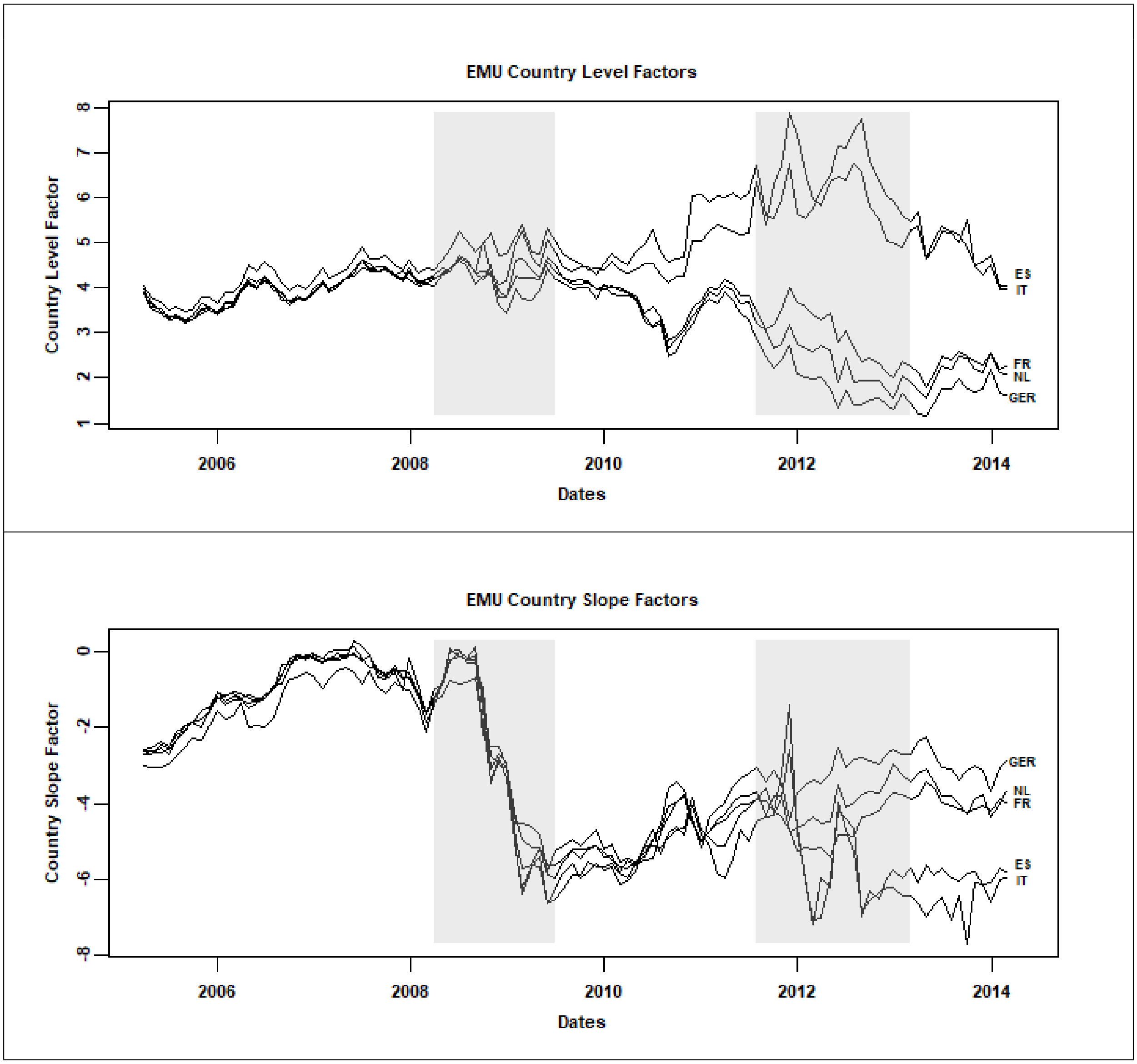


Table 3.2: Estimated level and slope factors of the country specific reduced independent DNS for Germany, France, Netherlands, Italy and Spain between 03/2005 and 02/2014.

increases until the end of the first EMU recession phase in Q2/2009. Thereafter the EMU level factor shows a declining pattern. The EMU's slope factor reflects the sharp decline of the country specific slope factors in reaction to the sharp decrease of the ECB's main refinancing operations rate beginning in Q4/2008. As for the slope factors of Germany, France and the Netherlands in the first half of 2010 the EMU's common slope factor starts increasing. Table 3.4 lists the estimated parameters of Equations 3.4, 3.5 and 3.6. Both the EMU common level and slope factor are in their dynamics highly autocorrelated reflected

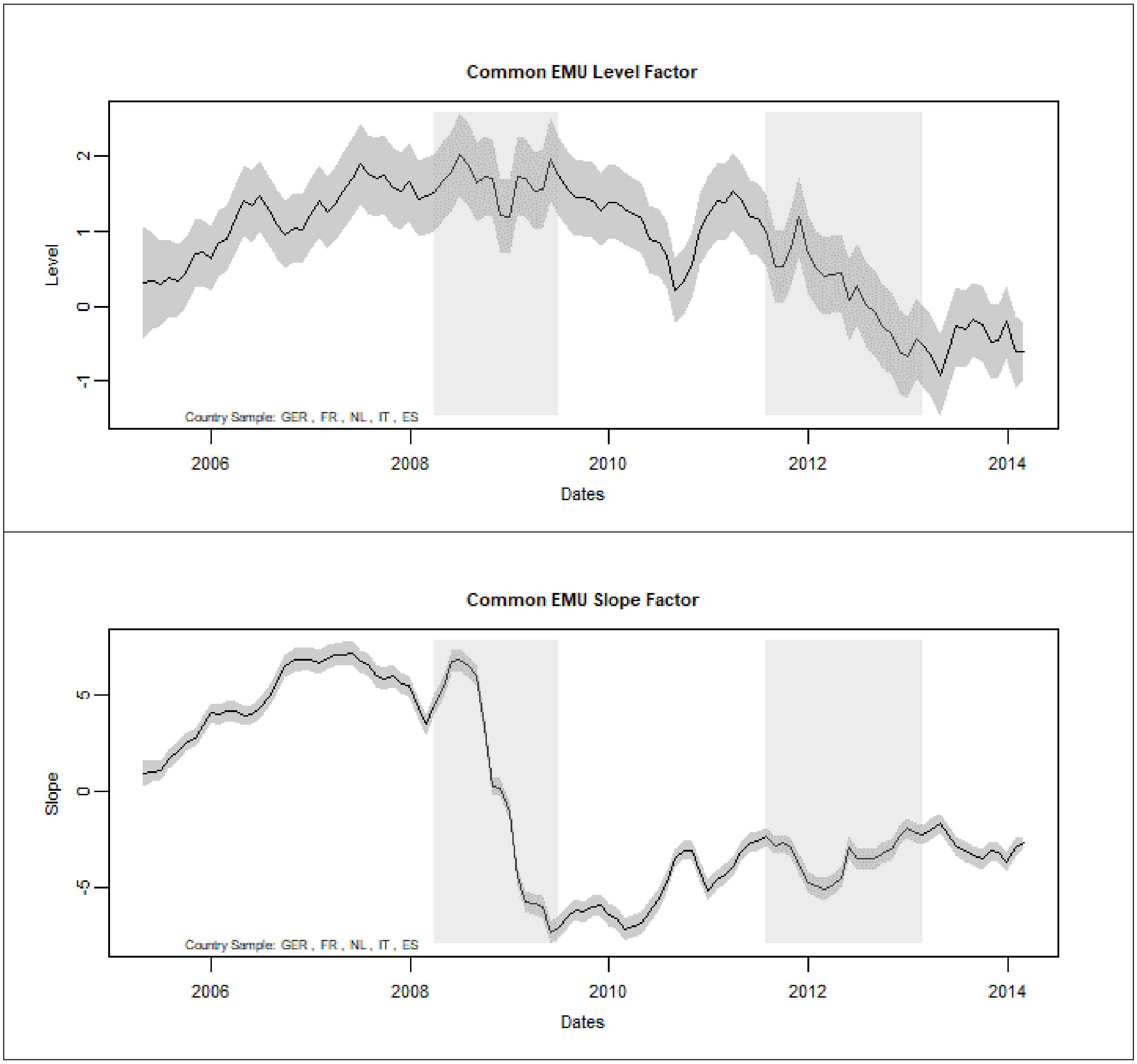


Table 3.3: Posterior mean of the common level and slope factor of Germany, France, Netherlands, Italy and Spain between 03/2005 and 02/2014. The shaded area around the mean reflects the factor estimate's [0.05, 0.95] confidence interval calculated from the posterior.

by the estimated values of above 0.9 of the autoregressive coefficients in 3.4. The country specific level and slope factors show a similar structure in their loadings β_i^l and β_i^s. For all country specific level and slope factors the loadings are positive and compared to Germany, France and the Netherlands are higher for Italy and Spain. The intercepts α_i^l and α_i^s in 3.5 reflect the systemic but EMU factor independent contributions to the country specific level and slope factors for Italy and Spain are in absolute terms even larger than those of

Common EMU level factor	Common EMU slope factor
$L_t = 0.904L_{t-1} + \epsilon_t^l$	$S_t = 0.978S_{t-1} + \epsilon_t^s$
(0.051)	(0.010)
Country specific level factors	
$l_{GER,t} = 2.529 + 0.936L_t + \epsilon_{GER,t}^l$	$\varepsilon_{GER,t}^l = 0.819\varepsilon_{GER,t-1}^l + 0.165u_{GER,t}^l$
(0.333) (0.230)	(0.087) (0.045)
$l_{FR,t} = 2.896 + 0.731L_t + \epsilon_{FR,t}^l$	$\varepsilon_{FR,t}^l = 0.796\varepsilon_{FR,t-1}^l + 0.102u_{FR,t}^l$
(0.276) (0.224)	(0.110) (0.027)
$l_{NL,t} = 2.731 + 0.902L_t + \epsilon_{NL,t}^l$	$\varepsilon_{NL,t}^l = 0.786\varepsilon_{NL,t-1}^l + 0.149u_{NL,t}^l$
(0.323) (0.224)	(0.107) (0.042)
$l_{IT,t} = 3.417 + 1.320L_t + \epsilon_{IT,t}^l$	$\varepsilon_{IT,t}^l = 0.899\varepsilon_{IT,t-1}^l + 0.387u_{IT,t}^l$
(0.436) (0.157)	(0.040) (0.098)
$l_{ES,t} = 3.415 + 1.337L_t + \epsilon_{ES,t}^l$	$\varepsilon_{ES,t}^l = 0.915\varepsilon_{ES,t-1}^l + 0.402u_{ES,t}^l$
(0.439) (0.153)	(0.103) (0.051)
Country specific slope factors	
$s_{GER,t} = -2.525 + 0.360S_t + \epsilon_{GER,t}^s$	$\varepsilon_{GER,t}^s = 0.748\varepsilon_{GER,t-1}^s + 0.111u_{GER,t}^s$
(0.079) (0.020)	(0.042) (0.014)
$s_{FR,t} = -2.924 + 0.374S_t + \epsilon_{FR,t}^s$	$\varepsilon_{FR,t}^s = 0.682\varepsilon_{FR,t-1}^s + 0.066u_{FR,t}^s$
(0.087) (0.020)	(0.060) (0.009)
$s_{NL,t} = -2.893 + 0.388S_t + \epsilon_{NL,t}^s$	$\varepsilon_{NL,t}^s = 0.588\varepsilon_{NL,t-1}^s + 0.084u_{NL,t}^s$
(0.088) (0.021)	(0.061) (0.013)
$s_{IT,t} = -3.430 + 0.484S_t + \epsilon_{IT,t}^s$	$\varepsilon_{IT,t}^s = 0.791\varepsilon_{IT,t-1}^s + 0.397u_{IT,t}^s$
(0.087) (0.028)	(0.034) (0.041)
$s_{ES,t} = -3.292 + 0.498S_t + \epsilon_{ES,t}^s$	$\varepsilon_{ES,t}^s = 0.748\varepsilon_{ES,t-1}^s + 0.323u_{ES,t}^s$
(0.091) (0.028)	(0.034) (0.010)

Table 3.4: Posterior means of the common term structure of interest rates model's parameters for Germany, France, Netherlands, Italy and Spain between 03/2005 - 02/2014. The posterior standard deviations of the parameter estimates are listed in parenthesis.

Germany, France and the Netherlands. Interestingly compared to Germany, France and the Netherlands the idiosyncratic disturbance terms $\varepsilon_{i,t}^{l}$ and $\varepsilon_{i,t}^{s}$ in 3.5 and 3.6 show for Italy and Spain at least a three times higher volatility measured by the estimates of $\left(\sigma_{u,i}^{l}\right)^2$ and $\left(\sigma_{u,i}^{s}\right)^2$. In Figure 3.5 we plot the variance decomposition of the country specific yields with maturities $\tau = 12, 24, ..., 120$ month. Variance decomposition is calculated by regressing the EMU factor's variance on the variance of the country specific factors, determining the share of the country factors variance explained by the EMU factors. Figure 3.5 makes clear that for all five countries the extracted common EMU factors systematically explain larger portions of the country specific yield dynamics. For short to middle term maturities ranging between 12 to 60 month the EMU factors are responsible for a share of at least 50% in the variation of country specific yields. With longer maturities for all countries this share is decreasing but still stays above 40% . The dominance of the subgroup with Germany, France and

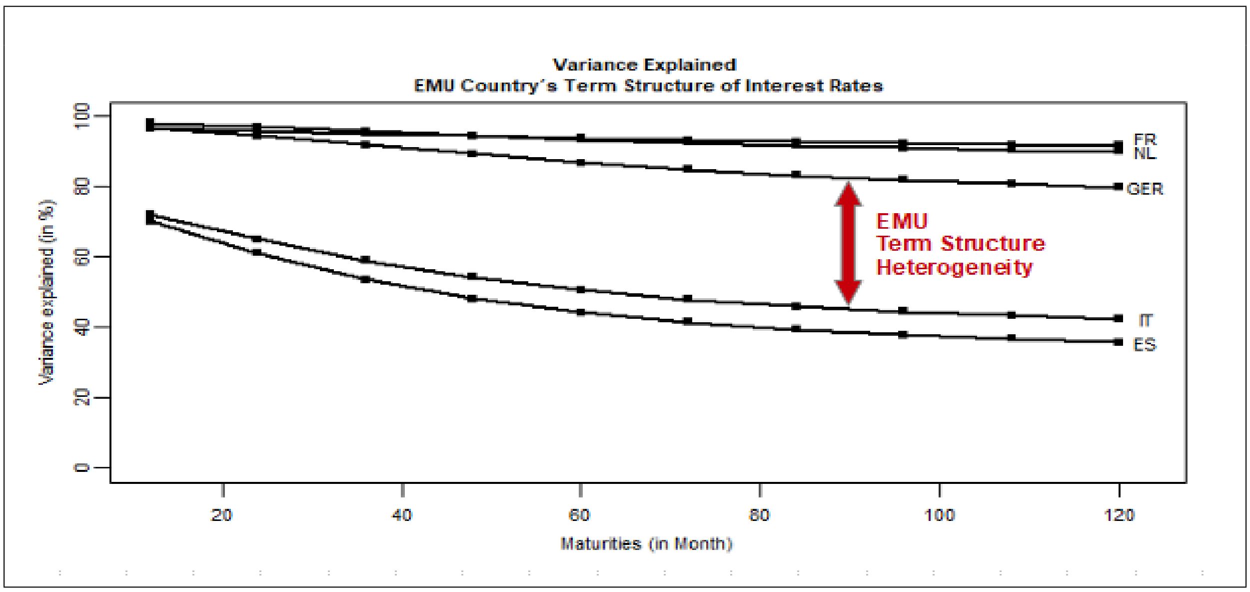


Table 3.5: Variance decomposition of the term structure of interest rates for Germany, France, Netherlands, Italy and Spain. (Decomposed variances are calculated with the posterior means of the MCMC generated parameter distributions).

the Netherlands is even enforced by the variance decomposition of the country specific term structure of interest rates plotted in Figure 3.5. Here too there is a separation of the five EMU countries into two subgroups consisting of Germany, France and the Netherlands and of Italy and Spain. For the first subgroup the share of variance explained by the common EMU factors - especially for long term yields - is considerably higher than for Italy and Spain. For Germany, France and the Netherlands the share of variation explained by the extracted EMU factors is above 80% over the whole maturity spectrum. Compared to this the share of explained variance of Spanish long term yields is as half as the share of explained long term yield variation of the first subgroup.

3.3.3 Term structure slope factor and macroeconomic recessions

Based on the expectations hypothesis Estrella and Trubin [2006] argue that expected economic recession phases are related to lower expected inflation rates and a more expansive future monetary policy with lower central bank controlled short term rates. According to the (pure) expectations hypothesis discussed in detail in Campbell, Lo and MacKinlay [1997] and in Cox, Ingersoll and Ross [1981], where the return of an $n > 1$ periods bond is determined by:

$$\begin{aligned} (1 + y(t,n))^n &= (1 + y(t,1))\, \mathbb{E}_t \left[(1 + y(t+1,1))\,(1 + y(t+2,1)...\,(1 + y(t+n,1)))\right] \\ &= (1 + y(t,1))\, \mathbb{E}_t \left[\prod_{i=1}^{n} (1 + y(t+i,1))\right] \end{aligned} \tag{3.9}$$

stays that lower expected future short term rates caused by an expected recessive economic environment and an expansive future monetary policy lead to a reduction of the slope of the term structure of interest rates expressed in a narrowing of the long-short yield spread. For our macroeconomic analysis in the following sections we use a monetary policy rule in which a common EMU measure of the slope of the term structure of interest rate is integrated. To motivate this integration in Figure 3.6 we plot the 12 month ahead recession probabilities $P\left(I_{GER}\left(t+12\right) = 1|\Delta_y\left(\hat{s}_{GER,t}\right)\right)$ indicating a recession phase in $t + 12$ conditional to the extracted DNS slope factor $\hat{s}_{GER,t}$ on a longer horizon ranging from 09/1972 to 02/2014. Because of the data availability we calculated $P\left(I_{GER}\left(t+12\right) = 1|\Delta_y\left(\hat{s}_{GER,t}\right)\right)$ only for Germany, for which we used the monthly historic term structure of interest rate data provided by the German Bundesbank. Obviously in six of the eight recession phases in Figure 3.6 the recession probability conditional to $\hat{s}_{GER,t}$ is above 50.0% revealing the mentioned informational content of $\hat{s}_{GER,t}$ with respect to the future macroeconomic development. Details about determining $P\left(I_{GER}\left(t+12\right) = 1|\Delta_y\left(\hat{s}_{GER,t}\right)\right)$ based on a Probit model are outlined in Appendix B.3.

3.4 EMU term structure embedded in the EMU's macroeconomy

To model the EMU wide aggregated macroeconomic development we use the large-scale open economy New Area-Wide Model (NAWM) proposed by Christoffel, Coenen and Warne [2008]. As outlined by the ECB [2016] the NAWM is of high practical relevance for the monetary policy decisions made by the ECB. The NAWM is currently used by the ECB for their EMU wide economic policy analysis and their macroeconomic staff projections. The EMU wide modeling characteristics and the practical importance of the NAWM for the EMU are the reasons we use the macroeconomic modeling framework.

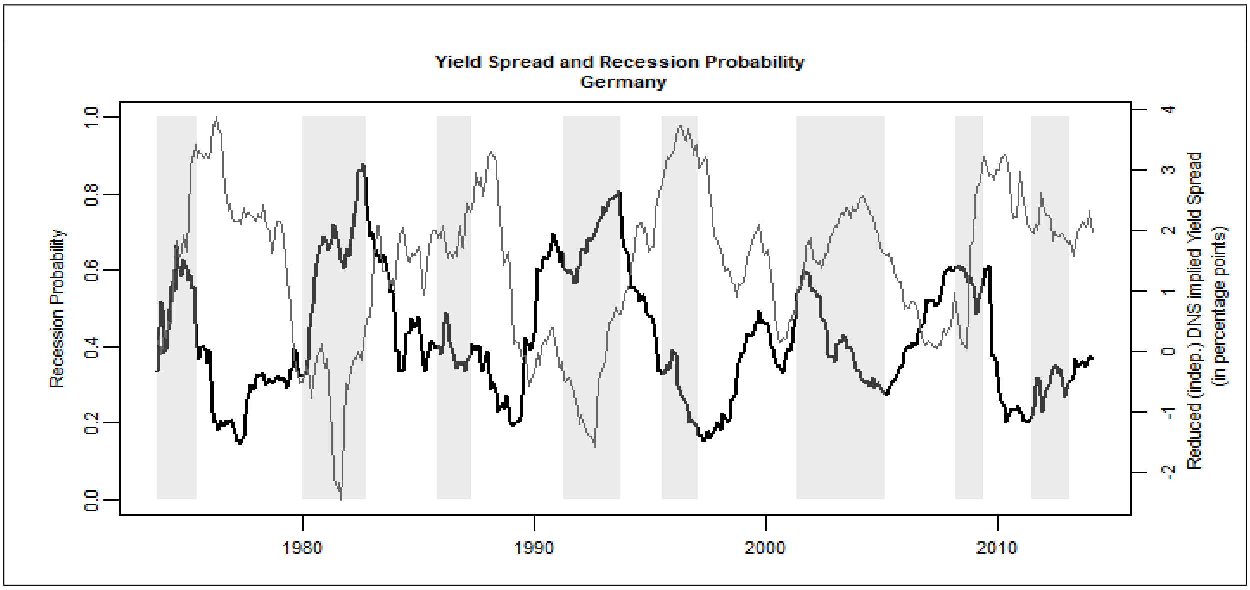


Table 3.6: Recession probabilities (black) conditional to the term spread (gray) determined by the slope factor $\hat{s}_{GER,t}$ extracted by the reduced independent DNS for Germany between 09/1972 to 02/2014.

3.4.1 EMU wide macroeconomic modeling framework

In this section we outline the log-linearized modeling equations of the different sectors of the aggregated EMU economy of the NAWM.

3.4.1.1 Households

The NAWM households maximize their expected lifetime utility with respect to their budget constraint at a given time t. Therefore they have to optimally allocate their recources into consumption $\hat{c}_t$, investment $\hat{i}_t$ and domestic and foreign government bonds B_{t+1} and B^*_{t+1} respectively. Choosing the resources allocated into the purchase of investment goods $\hat{i}_t$ directly determines the periods $t+1$ accumulated capital stock k_{t+1}. Households also have to decide about u_t, the intensity with which the economy's capital stock is utilized in period t . The FOC in log-linearized form for the households optimal choice of allocation are:

$$\begin{aligned}\hat{c}_t = &\frac{1}{(1+\kappa g_z^{-1})}\mathbb{E}_t\left[\hat{c}_{t+1}\right] + \frac{\kappa g_z^{-1}}{(1+\kappa g_z^{-1})}\hat{c}_{t-1} - \frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})}\left(\hat{r}_t - \mathbb{E}_t\left[\hat{\pi}_{c,t+1}\right] + \hat{\epsilon}_t^{RP}\right)\\ &- \frac{1}{(1+\kappa g_z^{-1})}\left(\mathbb{E}_t\left[\hat{g}_{z,t+1}\right] - \kappa g_z^{-1} g_{z,t}\right) + \frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})(1+\tau^c)}\left(\mathbb{E}_t\left[\hat{\tau}_{t+1}^c\right] - \hat{\tau}_t^c\right)\\ &- \frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})}\left(\mathbb{E}_t\left[\hat{\epsilon}_{t+1}^c\right] - \hat{\epsilon}_t^c\right)\end{aligned} \tag{3.10}$$

where κ signals the households degree of external habit formation, g_z is the NAWM's steady-state annual growth rate of labor productivity, whereas $g_{z,t} = z_t/z_{t-1}$ is a shock variable representing deviations from the steady-state labor productivity growth rate coming from changes in the long-lasting permanent technology shock z_t affecting the economy's labor productivity. $\hat{\pi}_{c,t}$ with $\hat{\pi}_{c,t} = ln\left(\Pi_{c,t}/\bar{P}i\right)$ where $\Pi_{c,t} = P_{c,t}/P_{c,t-1}$ and τ_t^c are the time t inflation rate for consumption goods and the consumption tax on purchases of consumption goods respectively. $\bar{\Pi}$ is the ECB's long-term inflation target. $\hat{\epsilon}_t^r$ and $\hat{\epsilon}_t^{RP}$ denoting the economy's monetary policy and risk premium shocks. As in the economy described by Smets and Wouters [2003, 2007] last one drives the wedge between the interest rate controlled by the ECB and the return the households require for buying domestic government bonds. Investment $\hat{i}_t$ in log-linearized form is given by:

$$\hat{i}_t = \frac{\beta}{(1+\beta)}\,\mathbb{E}_t\left[\hat{i}_{t+1}\right] + \frac{\beta}{(1+\beta)}\hat{i}_{t-1} + \frac{1}{\gamma_i g_z^2\,(1+\beta)}\left(\hat{Q}_t - \hat{p}_{i,t} + \hat{\epsilon}_t^i\right) + \frac{1}{(1+\beta)}\left(\beta\,\mathbb{E}_t\left[\hat{g}_{z,t+1}\right] - \hat{g}_{z,t}\right) \tag{3.11}$$

where $\hat{p}_{i,t} = log\left(p_{i,t}/p_i\right)$ is the (log) deviation of the price index of investment goods expressed in relative terms to the price index of private consumption goods $p_{i,t} = P_{i,t}/P_{c,t}$ from its steady-state value p_i. β denotes the household's discount factor and Tobin's $\hat{Q}_t$ is interpreted as the households marginal utility of a unit of investment goods. Last two state variables are determined by:

$$\begin{aligned}\hat{Q}_t = &\frac{\beta\,(1-\delta)}{g_z}\,\mathbb{E}_t\left[\hat{Q}_{t+1}\right] + \mathbb{E}_t\left[\hat{\lambda}_{t+1}\right] - \hat{\lambda}_t - \mathbb{E}_t\left[\hat{g}_{z,t+1}\right] - \frac{\beta\left(1-\tau^k\right)r_k}{g_z\left(1-\tau^k\right)}\,\mathbb{E}_t\left[\hat{\tau}_{t+1}^k\right] \\ &- \frac{\beta\left(1-\tau^k\right)r_k}{g_z}\,\mathbb{E}_t\left[\hat{r}_{k,t+1}\right] + \frac{\beta\delta p_i}{g_z}\left(\mathbb{E}_t\left[\hat{\tau}_{t+1}^k \hat{p}_{i,t+1}\right] + \mathbb{E}_t\left[\hat{\tau}_{t+1}^k\right]\right)\end{aligned} \tag{3.12}$$

and

$$\hat{p}_{i,t} = \hat{Q}_t + \hat{\epsilon}_t^i + \gamma_i g_z^2\left[\beta\left(\mathbb{E}_t\left[\hat{i}_{t+1}\right] - \hat{i}_t\right) - \left(\hat{i}_t - \hat{i}_{t-1}\right) + \beta\,\mathbb{E}_t\left[\hat{g}_{z,t+1}\right] - \hat{g}_{z,t}\right] \tag{3.13}$$

where $\hat{\tau}_t^k$ and τ^k are the tax and the tax rate levied on the household's capital income and $\gamma_i > 0$ in 3.11 and 3.13 is a parameter for the investment adjustment costs reflecting the costs for inserting the investments at time t into the economy's capital stock $\hat{k}_{t+1}$ with $\hat{k}_t = ln\left(k_t/k\right)$ where $k_t = K_t/z_{t-1}$. A larger value of γ_i indicates higher investment adjustment costs. Capital accumulation $\hat{k}_{t+1}$ in log-linearized form is determined by:

$$\hat{k}_{t+1} = (1-\delta)\,g_z^{-1}\hat{k}_t - (1-\delta)\,g_z^{-1}\hat{g}_{z,t} + \left[1-(1-\delta)\,g_z^{-1}\right]\left(\hat{i}_t + \hat{\epsilon}_t^i\right) \tag{3.14}$$

where δ is the economy's depreciation rate. The capital stock generates a rental rate $\hat{r}_{k,t}$ for the effective capital services rented to firms, which is described by:

$$\hat{r}_{k,t} = \frac{\gamma_{u,2}}{\gamma_{u,1}}\hat{u}_t + \hat{p}_{i,t} \tag{3.15}$$

where $\gamma_{u,1}, \gamma_{u,2} > 0$ are parameters determining the costs of varying the intensity of the economy's capital stock utilization $\hat{u}_t$. $\hat{r}_{k,t} = ln\left(r_{k,t}/r_k\right)$ with $r_{k,t} = R_{k,t}/P_{c,t}$. Analogue to Tobin's Q $\hat{\lambda}_t$ represents the households marginal utility of a unit of consumption goods, which are part of the remaining three log-linearized FOC's of the NAWM's household sector:

$$\hat{\lambda}_t = \frac{1}{(1-\kappa g_z^{-1})}\hat{c}_t + \frac{\kappa g_z^{-1}}{(1-\kappa g_z^{-1})}\hat{c}_{t-1} - \frac{\kappa g_z^{-1}}{(1-\kappa g_z^{-1})}\hat{g}_{z,t} - \frac{1}{(1+\tau^c)}\hat{\tau}_t^c + \hat{\epsilon}_t^c \tag{3.16}$$

$$\mathbb{E}_t\left[\hat{\lambda}_{t+1}\right] - \hat{\lambda}_t - \mathbb{E}_t\left[\hat{g}_{z,t+1}\right] + \hat{r}_t - \mathbb{E}_t\left[\hat{\pi}_{c,t+1}\right] + \hat{\epsilon}_t^{RP} = 0 \tag{3.17}$$

and

$$\begin{aligned}\mathbb{E}_t\left[\hat{\lambda}_{t+1}\right] - \hat{\lambda}_t - \mathbb{E}_t\left[\hat{g}_{z,t+1}\right] + \hat{r}_t^* - \mathbb{E}_t\left[\hat{\pi}_{c,t+1}\right] + \mathbb{E}_t\left[\hat{s}_{t+1}\right] - \hat{s}_t \\ + \mathbb{E}_t\left[\hat{\pi}_{y,t+1} - \hat{\pi}_{y,t+1}^*\right] - \gamma_{B^*}\hat{s}_{B^*,t+1} - \hat{\epsilon}_t^{RP^*} = 0\end{aligned} \tag{3.18}$$

where $\hat{s}_t$ is the real exchange rate (expressed as units of the domestic currency per unit of foreign currency). $\hat{s}_t = ln\left(s_t/s\right)$ with $s_t = S_t P_{y,t}^*/P_{y,t}$, where $P_{y,t}$ and $P_{y,t}^*$ denote the domestic and foreign output deflators. $\hat{s}_{B^*,t+1}$ with $\hat{s}_{B^*,t+1} = S_t P_{y,t}^*/(P_{y,t} y_t)$ reflects the EMU wide (net) holding ratio of internationally traded foreign bonds B_{t+1}^* (corrected by domestic government bonds hold by foreign investors) in domestic currency relative to the EMU's nominal output measured at time t. A negative sign of $\hat{s}_{B^*,t+1}$ stands for a (net) creditor and a positive sign for a (net) debtor position of the EMU. $\hat{\pi}_{y,t} = ln\left(\Pi_{y,t}/\Pi_y\right)$ and $\hat{\pi}_{y,t}^* = ln\left(\Pi_{y,t}^*/\Pi_y^*\right)$ with $\Pi_{y,t} = P_{y,t}/P_{y,t-1}$ and $\Pi_{y,t}^* = P_{y,t}^*/P_{y,t-1}^*$ are the (log) deviations of domestic and foreign gross inflation rates from its steady-state values respectively. In the NAWM the steady-state values determined by the long-run central bank's inflation target $\Pi_y = \Pi_y^* = \bar{\Pi}$. ϵ_t^c is a consumption related shock variable. Households in the NAWM faces an external financial intermediation premium, which is positively driven by the parameter $\gamma_{B^*>0}$ and the external risk premium shock $\hat{\epsilon}_t^{RP^*}$. Combining $\hat{r}_t$ and $\hat{r}_t^*$, the nominal exchange rate $\hat{s}_t$, the EMU and foreign world economy inflation $\hat{\pi}_{y,t}$ and $\hat{\pi}_{y,t}^*$ as well as EMU's net holding ratio of foreign bonds $\hat{s}_{B^*,t+1}$ and the shock variables $\hat{\epsilon}_t^{RP}$ and $\hat{\epsilon}_t^{RP^*}$ respectively, the NAWM household sector FOC's deliver the risk-adjusted uncovered interest parity condition:

$$\hat{r}_t - \hat{r}_t^* + \hat{\epsilon}_t^{RP} = \mathbb{E}_t\left[\hat{s}_{t+1}\right] - \hat{s}_t + \mathbb{E}_t\left[\hat{\pi}_{y,t+1} - \hat{\pi}_{y,t+1}^*\right] - \gamma_{B^*}\hat{s}_{B^*,t+1} - \hat{\epsilon}_t^{RP^*} \tag{3.19}$$

3.4.1.2 Labor market

Similar to the economy proposed by Smets and Wouters [2003, 2007] the wage-setting in the NAWM is described by using the Calvo scheme developed by Calvo [1983] with partial indexation and staggered wages, where $(1-\xi_w)$ with $0 \leq \xi_w \leq 1$ is the probability for the labor unions to actively readjust their members wages in a given period t. Combining the Calvo wage setting with the FOC's of the NAWM household sector leads to the loglinear wage equation:

$$
\begin{aligned}
\hat{w}_t = & \frac{\beta}{(1+\beta)} \mathbb{E}_t \left[\hat{w}_{t+1}\right] + \frac{1}{(1+\beta)} \hat{w}_{t-1} \\
& + \frac{\beta}{(1+\beta)} \mathbb{E}_t \left[\hat{\pi}_{c,t+1}\right] - \frac{(1+\beta\chi_w)}{(1+\beta)} \hat{\pi}_{c,t} + \frac{\chi_w}{(1+\beta)} \hat{\pi}_{c,t-1} - \frac{\beta(1-\chi_w)}{(1+\beta)} \mathbb{E}_t \left[\hat{\bar{\pi}}_{c,t+1}\right] \\
& + \frac{(1-\chi_w)}{(1+\beta)} \hat{\bar{\pi}}_{c,t} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1-\beta)\xi_w \left(1+\varphi^w(\varphi^w-1)^{-1}\zeta\right)} \left(\hat{w}_t^{\tau} - m\hat{r}s_t - \hat{\varphi}_t^w\right)
\end{aligned}
\tag{3.20}
$$

where:

$$
\hat{w}_t^{\tau} = \hat{w}_t - \frac{\left(\hat{\tau}_t^N + \hat{\tau}_t^{w_h}\right)}{\left(1 - \hat{\tau}_t^N + \hat{\tau}_t^{w_h}\right)} \tag{3.21}
$$

and

$$
m\hat{s}r_t = \zeta \hat{N}_t - \hat{\lambda}_t + \hat{\epsilon}_t^N \tag{3.22}
$$

are the tax-adjusted real wage and the household's marginal rate of substitution between consumption and leisure respectively. $\hat{w}_t = ln\,(w_t/w)$ is the real wages (log) deviation from its steady-state with $w_t = W_t/(z_t P_{c,t})$. ζ defines the inverse of the Frisch elasticity of labor supply and φ^w denotes the wage mark-up. τ_t^N and $\tau_t^{w_h}$ the tax on wage income as well as the household's wage income contribution to social security programs.$(1-\chi_w)$ with $\chi_w \leq 1$ are the weighting factor of the ECB's current and future inflation path π_{t+s} in the ξ_w probability non-active union's passive wage setting with $s = 1, ..., k$ for some $k \geq 1$. $\hat{\pi}_{c,t} = ln\left(\Pi_{c,t}/\bar{\Pi}\right)$ represents the (log) deviation of the observed quarter-on-quarter consumer price inflation $\Pi_{c,t} = P_{c,t}/P_{c,t-1}$ from the ECB's long-term inflation target $\bar{\Pi}$ in the economy's steady-state. $\hat{N}_t$ is the economy's state variable indicating the number of hours worked. $\hat{\varphi}_t^w$ and $\hat{\epsilon}_t^N$ denote the NAWM's wage mark-up and the labor supply shocks.

In the NAWM the number of hours worked $\hat{N}_t$ is related to total employment $\hat{E}_t$ by:

$$
\hat{E}_t = \frac{\beta}{(1+\beta)} \mathbb{E}_t \left[\hat{E}_{t+1}\right] + \frac{\beta}{(1+\beta)} \hat{E}_{t-1} + \frac{(1+\beta\xi_E)(1-\xi_E)}{(1+\beta)\xi_E} \left(\hat{N}_t - \hat{E}_t\right) \tag{3.23}
$$

where ξ_E is a measure of the sensitivity of employment $\hat{E}_t$ with respect to hours worked $\hat{N}_t$.

3.4.1.3 Firms

Domestic and foreign intermediate-goods producing firms

The goods producing sector of the NAWM is organized in an intermediate and a final goods sector. In the intermediate goods sector the firms produce differentiated intermediate goods for domestic and foreign markets. Beside the domestic producers there are foreign firms also producing differentiated intermediate goods and selling them in the markets of the EMU.

The domestic intermediate-good producers maximize their profits subject to their increasing-returns-to-scale Cobb-Douglas production technology in monopolistically competitive markets. Price setting of the domestic intermediate-goods producer for selling in the domestic and foreign market as well as the foreign firms selling in the domestic EMU market follow the Calvo scheme with indexation and staggered prices. The log-linearized Cobb-Douglas production technology is:

$$\hat{y}_t = \left(1 + \psi y^{-1}\right) \left[\alpha \left(\hat{k}^s_t - \hat{g}_{z,t}\right) + (1-\alpha)\, \hat{N}_t + \hat{\epsilon}_t\right] \tag{3.24}$$

where α indicates the capital share used in the domestic intermediate-goods production process. ψ determines the fixed costs of the domestic intermediate-goods producing firms. $\hat{y}_t = ln\,(y_t/y)$ denotes the (log) deviation of the real output $y_t = Y_t/z_t$ adjusted by the NAWM economy's productivity level from its steady-state value y. $\hat{k}^s_t$ stands for the capital stock that directly serves in the production process of the economy. $\hat{\epsilon}_t$ is a further technology shock affecting the economy's total factor productivity. The domestic intermediate-goods firm's profit maximization implies the following two additional equations with respect to the capital stock's rental rate $\hat{r}_{k,t}$ and the firm's marginal costs $\hat{mc}_t$:

$$\hat{r}_{k,t} = \hat{N}_t + \left(1 + \tau^{w_f}\right)^{-1} \hat{\tau}^{w_f}_t + \hat{w}_t + \hat{k}^s_t + \hat{g}_{z,t} \tag{3.25}$$

$$\hat{mc}_t = \alpha \hat{r}_{k,t} + (1-\alpha)\left[\left(1 + \tau^{w_f}\right)^{-1} \hat{\tau}^{w_f}_t + \hat{w}_t\right] - \hat{\epsilon}_t \tag{3.26}$$

where τ^{w_f} is the firm's contributions rate on wage income to social security programs. From the optimal price setting in the domestic market for intermediate goods according to the Calvo scheme the NAWM implies the domestic price forward-backward looking New-Keynesian Phillips curve:

$$\begin{aligned}\hat{\pi}_{h,t} - \hat{\bar{\pi}}_t = {} & \frac{\beta}{(1+\beta\chi_h)}\, \mathbb{E}_t\left[\hat{\pi}_{h,t+1} - \hat{\bar{\pi}}_{t+1}\right] + \frac{\chi_h}{(1+\beta\chi_h)}\left(\hat{\pi}_{h,t-1} - \hat{\bar{\pi}}_t\right) \\ & + \frac{\beta\chi_h}{(1+\beta\chi_h)}\left(\mathbb{E}_t\left[\hat{\bar{\pi}}_{t+1}\right] - \hat{\bar{\pi}}_t\right) + \frac{(1-\beta\xi_h)(1-\xi_h)}{\xi_h\,(1+\beta\xi_h)}\left(\hat{mc}^h_t + \hat{\varphi}^h_t\right)\end{aligned} \tag{3.27}$$

where:

$$\hat{mc}^h_t = \hat{mc}_t - \hat{p}_{h,t} \tag{3.28}$$

are the average real marginal costs of the domestic intermediate-goods producers selling in the domestic market with $mc_t = MC_t/P_{c,t}$ and $p_{h,t} = P_{h,t}/P_{c,t}$. $(1-\xi_h)$ is the probability for an intermediate goods producer to actively set the price on the domestic market for intermediate goods. The passive price setting producers use for their price setting the weighting factor $(1-\chi_h)$ with which they take the ECB's future inflation path into account. $\xi_h \leq 1$ and $\chi_h \leq 1$. Calvo price setting of the domestic intermediate goods producers in the foreign market leads to the export price forward-backward looking Phillips curve:

$$\begin{aligned}\hat{\pi}_{x,t} - \hat{\bar{\pi}}_t = {} & \frac{\beta}{(1+\beta\chi_x)}\, \mathbb{E}_t\left[\hat{\pi}_{x,t+1} - \hat{\bar{\pi}}_{t+1}\right] + \frac{\chi_x}{(1+\beta\chi_x)}\left(\hat{\pi}_{x,t-1} - \hat{\bar{\pi}}_t\right) \\ & + \frac{\beta\chi_x}{(1+\beta\chi_x)}\left(\mathbb{E}_t\left[\hat{\bar{\pi}}_{t+1}\right] - \hat{\bar{\pi}}_t\right) + \frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x\,(1+\beta\xi_x)}\left(\hat{mc}^x_t + \hat{\varphi}^x_t\right)\end{aligned} \tag{3.29}$$

where:

$$\hat{mc}_t^x = \hat{mc}_t - \hat{p}_{x,t} \tag{3.30}$$

represents the average real marginal costs of the domestic intermediate goods producing firms selling in foreign markets. Export prices $p_{x,t} = P_{x,t}/P_{c,t}$ are expressed in terms of consumption goods prices. $\xi_x \leq 1$ and $\chi_x \leq 1$ have an analogue interpretation to the parameters ξ_h and χ_h in the domestic market for intermediate goods. For the importing foreign intermediate-goods producing firms the NAWM implies the import price forward-backward New-Keynesian Phillips curve:

$$\begin{aligned}\hat{\pi}_{im,t} - \hat{\bar{\pi}}_t &= \frac{\beta^*}{(1+\beta^*\chi^*)}\,\mathbb{E}_t\left[\hat{\pi}_{im,t+1} - \hat{\pi}_{t+1}\right] + \frac{\chi^*}{(1+\beta^*\chi^*)}\left(\hat{\pi}_{im,t-1} - \hat{\pi}_t\right) \\ &\quad + \frac{\beta^*\chi^*}{(1+\beta^*\chi^*)}\left(\mathbb{E}_t\left[\hat{\bar{\pi}}_{t+1}\right] - \hat{\bar{\pi}}_t\right) + \frac{(1-\beta^*\xi^*)(1-\xi^*)}{\xi^*(1+\beta^*\xi^*)}\left(\hat{mc}_t^* + \hat{\varphi}_t^*\right)\end{aligned} \tag{3.31}$$

with:

$$\hat{mc}_t^* = \hat{s}_t + \hat{p}_{y,t} + \hat{p}_{im,t} + \omega^*\hat{p}_{o,t} \tag{3.32}$$

representing the average real marginal costs of the foreign intermediate-goods producing firms selling their goods in the EMU's markets. $\hat{p}_{o,t}$ and ω^* in the marginal costs of foreign firms are the price of oil and the oil's share in EMU imports. $p_{im,t} = P_{im,t}/P_{c,t}$ and $p_{y,t} = P_{y,t}/P_{c,t}$ are expressed in terms of consumption goods prices, whereas the oil price is expressed in terms of foreign output prices $p_{o,t} = P_{o,t}/P_{y^*,t}$. $\hat{\varphi}_t^h, \hat{\varphi}_t^k$ and $\hat{\varphi}_t^*$ are price-markup shocks of domestically produced intermediate-goods for sell in the domestic and foreign markets and abroad produced intermediate-goods for sell in the EMU markets.

Domestic and foreign final-goods producing firms

In the NAWM the market of final-goods is fragmented into three different segments. These are the market segments for final private consumption $\hat{q}_t^c$, investment $\hat{q}_t^i$ and public consumption goods $\hat{q}_t^g$. For producing the quantities $\hat{q}_t^c$, $\hat{q}_t^i$ and $\hat{q}_t^g$ the representative domestic final-goods producer combines quantities of intermediate-goods produced by domestic and foreign firms. The final-goods producers demand for the domestic and foreign intermediate-goods are denoted as $\hat{h}_t^c, \hat{im}_t^c$ and $\hat{h}_t^i, \hat{im}_t^i$ for producing $\hat{q}_t^c$ and $\hat{q}_t^i$ respectively. For the production of public consumption goods $\hat{q}_t^g$ the final-goods producing firms only use domestically produced intermediate-goods $\hat{h}_t^g$. The final-goods firm's production technology in the private consumption $\hat{q}_t^c$ and investment segment $\hat{q}_t^i$ is a constant returns-to-scale CES technology, that is in log-linearized form expressed as:

$$\begin{aligned}\hat{q}_t^c &= v_c^{\frac{1}{\mu_c}}\left(\frac{h^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)}\hat{h}_t^c + (1-v_c)^{\frac{1}{\mu_c}}\left(\frac{im^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)}\hat{im}_t^c \\ &\quad + \frac{1}{(\mu_c-1)}\left[v_c^{\frac{1}{\mu_c}}\left(\frac{h^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)} - \frac{v_c}{(1-v_c)}(1-v_c)^{\frac{1}{\mu_c}}\left(\frac{im^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)}\right]\hat{v}_{c,t}\end{aligned} \tag{3.33}$$

$$
\begin{aligned}
\hat{q}_t^i &= v_i^{\frac{1}{\mu_i}} \left(\frac{h^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)} \hat{h}_t^i + (1-v_i)^{\frac{1}{\mu_i}} \left(\frac{im^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)} \hat{im}_t^i \\
&+ \frac{1}{(\mu_i - 1)} \left[v_i^{\frac{1}{\mu_i}} \left(\frac{h^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)} - \frac{v_i}{(1-v_i)} (1-v_i)^{\frac{1}{\mu_i}} \left(\frac{im^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)} \right] \hat{v}_{i,t}
\end{aligned}
\tag{3.34}
$$

where μ_c and μ_i are the intratemporal elasticity of substitution between domestic and foreign intermediate-goods whereas $\hat{v}_{c,t}$ and $\hat{v}_{i,t}$ are the NAWM economy's state variables indicating the home bias in the production of private consumption and investment goods respectively. The remaining constants in the CES production functions are the steady state values of the corresponding time varying state variables. The final-goods producers take the prices of the intermediate-goods $\hat{p}_{h,t}$ and $\hat{p}_{im,t}$ as given. They maximize their profits in a monopolistically competitive market in minimizing their expenditures for the intermediate goods in optimally choosing their demand for domestic and foreign intermediate-goods $\hat{h}_t^c, \hat{h}_t^i$ and $\hat{h}_t^g$ and $\hat{im}_t^c$ and $\hat{im}_t^i$ respectively. For private consumption and investment these are in log-linearized form:

$$
\hat{h}_t^c = \hat{v}_{c,t} - \mu_c \left(\hat{p}_{h,t} - \hat{p}_{c,t}\right) + \hat{q}_t^c \tag{3.35}
$$

$$
\hat{im}_t^c = -\frac{v_c}{(1-v_c)} \hat{v}_{c,t} - \mu_c \left(\hat{p}_{im,t} - \hat{p}_{c,t} - \hat{\Gamma}_{im^c,t}\right) + \hat{q}_t^c \tag{3.36}
$$

and

$$
\hat{h}_t^i = \hat{v}_{i,t} - \mu_i \left(\hat{p}_{h,t} - \hat{p}_{i,t}\right) + \hat{q}_t^i \tag{3.37}
$$

$$
\hat{im}_t^i = -\frac{v_i}{(1-v_i)} \hat{v}_{i,t} - \mu_i \left(\hat{p}_{im,t} - \hat{p}_{i,t} - \hat{\Gamma}_{im^i,t}\right) + \hat{q}_t^i \tag{3.38}
$$

$\hat{\Gamma}_{im^c,t}$ and $\hat{\Gamma}_{im^i,t}$ in the demand functions for foreign intermediate-goods are the adjustment costs for varying the amount of imported intermediate-goods in the production of final private consumption and investment goods. For private consumption and investment goods the import adjustment costs are determined by:

$$
\hat{\Gamma}_{im^c,t} = -\gamma_{im^c} \left[\left(\hat{im}_t^c - \hat{q}_t^c\right) - \left(\hat{im}_{t-1}^c - \hat{q}_{t-1}^c\right) \right] + \hat{\epsilon}_t^{im} \tag{3.39}
$$

$$
\hat{\Gamma}_{im^i,t} = -\gamma_{im^i} \left[\left(\hat{im}_t^i - \hat{q}_t^i\right) - \left(\hat{im}_{t-1}^i - \hat{q}_{t-1}^i\right) \right] + \hat{\epsilon}_t^{im} \tag{3.40}
$$

where $\gamma_{im^c}, \gamma_{im^i} > 0$ positively determine the import adjustment costs producing $\hat{q}_t^c$ and $\hat{q}_t^i$. The final-goods producer's price setting yields for private consumption and investment goods the following price indices:

$$
\begin{aligned}
\hat{p}_{c,t} &= v_c \left(\frac{p_h}{p_c}\right)^{(1-\mu_c)} \hat{p}_{h,t} + (1-v_c) \left(\frac{p_{im}}{p_c}\right)^{(1-\mu_c)} \left(\hat{p}_{im,t} - \hat{\Gamma}_{im^c,t}\right) \\
&+ \frac{v_c}{(1-\mu_c)} \left[\left(\frac{p_h}{p_c}\right)^{(1-\mu_c)} - \left(\frac{p_{im}}{p_c}\right)^{(1-\mu_c)} \right] \hat{v}_{c,t}
\end{aligned}
\tag{3.41}
$$

and:

$$
\begin{aligned}
\hat{p}_{i,t} = {} & v_i \left(\frac{p_h}{p_i}\right)^{(1-\mu_i)} \hat{p}_{h,t} + (1-v_i)\left(\frac{p_{im}}{p_i}\right)^{(1-\mu_i)}\left(\hat{p}_{im,t} - \hat{\Gamma}_{im^i,t}\right) \\
& + \frac{v_i}{(1-\mu_i)}\left[\left(\frac{p_h}{p_i}\right)^{(1-\mu_i)} - \left(\frac{p_{im}}{p_i}\right)^{(1-\mu_i)}\right]\hat{v}_{i,t}
\end{aligned}
\tag{3.42}
$$

For the production of the public consumption goods it is assumed that:

$$
\hat{q}^g_t = \hat{h}^g_t \tag{3.43}
$$

and:

$$
\hat{p}_{g,t} = \hat{p}_{h,t} \tag{3.44}
$$

The aggregated demand for domestically produced intermediate-goods in the EMU is:

$$
\hat{h}_t = \frac{h^c}{h}\hat{h}^c_t + \frac{h^i}{h}\hat{h}^i_t + \frac{h^g}{h}\hat{h}^g_t \tag{3.45}
$$

whereas the aggregated EMU's demand for intermediate-goods produced abroad is:

$$
\hat{im}_t = \frac{im^c}{im}\hat{im}^c_t + \frac{im^i}{im}\hat{im}^i_t \tag{3.46}
$$

Domestic export oriented firms

In the NAWM economy there are domestic foreign retail firms, which bundle the domestically produced intermediate-goods into an export bundle $\hat{x}_t$ satisfying the foreign demand. EMU exports $\hat{x}_t$ are described by:

$$
\hat{x}_t = \hat{v}^*_t - \mu^*\left(\hat{p}_{x,t} - \hat{p}_{y,t} - \hat{s}_t - \hat{p}^c_{x,t} - \hat{\Gamma}_{x,t}\right) + \hat{y}^*_t + \hat{\hat{z}}_t \tag{3.47}
$$

where $\hat{\Gamma}_{x,t}$ are the export adjustment costs determined by:

$$
\hat{\Gamma}_{x,t} = -\gamma^*\left[\left(\hat{x}_t - \hat{y}^*_t - \hat{\hat{z}}_t\right) - \left(\hat{x}_{t-1} - \hat{y}^*_{t-1} - \hat{\hat{z}}_{t-1}\right)\right] + \hat{\epsilon}^x_t \tag{3.48}
$$

With the export preference and export demand shocks $\hat{v}^*_t$ and $\hat{\epsilon}^x_t$ EMU's export oriented firms face two different kinds of shocks. μ^* reflects the price elasticity of exports and γ^* positively determines the export adjustment costs. The NAWM state variables $\hat{y}^*_t$ and $\hat{v}^*_t$ are the overall foreign demand and the export share of domestic intermediate-goods capturing foreign non-price related preferences for domestic intermediate-goods. $\hat{\hat{z}}_t$ with $\hat{z}_t = z_t/z^*_t$ captures the (log) productivity differential between the domestic and the foreign economy.

3.4.1.4 Fiscal and Monetary Authorities

According to the NAWM the ECB's monetary policy reaction function is:

$$\begin{aligned}\hat{r}_t = \phi_r \hat{r}_{t-1} + (1-\phi_r)\left[\hat{\bar{\pi}}_t + \phi_\pi\left(\hat{\pi}_{c,t} - \hat{\bar{\pi}}_t\right) + \phi_y \hat{y}_t\right] \\ + \phi_{\Delta\pi}\left(\hat{\pi}_{c,t} - \hat{\pi}_{c,t-1}\right) + \phi_{\Delta yi}\left(\hat{y}_t - \hat{y}_{t-1}\right) + \hat{\epsilon}_t^r\end{aligned} \tag{3.49}$$

where $\hat{r}_t = ln\left(R_t/R\right)$ is the (log) deviation of the observed risk-less short rate R_t from its steady-state value, whereas $\hat{\bar{\pi}}_t = ln\left(\bar{\Pi}_t/\bar{\Pi}\right)$ denotes the (log) deviation of the ECB's inflation objective from the ECB's long-term target inflation $\bar{\Pi}$. The dynamics of $\bar{\bar{\pi}}_t$ are determined by:

$$\hat{\bar{\pi}}_t = \phi_{\bar{\pi}} \hat{\bar{\pi}}_{t-1} + \hat{\epsilon}_t^{\bar{\pi}} \tag{3.50}$$

The government budget variables fiscal spending $s_{G,t} = P_{G,t}G_t/P_{y,t}Y_t$ and fiscal revenues from lump-sum taxes $s_{T,t} = T_t/P_{y,t}Y_t$ are expressed in its share of nominal output. For financing budget deficits it is assumed that Riccardian equivalence holds such that in the long run there is no difference in financing the budget deficit by issuing new bonds or levying lump-sum taxes. Keeping this assumption as given the government balances its budget by using lump-sum taxes T_t, such that $B_t = 0$. The NAWM's log-linearized balanced budget is expressed as:

$$\begin{aligned}\hat{s}_{g,t} = & \frac{p_c c}{p_y y}\left[\hat{\tau}_t^c + \tau^c\left(\hat{p}_{c,t} + \hat{c}_t - \hat{p}_{y,t} - \hat{y}_t\right)\right] \\ & + \frac{wN}{p_y y}\left[\hat{\tau}_t^N + \hat{\tau}_t^{w_h} + \hat{\tau}_t^{w_f} + \left(\hat{\tau}^N + \hat{\tau}^{w_h} + \hat{\tau}^{w_f}\right)\left(\hat{w}_t + \hat{N}_t - \hat{p}_{y,t} - \hat{y}_t\right)\right] \\ & + \frac{r_k k g_z^{-1}}{p_y y}\left[\hat{\tau}_t^k + \tau^k\left(\hat{u}_t + \hat{r}_{k,t} + \hat{k}_t - \hat{g}_{z,t} - \hat{p}_{y,t} - \hat{y}_t\right)\right] \\ & + \frac{p_i k g_z^{-1}}{p_y y}\left[\delta\hat{\tau}_t^k + \tau^k \gamma_{u,1}\hat{u}_t + \tau^k\delta\left(\hat{p}_{i,t} + \hat{k}_t - \hat{g}_{z,t} - \hat{p}_{y,t} - \hat{y}_t\right)\right] \\ & + \tau^d \hat{s}_{d,t} + s_d \hat{\tau}_t^d + \hat{s}_{T,t}\end{aligned} \tag{3.51}$$

where τ^d is the government tax rate levied on household's dividend income. The government share of the economy's overall output in log-linearized form is given by:

$$\hat{s}_{g,t} = s_g\left(\hat{p}_{g,t} + \hat{g}_t - \hat{p}_{y,t} - \hat{y}_t\right) \tag{3.52}$$

where $\hat{g}_t = ln\left(g_t/g\right)$ with $g_t = G_t/z_t$ is the (log) difference of the observed real government expenditures from its steady-state g. $s_{D,t} = D_t/P_{y,t}Y_t$ denotes the economy's profit share of nominal output and is expressed in log-linearized form as:

$$\hat{s}_{d,t} = -\frac{1}{\varphi}\left(1 + \psi y^{-1}\right)\left(\hat{mc}_t - \hat{p}_{y,t}\right) - \frac{1}{\varphi}\left(\frac{h}{y}\hat{h}_t + \frac{x}{y}\hat{x}_t - \frac{(h+x+\psi)}{y}\hat{y}_t\right) \tag{3.53}$$

with the steady-state price-markup $\varphi = \varphi^h = \varphi^x$.

3.4.1.5 Net foreign Assets, Trade Balance and Terms of Trade

The EMU's (net) holding of foreign bonds $\hat{b}^*_{t+1}$ expressed in log-linearized form as:

$$\begin{aligned}\hat{b}^*_{t+1}R^{*\,-1} &= g_z^{-1}\bar{\Pi}_y^{*\,-1}\hat{b}^*_t + \frac{p_x x}{s p_y}\left(\hat{p}_{x,t} + \hat{x}_t - \hat{s}_t - \hat{p}_{y,t} - \hat{\tilde{z}}_t\right) \\ &- \frac{p_{im} im}{s p_y}\left(\hat{p}_{im,t} + \hat{im}_t - \hat{s}_t - \hat{p}_{y,t} - \hat{\tilde{z}}_t\right)\end{aligned} \tag{3.54}$$

is determined by the current amount $\hat{b}^*_t$ in the first and the trade balance denominated in foreign currency and adjusted by the productivity differential in the second and third term. The (net) bond holdings $\hat{b}^*_{t+1} = B^*_{t+1}I\left(z^*_t P^*_{y,t}\right)$ are adjusted by the foreign price-index $P^*_{y,t}$ and the foreign productivity level z^*_t For the foreign long-term inflation target it is assumed $\bar{\Pi}^* = \bar{\Pi}$. The log-linearized share of foreign assets (net) holdings to EMU's nominal output expressed in domestic currency is determined by:

$$\hat{s}_{B^*,t+1} = s\tilde{z}y^{-1}\hat{b}^*_{t+1} \tag{3.55}$$

whereas the NAWM economy's log-linearized trade balance expressed in shares of nominal output is defined as:

$$\hat{s}_{TB^*,t} = \hat{s}_{x,t} - \hat{s}_{im,t} \tag{3.56}$$

with the export and import shares $s_{x,t} = P_{x,t}X_t/P_{y,t}Y_t$ and $s_{im,t} = P_{im,t}IM_t/P_{y,t}Y_t$ (log) deviations from their steady-state values:

$$\hat{s}_{x,t} = s_x\left(\hat{p}_{x,t} + \hat{x}_t - \hat{p}_{y,t} - \hat{y}_t\right) \tag{3.57}$$

and

$$\hat{s}_{im,t} = s_x\left(\hat{p}_{im,t} + \hat{im}_t - \hat{p}_{y,t} - \hat{y}_t\right) \tag{3.58}$$

The economy's log-linearized terms of trade are determined by:

$$\hat{ToT}_t = \hat{p}_{im,t} - \hat{p}_{x,t} \tag{3.59}$$

3.4.2 Integration of the EMU yield factors into the macroeconomic framework

The integration of the term structure of interest rates into a larger macroeconomic modeling framework is addressed by Andreasen, Fernandez-Villaverde and RubioRamirez [2018], De Greave, Emiris and Wouters [2009], Rudebusch and Swanson [2008, 2012], Beakert, Cho and Moreno [2010], van Binsenberg, Fernandez-Villaverde, Koijen and Rubio-Ramirez [2012] or Kliem and Meyer-Gohde [2017]. Our term structure integration is done by extending the monetary policy rule in 3.49 by:

$$\begin{aligned}\hat{r}_t &= \phi_r\hat{r}_{t-1} + \phi_l\hat{L}_{t-1} + \phi_s\hat{S}_{t-1} + (1-\phi_r)\left[\hat{\pi}_t + \phi_\pi\left(\hat{\pi}_{c,t} - \hat{\pi}_t\right) + \phi_y\hat{y}_t\right] \\ &+ \phi_{\Delta\pi}\left(\hat{\pi}_{c,t} - \hat{\pi}_{c,t-1}\right) + \phi_{\Delta yi}\left(\hat{y}_t - \hat{y}_{t-1}\right) + \hat{\epsilon}^r_t\end{aligned} \tag{3.60}$$

where the three state variables $\hat{r}_{t-1}$, $\hat{L}_{t-1}$ and $\hat{S}_{t-1}$ of the extended monetary policy rule determine the very short and as well as the middle to long term maturity segment and further the steepness of the EMU's term structure of interest rates indicating the interest rate differences between long and short term rates. With the factors $\hat{L}_{t-1}$ and $\hat{S}_{t-1}$ in the monetary policy rule the extension endogenizes the EMU's term structure issues into the EMU's monetary policy decisions framework. Integrating the EMU's common slope factor $\hat{S}_{t-1}$ in the monetary policy rule is motivated by the discussions in Estrella and Mishkin [1996] and Estrella and Trubin [2006] and our empirical findings in section 3.3.3, where we revealed the informational content of $\hat{S}_{t-1}$ with respect to the economy's future development.

3.4.3 Calibration and estimation of the EMU wide macroeconomic model

3.4.3.1 Calibration and steady state

In line to Kydland and Prescott [1982] parts of the NAWM's parameters are calibrated. In calibrating the parameters of the NAWM we follow Christoffel, Coenen and Warne [2008]. With respect to the model's steady-state values we reduce the steady-state version of the NAWM analogue to Christoffel et. al. [2008] to a system consisting of four equations expressing the equilibrium relations in the labor-, the capital-, and the goods-markets with their relative prices respectively. We simultaneously solve this equation system, receiving the steady-state values of k, c, N and π where the last one is the price of the investment good expressed relative to the price of the consumption good. Similar to Christoffel, Coenen and Warne [2008] for the global economy related variables $\hat{p}^c_t, \hat{y}^*_t, \hat{r}^*_t, \hat{p}_{o,t}, \hat{p}_{y^*,t}$ we estimate a structural VAR (SVAR). (See remarks in footnote 31 in Christoffel, Coenen and Warne [2008] for further details). Also as done by Christoffel, Coenen and Warne [2008] we estimate an AR[1] process for government spending $\hat{g}_t$. Both the paramters of the SVAR and the AR are kept fixed throughout the estimation of the remaining parameters of the NAWM.

3.4.3.2 Canonical rational expectations form of the implemented NAWM

Implementation of the baseline NAWM

Following Herbst and Schorfheide [2016] or Dejong and Dave [2011] to determine the agent's rational expectations in a first step the 62 log-linearized equations outlined in 3.4.1 and the market clearing and aggregate constraints outlined in Appendix B.4.1 combined with 50 additional necessary (log) linear equations for the 112 endogenous macroeconomic variables of the NAWM are transferred into the DSGE's canonical linear rational expectations form:

$$\boldsymbol{\Gamma}_0 \boldsymbol{s}_t = \boldsymbol{\Gamma}_1 \boldsymbol{s}_{t-1} + \boldsymbol{\Psi} \boldsymbol{\varepsilon}_t + \boldsymbol{\Pi} \boldsymbol{\eta}_t \tag{3.61}$$

where:

$$
\begin{aligned}
\boldsymbol{s}_t^T = [& \hat{\lambda}_t, \hat{c}_t, \hat{Q}_t, \hat{p}_{i,t}, \hat{p}_{k,t}, \hat{p}_{x,t}, \hat{p}_{y,t}, \hat{p}_{im,t}, \hat{p}_{c,t}, \hat{p}_{g,t}, \hat{p}_t^c, \hat{i}_t, \hat{r}_{k,t}, \hat{r}_t, \hat{u}_t, \hat{\pi}_{c,t}, \hat{\bar{\pi}}_{c,t}, \hat{\pi}_{k,t}, \hat{\pi}_t, \hat{\pi}_{x,t}, \hat{\pi}_{im,t}, \hat{\pi}_{y,t}, \\
& \hat{\pi}_{i,t}, \hat{s}_t, \hat{s}_{g^*,t}, \hat{s}_{g,t}, \hat{s}_{d,t}, \hat{s}_{T,t}, \hat{s}_{TB,t}, \hat{s}_{x,t}, \hat{s}_{im,t}, T\hat{o}T_t, \hat{k}_t, \hat{k}_t^s, \hat{w}_t, \hat{w}_t^{\tau}, m\hat{r}s_t, m\hat{c}_t, m\hat{c}_t^k, m\hat{c}_t^x, \hat{N}_t, \hat{y}_t, \hat{x}_t, \\
& \hat{q}_t^c, \hat{q}_t^i, \hat{q}_t^g, \hat{h}_t^c, \hat{h}_t^i, \hat{h}_t^g, \hat{h}_t, i\hat{m}_t^c, i\hat{m}_t^i, i\hat{m}_t, \hat{v}_{c,t}, \hat{v}_{i,t}, \hat{\Gamma}_{im^c,t}, \hat{\Gamma}_{im^i,t}, \hat{\Gamma}_{im^*,t}, \hat{\bar{z}}_t, \hat{E}_t, \mathbb{E}_t\left[\hat{\lambda}_{t+1}\right], \mathbb{E}_t\left[\hat{c}_{t+1}\right], \\
& \mathbb{E}_t\left[\hat{Q}_{t+1}\right], \mathbb{E}_t\left[\hat{p}_{i,t+1}\right], \mathbb{E}_t\left[\hat{i}_{t+1}\right], \mathbb{E}_t\left[\hat{r}_{k,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{c,t+1}\right], \mathbb{E}_t\left[\hat{\bar{\pi}}_{c,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{y,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{k,t+1}\right], \\
& \mathbb{E}_t\left[\hat{\pi}_{t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{x,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{im,t+1}\right], \mathbb{E}_t\left[\hat{w}_{t+1}\right], \mathbb{E}_t\left[\hat{s}_{t+1}\right], \mathbb{E}_t\left[\hat{\epsilon}_{t+1}^c\right], \mathbb{E}_t\left[\hat{g}_{z,t+1}\right], \mathbb{E}_t\left[\hat{E}_{t+1}\right], m\hat{c}_t^*, \\
& \hat{b}_t^*, \hat{\epsilon}_t^c, \hat{g}_{z,t}, \hat{\epsilon}_t^i, \hat{\epsilon}_t^{RP}, \hat{\epsilon}_t^{RP^*}, \hat{\varphi}_t^w, \hat{\epsilon}_t^N, \hat{\varphi}_t^k, \hat{\varphi}_t^x, \hat{\varphi}_t^*, \hat{\epsilon}_t^{im}, \hat{\eta}_t^r, \hat{\eta}_t^{\bar{\pi}}, \hat{\epsilon}_t^x, \hat{\epsilon}_t, \hat{v}_t^*, \mathbb{E}_t\left[\hat{\pi}_{y^*,t+1}\right], \hat{y}_t^*, \hat{r}_t^*, \\
& \hat{p}_{o,t}, \hat{p}_{y^*,t}, \hat{g}_t, \hat{y}_{t-1}, \hat{c}_{t-1}, \hat{i}_{t-1}, \hat{x}_{t-1}, \hat{p}_{im,t-1}, \hat{p}_{t-1}^c, i\hat{m}_{t-1}, \hat{w}_{t-1}, \hat{y}_{t-1}^*, \hat{p}_{y^*,t-1}]
\end{aligned}
$$

defines the 112×1 state vector.

$$
\boldsymbol{\varepsilon}_t^T = \left[\varepsilon_t^c, \varepsilon_t^{g_z}, \varepsilon_t^i, \varepsilon_t^{RP}, \varepsilon_t^{RP^*}, \varepsilon_t^{\varphi^w}, \varepsilon_t^N, \varepsilon_t^{\varphi^k}, \varepsilon_t^{\varphi^x}, \varepsilon_t^{\varphi^*}, \varepsilon_t^{im}, \varepsilon_t^r, \varepsilon_t^{\bar{\pi}}, \varepsilon_t^x, \varepsilon_t, \varepsilon_t^{v^*}\right]
$$

is the 16×1 vector of innovations and

$$
\begin{aligned}
\boldsymbol{\eta}_t^T = [& \hat{\lambda}_t - \mathbb{E}_{t-1}\left[\hat{\lambda}_t\right], \hat{c}_t - \mathbb{E}_{t-1}\left[\hat{c}_t\right], \hat{Q}_t - \mathbb{E}_{t-1}\left[\hat{Q}_t\right], \hat{p}_{i,t} - \mathbb{E}_{t-1}\left[\hat{p}_{i,t}\right], \hat{i}_t - \mathbb{E}_{t-1}\left[\hat{i}_t\right], \\
& \hat{r}_{k,t} - \mathbb{E}_{t-1}\left[\hat{r}_{k,t}\right], \hat{\pi}_{c,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{c,t}\right], \hat{\bar{\pi}}_{c,t} - \mathbb{E}_{t-1}\left[\hat{\bar{\pi}}_{c,t}\right], \hat{\pi}_{y,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{y,t}\right], \hat{\pi}_{h,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{h,t}\right], \\
& \hat{\bar{\pi}}_t - \mathbb{E}_{t-1}\left[\hat{\bar{\pi}}_t\right], \hat{\pi}_{x,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{x,t}\right], \hat{\pi}_{im,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{im,t}\right], \hat{w}_t - \mathbb{E}_{t-1}\left[\hat{w}_t\right], \hat{s}_t - \mathbb{E}_{t-1}\left[\hat{s}_t\right], \\
& \hat{\epsilon}_t^c - \mathbb{E}_{t-1}\left[\hat{\epsilon}_t^c\right], \hat{g}_{z,t} - \mathbb{E}_{t-1}\left[\hat{g}_{z,t}\right], \hat{E}_t - \mathbb{E}_{t-1}\left[\hat{E}_t\right], \hat{\pi}_{y^*,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{y^*,t}\right]]
\end{aligned}
$$

is the 19×1 vector of expectation errors. $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_1$ are 112×112 matrices, and $\boldsymbol{\Psi}$ and $\boldsymbol{\Pi}$ are 112×16 and 112×19 matrices relating the vectors of innovations and expectation errors to the dynamics of the state variables. In Appendix B.4.2 the row-wise specification of the matrices $\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}$ and $\boldsymbol{\Pi}$ is outlined in detail.

Implementation of the NAWM extended by the EMU's term structure of interest rates

The canonical rational expectations form of the NAWM with the integrated EMU's term structure of interest rates is based on the canonical form of the baseline NAWM and is specified as:

$$
\boldsymbol{\Gamma}_0^{TS} \tilde{\boldsymbol{s}}_t = \boldsymbol{\Gamma}_1^{TS} \tilde{\boldsymbol{s}}_{t-1} + \boldsymbol{\Psi}^{TS} \tilde{\boldsymbol{\varepsilon}}_t + \boldsymbol{\Pi}^{TS} \tilde{\boldsymbol{\eta}}_t \tag{3.62}
$$

where the vector of state variables becomes $\tilde{\boldsymbol{s}}_t^T = \left[\boldsymbol{s}_t, \hat{\boldsymbol{F}}_t, \hat{\boldsymbol{\epsilon}}_t\right] = \left[\boldsymbol{s}_t, \hat{L}_t, \hat{S}_t, \hat{\epsilon}_t^l, \hat{\epsilon}_t^s\right]$ with the EMU's common level and slope factors and their disturbance terms $\mathbf{F}_t^T = [L_t, S_t]$ and $\boldsymbol{\epsilon}_t^T = \left[\epsilon_t^l, \epsilon_t^s\right]$ from the EMU's factor dynamics in 3.4 respectively. The system's innovations $\boldsymbol{\varepsilon}_t$ become $\tilde{\boldsymbol{\varepsilon}}_t^T = \left[\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_t^{TS}\right]$ where $\boldsymbol{\varepsilon}_t^{TS} = \left[\varepsilon_t^l, \varepsilon_t^s\right]$ are standardized Gaussian $\boldsymbol{\varepsilon}_t^{TS} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_\epsilon\right)$. The

116×116 matrices $\mathbf{\Gamma}_0^{TS}, \mathbf{\Gamma}_1^{TS}$ and the 116×116 matrix $\mathbf{\Psi}^{TS}$ are specified as:

$$\mathbf{\Gamma}_0^{TS} = \begin{bmatrix} \mathbf{\Gamma}_0 & \mathbf{0}_{112\times 2} & \mathbf{0}_{112\times 2} \\ \mathbf{0}_{2\times 112} & \mathbf{I}_{2\times 2} & -\mathbf{I}_{2\times 2} \\ \mathbf{0}_{2\times 112} & \mathbf{0}_{2\times 2} & \mathbf{I}_{2\times 2} \end{bmatrix} \quad \mathbf{\Gamma}_1^{TS} = \begin{bmatrix} \mathbf{\Gamma}_1 & \mathbf{\Delta}^{TS} & \mathbf{0}_{112\times 2} \\ \mathbf{0}_{2\times 112} & \mathbf{\Phi} & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 112} & \mathbf{0}_{2\times 2} & \mathbf{0}_{2\times 2} \end{bmatrix}$$
$$\mathbf{\Psi}^{TS} = \begin{bmatrix} \mathbf{\Psi} & \mathbf{0}_{112\times 2} \\ \mathbf{0}_{2\times 16} & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 16} & \mathbf{I}_{2\times 2} \end{bmatrix} \tag{3.63}$$

where $\mathbf{0}_{m\times n}$ are $m \times n$ matrices of zeros and $I_{n,n}$ is the $n \times n$ identity matrix. The matrices $\mathbf{\Gamma}_0$, $\mathbf{\Gamma}_1$ and $\mathbf{\Psi}$ are from 3.61. $\mathbf{\Phi}$ is the 2×2 diagonal matrix from the EMU factor dynamics defined in 3.4 and Δ^{TS} is a 112×2 matrix of zeros except for $\Delta_{39,1} = \phi_l$ and $\Delta_{39,2} = \phi_s$.

3.4.3.3 Log-linearized state space system

Log linear state space system of the baseline NAWM

For estimating the NAWM we formulate the model in state space form. The system's measurement equation is specified as follows:

$$\boldsymbol{y}_t = \boldsymbol{c} + \mathbf{M}\boldsymbol{s}_t + \boldsymbol{\vartheta}_t \tag{3.64}$$

where the 18×1 vector:

$$\boldsymbol{y}_t^T = [\Delta GDP_t^{EMU}, \Delta CONS_t, \Delta INV_t, ln(GOV_t), \Delta EXPORT_t, \Delta IMPORT_t, INF_{Y,t}^{EMU}, INF_{C,t}, INF_{IM,t}, ln(LABOR_{y,t}), \Delta WAGE_t, ECB_t, FX_t, \Delta GDP_t^{WORLD}, INF_{Y,t}^{WORLD}, LIBOR_t^{\$}, INF_{EXPORT,t}, PRICE_t^{OIL}]$$

contains the measurements of (EMU) GDP, consumption, investment, government spending, exports, imports, GDP-, consumption- and import-deflator based inflation rates, labor (measured in total employment), per head wages, the monetary policy rate set by the ECB (approximated by the EONIA swap rate), the effective exchange rate, (world) GDP and (world) GDP-deflator based inflation, the USD-LIBOR, export-deflator based inflation and the oil price (UK-Brent). Data details and details related to the preparation and transformation of the data are outlined in Appendix B.1. $\boldsymbol{c}$ is a 18×1 vector containing the state-variables' steady state values and $\mathbf{M}$ is a specified 18×112 matrix. $\boldsymbol{\vartheta}_t \sim N(\mathbf{0}, \mathbf{\Sigma})$ is the Gaussian measurement error with diagonal covariance $\mathbf{\Sigma}$. The system's transition equation for describing the dynamics of the state variables is determined by applying Sim's QZ algorithm outlined in chapter 2.2.2.2 to the NAWM's linear rational expectations form expressed in the foregoing section 3.4.3.2.

Log linear state space system of the term structure extended NAWM

As for the canonical rational expectations form the (log) linear state space system of the term structure extended NAWM is based on the state space model of the baseline NAWM. The extended measurement equation becomes:

$$\tilde{\boldsymbol{y}}_t = \boldsymbol{c}^{TS} + \mathbf{M}^{TS}\tilde{\boldsymbol{s}}_t + \tilde{\boldsymbol{\vartheta}}_t \tag{3.65}$$

where we extent the vector of EMU measurements by $\tilde{\boldsymbol{y}}_t^T = \left[\boldsymbol{y}_t, \hat{\mathbf{F}}_t, \hat{\boldsymbol{\epsilon}}_t\right]$. $\tilde{\boldsymbol{\vartheta}}_t$ are the Gaussian measurement errors with $\tilde{\boldsymbol{\vartheta}}_t \sim N\left(\mathbf{0}, \mathbf{\Sigma}^{TS}\right)$. The 22×116 and 22×22 matrices $\mathbf{M}^{TS}$ and $\mathbf{\Sigma}^{TS}$ are specified as:

$$\mathbf{M}^{TS} = \begin{bmatrix} \mathbf{M} & \mathbf{0}_{18\times 4} \\ \mathbf{0}_{4\times 18} & \mathbf{I}_{4\times 4} \end{bmatrix} \quad \mathbf{\Sigma}^{TS} = \begin{bmatrix} \mathbf{\Sigma} & \mathbf{0}_{18\times 4} \\ \mathbf{0}_{4\times 18} & \mathbf{I}_{4\times 4} \end{bmatrix} \tag{3.66}$$

3.4.3.4 Estimation remarks

Estimation of the baseline NAWM is done by a Markov Chain Monte Carlo (MCMC) procedure where in every iteration the Random-Walk Metropolis-Hastings (RW-MH) algorithm is applied. Because of the large number of parameters we additionally use the Random-Block RW-MH (RB-RW-MH) algorithm for drawing the parameters. For both the RW-MH and the RB-RW-MH the (log) posterior is computed by using the Kalman filter. Switching between the RW-MH and RBRW-MH is randomly, where by reasons of computational effort the RB-RW-MH is applied only in ten percent of the MCMC iterations. Because of the unidirectional causality estimation of the NAWM extended by the EMU term structure of interest rates is separated into two blocks: The EMU term structure block and the NAWM block. The NAWM block is conditional to the EMU term structure block, whereas the EMU term structure block is independent to the NAWM block. Therefore in a first step we have estimated the parameters of the EMU term structure of interest rates model by using a MCMC procedure outlined in section 3.2 and in Appendix B.2. In a second step for estimating the NAWM block, we integrate a Gibbs sampler in our MCMC procedure for the baseline NAWM estimation. Due to the unidirectional causality the Gibbs sampler draws in every MCMC iteration i the parameters of the NAWM block conditional to the i th MCMC iteration parameter draw of the EMU term structure block done in the first step.Table B.1 in the Appendix lists the priors we use in our MCMC estimation.

3.5 Empirical implications of the EMU macroeconomic framework

3.5.1 In-sample-fit

For estimating the baseline and term structure extended NAWM we use a MCMC procedure with 250000 iterations, where we cut off the first 50000 iterations as burn-in draws. The

Sample Mean (Q1/2005 – Q1/2014)													
	ΔGDP	$\Delta CONS$	ΔINV	GOV	ΔEXP	ΔIMP	INF_Y	INF_{OIL}	INF_{ι}	$LABOR$	$\Delta WAGE$	ECB	FX
Obs.	0.179	0.095	-0.084	-1.609	0.827	0.647	0.356	0.405	0.402	-1.117	0.186	1.998	0.007
NAWM	-0.160	0.227	0.271	-1.346	0.837	0.677	0.332	0.383	0.224	-1.119	0.222	2.022	0.041
Ext. NAWM	-0.096	0.213	0.364	-1.303	0.825	0.643	0.315	0.316	0.172	-1.134	0.185	1.973	0.005
Sample Standard Deviation (Q1/2005 – Q1/2014)													
	ΔGDP	$\Delta CONS$	ΔINV	GOV	ΔEXP	ΔIMP	INF_Y	INF_{OIL}	INF_{ι}	$LABOR$	$\Delta WAGE$	ECB	FX
Obs.	0.780	0.384	1.676	2.243	2.408	2.218	0.184	0.370	1.434	2.515	0.231	1.578	3.179
NAWM	0.419	0.397	1.413	1.690	2.419	2.086	0.233	0.236	0.285	2.522	0.274	1.569	3.096
Ext. NAWM	0.377	0.326	1.143	1.607	2.405	2.169	0.399	0.379	0.441	2.533	0.246	1.537	3.094

Table 3.7: Observed and NAWM implied unconditional first and second moments. (First and second moments of the NAWM are calculated at the mode of the model's posterior).

Sample Mean (Q1/2005 – Q1/2014)													
	ΔGDP	$\Delta CONS$	ΔINV	GOV	ΔEXP	ΔIMP	INF_Y	INF_{OIL}	INF_{ι}	$LABOR$	$\Delta WAGE$	ECB	FX
NAWM 05-14	0.339	0.132	0.355	0.263	0.010	0.029	0.023	0.021	0.178	0.003	0.035	0.025	0.034
NAWM 87-14	0.380	0.150	0.328	1.509	0.023	0.094	0.041	0.186	0.412	0.007	0.962	0.383	0.005
TS Ext. NAWM	0.275	0.118	0.449	0.306	0.002	0.004	0.040	0.088	0.230	0.017	0.001	0.025	0.002

Table 3.8: RMSE comparison of three different NAWM estimations. (The RMSEs are calculated at the mode of the model's posterior).

remaining 200000 draws are used for statistical inference. To verify the model's goodness-of-fit in Table 3.7 we list the unconditional first and second moments of the observed and NAWM implied macroeconomic state variables. In Table 3.8 we list the root mean squared errors (RMSE) of the model's time series estimates. In Figure 3.9 we additionally plot the 18 observed and model implied time series of our used macroeconomic variables. From Table 3.7 and 3.8 as well as from Figure 3.9 it becomes clear that our two different NAWM specifications fit the observed data very well. The mean absolute deviation between the baseline and the term structure extended NAWM implied first moments and the mean of the observed macro economic data series are only 0.111 and 0.120 percentage points respectively. With deviations of 0.355 and 0.448 percentage points from the observed first moment both specifications have their largest deviation in estimating the EMU's real investment activities. Their best first moment fit with absolute deviations of 0.002 implied by the baseline NAWM and 0.001 implied by the extended NAWM have the models for the employment rate and ECB's short term interest rate respectively. With respect to their second moment estimates both model specifications are very similar with average (absolute) deviations from the observed standard deviation of 0.216 and 0.235 percentage points respectively. The largest deviation for both the baseline NAWM and the extended NAWM are 1.149 and 0.993 percentage points respectively. Their best second moment fit with deviations of 0.007 and 0.003 percentage points respectively have the models for the employment rate and the for EMU's exports. For purposes concerning the model's robustness we have estimated the baseline NAWM with macroeconomic data between Q1/1987 and Q1/2014. In Figure B.6 in the Appendix, we plot the 18 observed and model implied time series over this long-term horizon. Compared to the in-sample-fit of the long term NAWM estimation figured out in Table 3.8 it becomes clear that our two models - estimated on the shorter horizon between Q1/2005 and Q1/2014 - show a similar good in-sample fit. zero. Looking at the parameter estimates listed in Tables B.3, B.4 and B.5 in Appendix B.6 reveals an average (relative) parameter variation between all three estimates of 62% . The estimates of the baseline and the term structure extended NAWM with data between Q1/2005 and Q1/2014 variate on average by only 8% and are therefore very stable. With respect to our extension of the baseline NAWM by integrating the EMU's level and slope term structure factors in the NAWM's policy decision rule Table B.5 reveals that only the EMU's slope factor is internalized into the ECB's monetary decision framework. The level factor's impact is not significantly different from zero.

3.5.2 EMU's historic shock decomposition

To get an impression about the underlying shock processes affecting the EMU's macroeconomic variables in the past in Figure 3.10 we plot the historical decomposition for the EMU's (real) growth rates of its aggregates GDP, consumption, investment, exports and imports as well as ECB's short term rate. Focusing on the economic breakdown following the Lehman bankruptcy in September 2008 reveals a very similar pattern for EMU's GDP, consumption, investments and exports. For all of these four aggregates in this phase shocks induced by monetary policy decisions have larger effects. In late 2008 and the first half of 2009 shocks effecting the firm's investment activities in the EMU are mainly composed by monetary policy related shocks. Beside shocks from monetary policy decisions, investment activities are strongly effected by shocks related to risk premiums investors demanded domestically and abroad. ECB's monetary policy decisions itself areeffected by shocks related to risk premiums required by EMU and abroad investors. Figure 3.10 additionally shows that the decisive decrease in the short term rate in Q4/2008 and Q1/2009 in which the ECB's main refinancing operations rate fell by 225 basis points from 3.75% in the beginning of November 2008 to 1.50% in March 2009 is also effected by a shock induced by a changing slope of the EMU's common term structure of interest rates.

3.5.3 Impulse Response

To find answers related to the inner economic structure of the EMU in Figure 3.11 we plot the responses for the three major EMU macroeconomic state variables GDP, investment and ECB's controlled short term interest rate to shocks coming from seven different economic sources, related to the EMU's production technology, the risk premiums EMU faces domestically and abroad, the ECB's monetary policy and its target inflation rate, our integrated EMU term structure slope factor and an export demand shock the EMU faces from abroad. In Figure B.7 in Appendix B.6 we further show the impulse responses of EMU's aggregated consumption and exports and EMU's real effective exchange rate. The largest effects in Figures 3.11 and B.7. are coming from shocks related to EMU's technology components. Here a one standard deviation shock leads to declining real marginal costs leading to sharp increases especially in EMU's GDP, exports and in EMU's real effective exchange rate. Because of the increased deflationary pressure caused by declining production costs the technology shock triggers an inducement of counter reacting measures by the ECB such that the ECB's controlled short term interest rate declines in tendency. The response pattern of EMU's investment and consumption activities are very similar to the patterns of EMU's GDP and exports. Risk premium shocks implying higher financing costs in the EMU and abroad lead in the case of domestically increased risk premiums to a decline in EMU's GDP and investment activities. In the case of increased risk premiums abroad, compared to there foreign competitors firms in the EMU have an advantage in there investment's risk-return profiles leading to an incentive for increasing their investment activities. For all shown response patterns there is a great uncertainty related to the EMU's macroeconomic state variable's responses to shocks coming from the foreign risk premium expressed in broader confidence intervals around the mean response. Looking at monetary policy induced shocks reveals a depressing reaction pattern for the EMU's GDP and investment activities. As in the case of higher risk premiums a direct increase in the short term interest rates leads to an increase in the EMU's real effective exchange rate. Interesting are the EMU economy's reactions on shocks coming from the ECB's inflation target. This monetary policy shock seems destabilizing in the sense, that it leads to decreasing EMU's GDP, investment, consumption and exports. Further a weakening of the ECB's target inflation rate leads to a devaluation of the Euro. Concerning the EMU's term structure of interest rates the ECB's monetary policy path shows an immediate increase in the ECB's controlled short term rate - initialized by an increase in the slope factor and therefore by a flattening of the term structure - expressing the term structure's flatting from its short end. In line with the expectation hypothesis ECB's short term interest rate's trajectory decreases in reaction to the flatting shock. In the middle term ECB's controlled short term policy rate reveals an upward directed trajectory. However the broader $[10\%, 90\%]$ confidence band around the policy rate's response path signals us, that we have to be aware about the greater deal of uncertainty the revealed ECB's short term interest rate's reaction path implies.

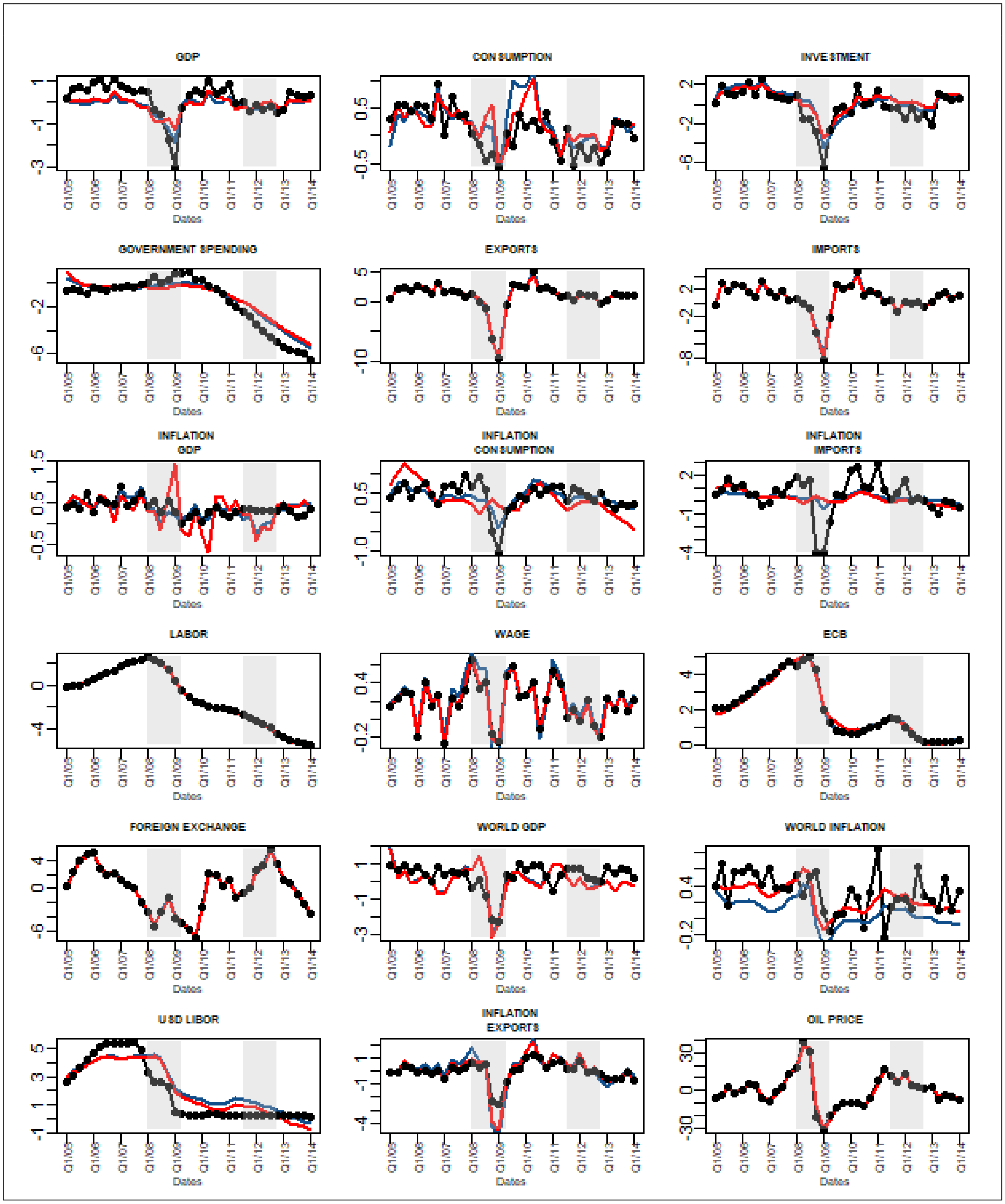


Table 3.9: Observed and NAWM implied aggregated area wide EMU macroeconomic variables between Q1/2005 and Q1/2014. The NAWM implied macroeconomic variables are evaluated at the mean of the model's posterior distribution. (blue baseline NAWM, red term structure extended NAWM).

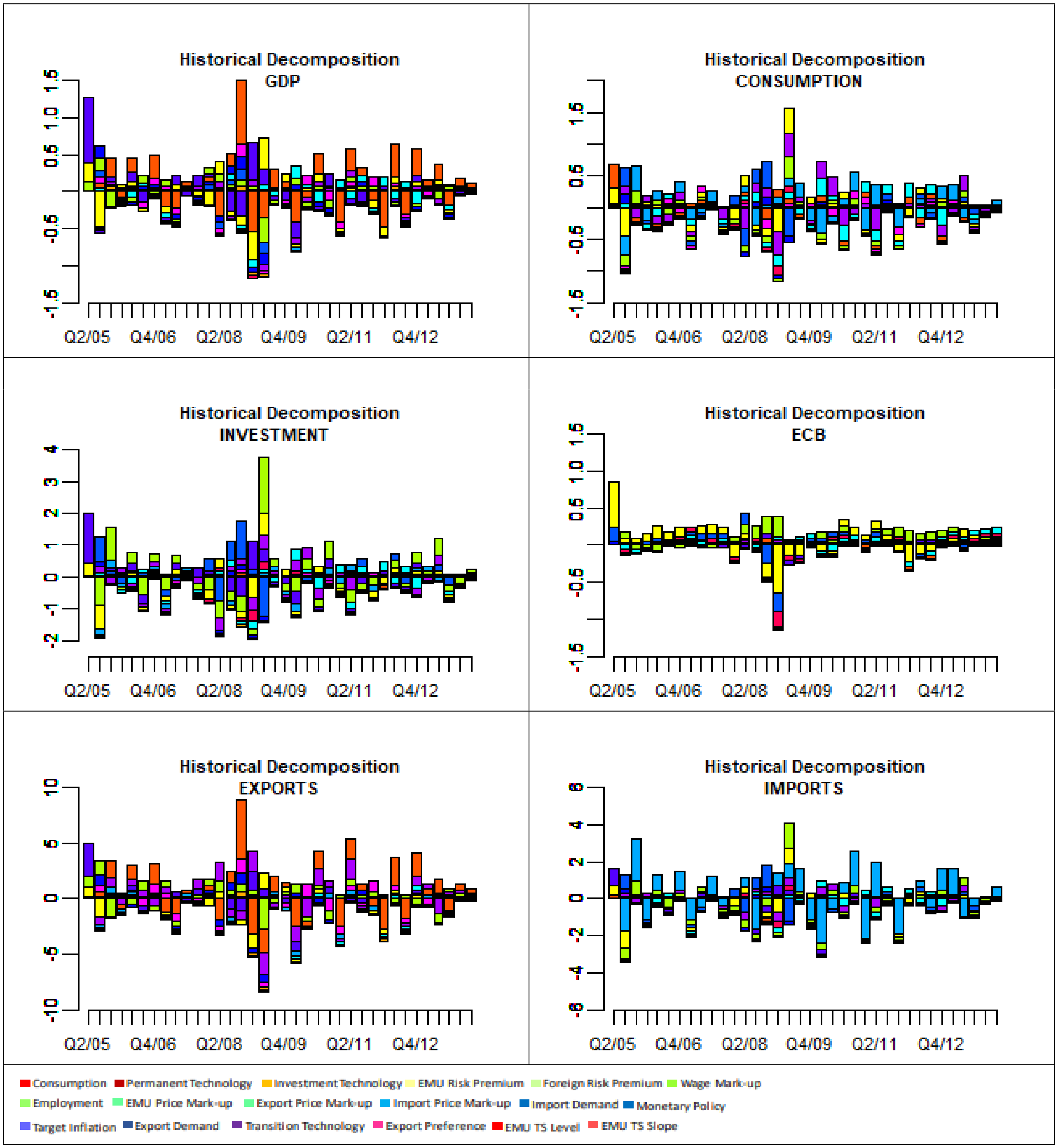


Table 3.10: Term structure extended NAWM's historic shock decomposition evaluated at the posterior's mode.

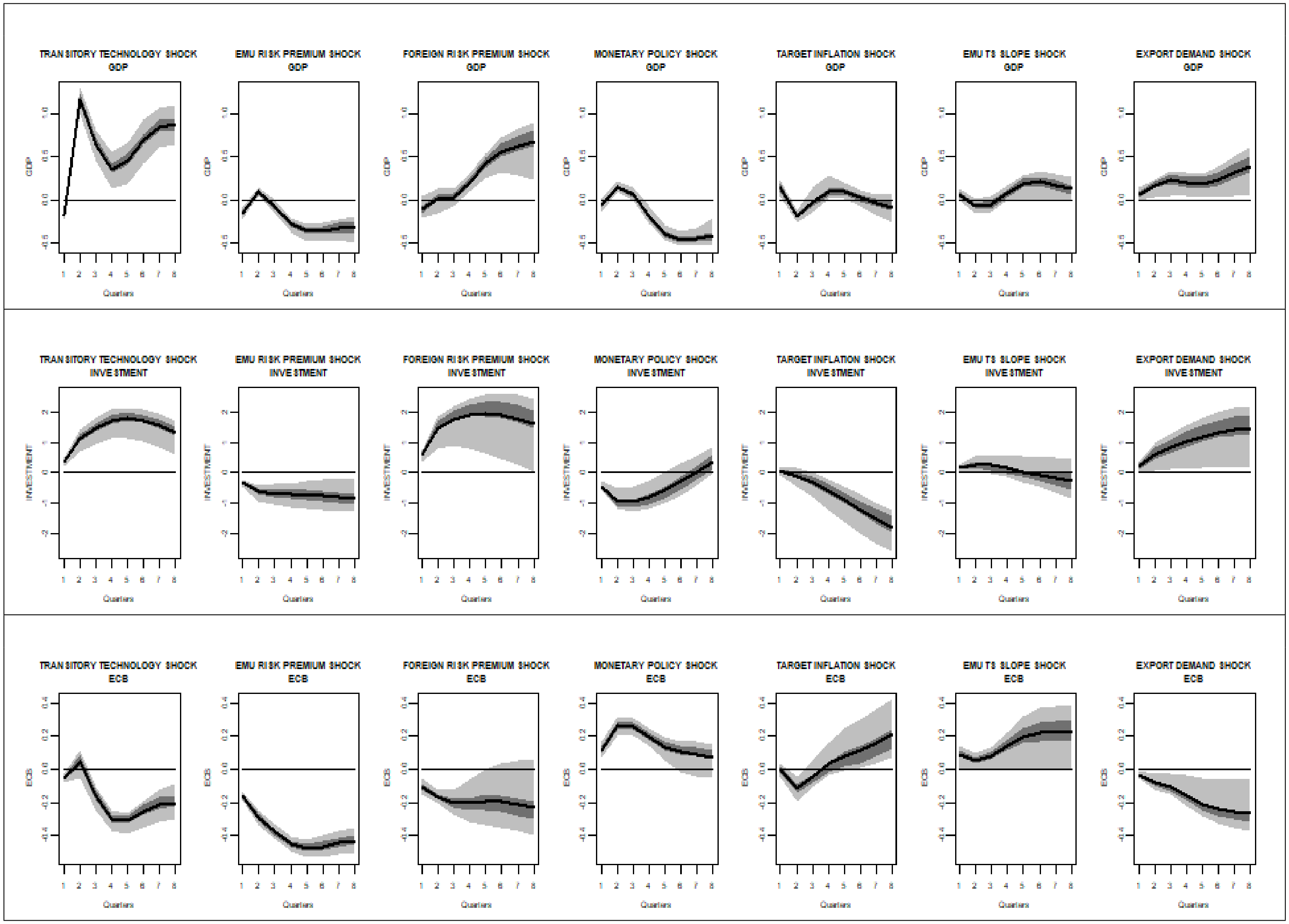


Table 3.11: Macroeconomic responses with respect to one standard deviation shocks coming from seven different economic sources. We show the responses of EMU's GDP y_t and investment i_t and ECB's short term interest rate r_t. Based on the models posterior distribution we compute 1000 impulse responses and report the mean and the [10% ,90%] and [30% ,70%] confidence intervals.

3.6 Conclusion

In this paper we have extended a large-scale open-economy DSGE of high practical relevance for ECB's and EMU's fiscal and monetary policy decisions by endognizing term structure of interest rate issues concerning EMU's economic development in the DSGE's implied monetary policy rule. Focusing on the time horizon ranging from Q1/2005 to Q1/2014 - before the ECB induced its expanded asset purchase program (EAPP) in Q4/2014 and the public sector purchase program (PSPP) in Q1/2015 - our findings are twofold. First in extracting country specific level and slope term structure factors for a larger subset of EMU member countries, we reveal the well known diverging development in the interest rates observed for the EMU's central and southern periphery members from a more unconventional perspective. Our factor extraction reveals the well known widening of the spreads in the interest rates' average levels among the EMU member countries. A not so well known aspect concerns the slope of the term structure. Here we find that after the economic breakdown following the Lehman bankruptcy in September 2008 the slope of the term structure sharply increases for all countries. For Germany, France and Italy the increase in the slope successively decreases whereas for Italy and Spain the interest rates differential between long and short term maturities becomes larger. This differential is essential for weighting long term investment decisions against their shorter counterparts. In our subsequent extraction of common EMU interest rate level and slope factors the increased heterogeneity among the EMU countries becomes again visible. In their country specific impact on these common EMU factors especially for longer maturities Italy and Spain fell behind the impact of Germany, France and the Netherlands on EMU's common interest rate developments. Second, with focus on the EMU's macroeconomic development we find that especially in the month around the collapse of Lehman Brother's in September 2008 monetary policy unfolded shocks to the EMU strongly effecting the development of EMU's macroeconomic aggregates such as GDP, consumption, investment, exports and imports. The monetary policy decisions by the ECB are itself affected largely by shocks concerning the risk-premiums paid to EMU's investors. Looking at the decisive decision of the sharp decrease in the short term interest rate in Q4/2008 and Q1/2009, where the ECB's main refinancing operations rate fell by 225 basis points from 3.75% in the beginning of November 2008 to 1.50% in March 2009 reveals, that this decision is also effected by shocks induced by a changing slope of the EMU's common term structure of interest rates. Clearly: With the decision made by the ECB the EMU's common term structure becomes much steeper, expressed in a decrease of the EMU's slope factor more or less synchronous to the ECB's short term rate decline. Related to the common EMU term structure factors we further find, that the EMU's common level factor has no significant effect on the EMU's macroeconomic development, whereas EMU's macroeconomic responses to common slope factor shocks are of the same magnitude as conventional short term interest rate shocks induced by monetary policy decisions. Our findings here should serve as an incentive for further research in this area. Especially a deeper understanding of the effects lying behind the response patterns to slope factor shocks concerning e.g. the balance sheet of the economy's financial intermediaries could be of special importance.

4. Macroeconomic Uncertainty and the Term Structure of InterestRates

4.1 Introduction

In this chapter we focus on the interdependencies between macroeconomic uncertainty and the uncertainty implied by the term structure of interest rates. Our key questions are: How can we extract the uncertainty patterns of the macroeconomy? Can we extract uncertainty patterns for a broader range of macroeconomic uncertainty sources? And most important: How can we combine these patterns of economic uncertainty with the term structure of interest rates? For answering these questions we combine a medium to large scale macroeconomic dynamic stochastic general equilibrium (DSGE) model extended by time-varying stochastic volatilites with an unspanned stochastic volatility affine term structure of interest rates model (USV-ATSM). The stochastic volatility DSGE allows us the extraction of uncertainty patterns from seven sources of macroeconomic uncertainty ranging from technology and productivity to the uncertainty about government spending and monetary policy activities. Our approach allows the spill over of the macroeconomic uncertainty measured by our stochastic volatility DSGE into the bond market for which we extract additional term structure specific uncertainty factors.

We are the first in giving an overall answer to the outlined questions where we focus here on Germany as a key member of the European Monetary Union (EMU) and Italy as the major country of the EMU's southern periphery states. Certain aspects of the outlined questions are central themes in the current research literature about macroeconomic and term-structure of interest rates modeling. If we split up these two topics as the two central strands of economic thinking in this paper, our macroeconomic strand of thought is related to the current work in the area of economic uncertainty in general e.g. by Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry [2018], Baker, Bloom and Davis [2016] and Bloom [2009] and to more specific non-linear macroeconomic DSGE models implying time-varying stochastic volatilities. Here our approach is related to the work by Justiano and Primiceri [2008], Fernandez-Villaverde and Rubio-Ramirez [2013] and Curdia, Del Negro and Greenwald [2014] and Diebold, Schorfheide and Shin [2017] where the first two cited works with its random walk postulation as the stochastic volatility's law of motions are very close to the approach we used here. The work by Curdia et. al. [2014] also uses a nonlinear DSGE with stochastic macroeconomic volatilities, but here the focus lies more on the usage of the t-distribution as a fat-tailed alternative to the Gaussian as the preferred distribution in modeling structural macroeconomic shocks. The methodological approach used by Diebold et. al. [2017] is similar to the one outlined in Justiano and Primiceri [2008], but here the three authors focus on the (point and density) forecasting performance of the methodology - an aspect we only have indirectly in mind by looking at the in-sample-fit performance, where we compare our approach especially with respect to our implied term structure modeling with a broad range of alternative term-structure of interest rate model implementations.

For combining the macroeconomic dynamics implied by our stochastic volatility DSGE model with the dynamics of the term structure of interest rates in an arbitrage-free model framework we use the (macro-finance) arbitrage-free modeling structure proposed by Ang and Piazzesi [2003] extended by Creal and Wu [2017] which allow (unspanned) stochastic volatility (USV) factors that (indirectly) affect the latent and observed factors driving the dynamics of the term structure of interest rates. Incorporating stochastic volatilities into the arbitrage-free modeling of the term structure of interest rates is a relatively novel field of research. Pioneering work was done by Cox, Ingersoll and Ross [1985] by assuming a stochastically driven volatility term in there one-factor dynamics underlying their arbitrage-free equilibrium term structure of interest rates. Later Collin-Dufresne and Goldstein [2002] proposed the class of USV models separating the dynamics of the term structure of interest rate factors from additional factors driving the stochastic volatilities of the term structure factors. More recent work done by Creal and Wu [2015] and by Cieslak and Povala [2016] who build on the work relating USV by endogenizing the whole term structure of interest rate volatilities in their arbitrage-free term structure of interest rate modeling. Beside the latent term structure of interest rate factors both works include latent term structure of interest rate volatility factors with unspanning and spanning characteristics for the model implied arbitrage-free bond pricing scheme. Different to our approach and the modeling approach proposed by Creal and Wu [2017] the cited works focus only on latent term structure factors without combining these with observed macroeconomic factors and their volatilities in describing the term structure of interest rates.

The integration of the term structure of interest rates into a larger macroeconomic modeling framework is addressed by Andreasen, Fernandez-Villaverde and RubioRamirez [2018], De Greave, Emiris and Wouters [2009], Rudebusch and Swanson [2008, 2012], Beakert, Cho and Moreno [2010], van Binsenberg, Fernandez-Villaverde, Koijen and Rubio-Ramirez [2012] or Kliem and Meyer-Gohde [2017]. Our modeling approach outlined here differs to these works in regarding time-varying volatility terms as additional endogenous components for depicting changes in the development of macroeconomic and term structure uncertainties, whereas the cited works only use constant volatility terms.

Our methodological approach outlined in this chapter is novel in twofold: First it extends the Smets-Wouters DSGE model by integrating the arbitrage-free affine term structure of interest rates model (ATSM), where instead of using constant interest rate volatilities a time-varying volatility approach is chosen. This is related to the second novelty of our approach. The second novelty lies in the modeling of time-varying volatilities in the combined DSGE and term structure approach. Here we methodologically introduce time-varying uncertainty patterns of both the macroeconomic as well as of the term structure determining variables.

This chapter is organized as follows: In the first part of section 4.2 we outline in detail the intertemporal decision problems the agents in the various sectors of our economy face. These are the sector specific decision problems formulated by Smets and Wouters (SW) [2003, 2007]. Using the SW-DSGE as the macroeconomic component of our DSGE-USV-ATSM is consequent because of its prototypical character in the current DSGE literature and its empirical success as documented in Smets and Wouters [2007], Edge and Gurkaynak [2010] or Del Negro and Schorfheide [2013]. In section 4.2 we introduce the DSGE implied stochastic volatilities of the structural macroeconomic shock variables. We further outline the rational expectations building for solving our extended DSGE component. The USV-ATSM component is outlined in the second half of section 4.2. Here we introduce the general law of motions of the term structure of interest rate factors and their volatility factors as well as the arbitrage-free bond pricing scheme from which the term structure of interest rates results. In section 4.3 we focus on the specification of our model implementation and the DSGE and ATSM blockwise Gibbs sampling Bayesian estimation procedure we apply. To keep our DSGE-USV-ATSM tractable we specify the DSGE-USV-ATSM in a monocausal way in assuming that developments in the macroeconomic environment directly spill over into the bond market, whereas specific developments of the bond markets do not directly effect our macroeconomic environment.

In section 4.4 we outline our empirical findings. Here we focus on the period between Q1/2005 and Q1/2014 - a phase where the Euro and the EMU institutions become more settled and before ECB initializes its unconventional expanded asset purchase program (EAPP) in Q4/2014 and the public sector purchase program (PSPP) in Q1/2015 - but with critical events such as the starting of the sharp decline of U.S. housing prices in Q1/Q2 2007 and the FED's intervention by lowering its federal funds rate from 5.25% in September 2007 to 0.25% in December 2008. Further there is the bankruptcy of Lehman Brothers in September 2008 and the spillover of the U.S. subprime and financial markets crisis to the EMU - becoming here a sovereign debt crisis with the ECB's short term interest rates lowering intervention especially since Q4/2008. Beside the evaluation of the quality and robustness of our estimation in the beginning of section 4.4, we further discuss in detail our extracted macroeconomic and term structure of interest rate volatility patterns as well as the patterns of economic uncertainty shocks occurring more or less simultaneously to critical economic and political events implied by our data sample. Both our macroeconomic as well as our term structure of interest rates uncertainty patterns show large peaks especially in the recession phase Q2/2008 and Q2/2009 with the bankruptcy of Lehman Brothers in September 2008 and the sharp 225 basis point decrease in the ECB's controlled short term main refinancing operations rate from 3.25% to 1.00% between Q4/2008 and Q2/2009. For Italy we find that especially in Q1/Q2 2007 - at the time of the decline in the U.S. housing prices and the FED's reaction in decreasing its federal funds rate following this decline - there are large macroeconomic and term structure uncertainty shocks strongly effecting the Italian economy and the Italian term structure of interest rates. We further find that the resignation of Mario Monti and his cabinet and the dissolution of the Italian parliament in December 2012, that interrupts the Italian reform program marks a further crucial event inducing high uncertainty to both the Italian economy as a whole and to the market of Italian government bonds in particular. Our conclusion in section 4.5 summarizes our findings and their respective political interpretations.

4.2 Macroeconomic and term structure modeling

4.2.1 Macroeconomy and its uncertainty

4.2.1.1 Final goods sector decisions

The final good Y_t is composed of a continuum of intermediate goods $Y_t(i)$ produced in the sector of intermediate goods i. The final goods producers sell their products to consumers, investors and the government and act as price takers in a perfectly competitive market, where they face the following profit maximization problem with respect to the decision about the amount Y_t to sell on the market for final goods and the amount $Y_t(i)$ to buy from the intermediate producers i:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \tag{4.1}$$

subject to the final goods production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{(1+\varepsilon_t^p)}} di \right)^{\left(1+\varepsilon_t^p\right)} \tag{4.2}$$

P_t and $P_t(i)$ are the prices in the final and intermediate goods sectors respectively. ε_t^P is a price mark-up shock which influences the production process of the final goods producers and is specified as:

$$ln\left(\varepsilon_t^p\right) = (1-\rho) ln\left(\epsilon_p\right) \rho_P ln\left(\varepsilon_{t-1}^p\right) + \sigma_p \epsilon_t^p \quad \epsilon_t^p \sim N\left(0,1\right) \tag{4.3}$$

4.2.1.2 Intermediate goods sector decisions

At every time t the intermediate goods producers i have to solve the following profit maximization problem:

$$\max_{Y_t(i), L_t(i), K_t^s(i)} P_t(i) Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i) \tag{4.4}$$

subject to i th intermediate producers used production technology:

$$Y_t(i) = \varepsilon_t^a K_t^s(i)^\alpha \left(\gamma^t L_t(i)\right)^{(1-\alpha)} - \gamma^t \phi \tag{4.5}$$

where the production factors are the capital service used in the economy's production process $K_t^s(i)$ and labour $L_t(i)$. W_t and R_t^k are the aggregated nominal wage and the rental rate on capital.γ^t is the labour augmented deterministic growth rate of the economy and ϕ is a general fixed cost factor which negatively effects the production process. The production process in the intermediate sector is disturbed by an exogenous log-normal process:

$$ln\left(\varepsilon_t^a\right) = \rho_a ln\left(\varepsilon_{t-1}^a\right) + \sigma_a \epsilon_t^a \tag{4.6}$$

where $\epsilon_t^a \sim N(0,1)$ is standard normal.

Price setting in the intermediate sector faces nominal rigidities. We consider price-setting as proposed by Calvo [1983] where only a fraction $(1-\xi_P)$ with $0 \leq \xi_P \leq 1$ of contracts expire each period and are renegotiated by the participants. The renegotiating firms set their prices according to their optimal nominal price $\tilde{P}_t(i)$. All other firms set their prices according to:

$$P_{t+s}(i) = \tilde{P}_t(i) X_{t,s} \tag{4.7}$$

where

$$X_{t,s} = \begin{cases} 1 \quad for \quad s = 0 \\ \prod_{m=1}^{s} \gamma \pi_{t+m-1}^{\iota_p} \bar{\pi}^{(1-\iota_p)} \quad \forall s \in \{1, 2, ..., \infty\} \end{cases} \tag{4.8}$$

so that the fraction ξ_p of firms in the intermediate sector, which are not part of the renegotiations passively adjust their prices according to a weighted average of the steady-state inflation rate $\bar{\pi}$, last period's inflation rate π_{t-1} and the general growth rate γ of the economy. The prices setting of the producers in the intermediate sector is described by the following optimization problem:

$$\max_{\tilde{P}_t(i)} \Lambda\left(\tilde{P}_t(i)\right) = \max_{\tilde{P}_t(i)} \mathbb{E}_t \left[\frac{\xi_p^s \beta^s \lambda_{t+s}^{HI} P_t}{\lambda_t^{HI} P_{t+s}} \left(\tilde{P}_t(i) X_{t,s} - \lambda_{t+s}^{IG}\right) Y_{t+s}(i)\right] \tag{4.9}$$

subject to the final goods producers optimal demand for intermediate goods:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1-\varepsilon_t^p)}{\varepsilon_t^p}} Y_t \tag{4.10}$$

where λ_{t+s}^{IG} are the marginal costs of the intermediate sector.

4.2.1.3 Household decisions

At every time t household j faces the following utility maximization problem:

$$\max_{C_t(j),L_t(j),B_t(j),I_t(j),Z_t(J)} U\left(C_t(j),C_{t-1}(j),L_t(j)\right) \tag{4.11}$$

where the household's time t maximization problem is embedded in the intertemporal optimization problem of maximizing the expected utility:

$$\mathbb{E}_t\left[\sum_{h=0}^{\infty}\beta^h U\left(C_{t+h}(j),C_{t+h-1}(j),L_{t+h}(j)\right)\right] \tag{4.12}$$

with the household's utility function specified as:

$$U\left(C_{t+h}(j),C_{t+h-1}(j),L_{t+h}(j)\right)=\frac{\left(C_{t+h}(j)-\lambda C_{t+h-1}(j)\right)^{(1-\sigma_c)}}{(1-\sigma_c)}exp\left(\frac{(\sigma_c-1)}{(1+\sigma_l)}L_{t+h}(j)^{(1+\sigma_l)}\right) \tag{4.13}$$

For this maximization problem the following two constraints hold:

$$\begin{aligned}C_{t+h}(j)+I_{t+h}(j)+\frac{B_{t+h}(j)}{\varepsilon_t^b R_{t+h}P_{t+h}}-T_{t+h}\leq&\frac{B_{t+h-1}(j)}{P_{t+h}}+\frac{W_{t+h}(j)L_{t+h}(j)}{P_{t+h}}\\&+\frac{R^k_{t+h}(j)Z_{t+h}(j)K_{t+h-1}(j)}{P_{t+h}}\\&-a\left(Z_{t+h}(j)\right)K_{t+h-1}(j)+\frac{D_{t+h}}{P_{t+h}}\end{aligned} \tag{4.14}$$

$$K_{t+h}(j)=(1-\delta)K_{t+h-1}+\varepsilon_t^i\left[1-S\left(\frac{I_{t+h}(j)}{I_{t+h-1}(j)}\right)\right]I_{t+h}(j) \tag{4.15}$$

where the first constraint is the household's budget restriction with respect to the household's consumption C_{t+h}, investment I_{t+h} and saving behavior (netted by regarding the lump sum tax T_{t+h}) on the one side and the income cash flows from saving, labor, direct capital investments and dividends D_{t+h} on the other side. Saving is done by buying one period bonds B_{t+h} with yield R_{t+h} and stochastic log-normal risk premium term:

$$ln\left(\varepsilon_t^b\right)=\rho_b ln\left(\varepsilon_{t-1}^b\right)+\sigma_b\epsilon_t^b \qquad \epsilon_t^b\sim N(0,1) \tag{4.16}$$

Labor income is determined by the working hours L_{t+h} and wage W_{t+h}. Capital income is determined by the effective capital service $K^s_{t+h}=Z_{t+h}K_{t+h-1}$ directly used in the production process and the cost of capital utilization $a\left(Z_{t+h}\right)K_{t+h-1}$. Z_{t+h} indicates the degree of the economy's capital utilization. The second constraint is the capital accumulation equation. δ is the depreciation ratio of capital and S (...) is the adjustment cost function, indicating the fraction of investment $S(...)I_{t+h}$ necessary to increase the economy's capital stock by the investments I_{t+h} done at time $t+h$ for some $h\geq 0$.

$$ln\left(\varepsilon_t^i\right)=\rho_i ln\left(\varepsilon_{t-1}^i\right)+\sigma_i\epsilon_t^i \qquad \epsilon_t^i\sim N(0,1) \tag{4.17}$$

is the exogenous log-normal shock on the investment component of the capital accumulation equation.

4.2.1.4 Labor market decisions

Demand and supply side of the labour market are organized as follows. The supply side consists of differentiated labour services $L_t(l)$ offered by the households. On the demand side there are the intermediate goods producer which are confronted with the various labour services. For reducing the complexity resulting from the labor fragmentation, there are labor packers as intermediaries between the households and the goods producers. The labor packers bundled the differentiated labor services to labor service packages L_t and offer them to the producers in the intermediate goods sector. For negotiation purposes every labor service l is represented by a union which negotiates their wages with the labor packers. Labor packers act profit orientated and therefore face the following profit maximization problem:

$$\max_{L_t, L_t(j)} W_t L_t - \int_0^1 L_t(l) W_t(l) dl \tag{4.18}$$

subject to:

$$L_t = \left(\int_0^1 L_t(l)^{\frac{1}{(1+\varepsilon_t^w)}} dl \right)^{(1+\varepsilon_t^w)} \tag{4.19}$$

where the exogenous shock process of ε_t^w is specified as:

$$ln\left(\varepsilon_t^w\right) = (1-\rho_w) ln\left(\varepsilon_w\right) + \rho_w ln\left(\varepsilon_{t-1}^w\right) \sigma_w \epsilon_t^w \qquad \epsilon_t^w \sim N(0,1) \tag{4.20}$$

In their wage negotiations labor unions face nominal wage rigidities. Union's wage negotiations are described by using a Calvo scheme with partial indexation, where $(1-\xi_w)$ with $0 \leq \xi_W \leq 1$ labor unions can actively readjust their wages and set them to $\tilde{W}_t(l)$ each period. On the contrary this implies that ξ_w unions do not readjust their wages. They passively set their prices $W_t(l)$ according to the growth rate γ and a weighted average of the steady-state inflation rate $\bar{\pi}$ and last period's inflation rate π_{t-1}, so that the wage setting decision between labor unions and labor packers is determined by:

$$\max_{\tilde{W}_t(l)} \Lambda\left(\tilde{W}_t(l)\right) = \max_{\tilde{W}_t(l)} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \frac{\xi_w^s \beta^s \lambda_{t+s}^{HI} P_t}{\lambda_t^{HI} P_{t+s}} \left(\tilde{W}_{t+s}(i) - W_{t+s} \right) L_{t+s} \right] \tag{4.21}$$

with the labor packers optimal demand for differentiated labor services:

$$L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\frac{(1+\lambda_{w,t})}{\varepsilon_t^w}} L_t \tag{4.22}$$

and the mentioned passive price-setting rule with respect to growth and inflation:

$$W_{t+s}(l) = X_{t,s} \tilde{W}_t(l) \tag{4.23}$$

with:

$$X_{t,s} = \left\{ \begin{array}{cc} 1 \quad for \quad s=0 & \\ \prod_{m=1}^{s} \gamma \pi_{t+m-1}^{\iota_w} \bar{\pi}^{(1-\iota_w)} & \forall s \in \{1,2,...,\infty\} \end{array} \right. \tag{4.24}$$

4.2.1.5 Monetary and fiscal policy decisions

The monetary decision rule is specified by a Taylor rule type monetary policy function in which the central bank sets the short term interest rate in reaction to the inflation and output gap and to the change in the output gap so that the central bank's decision rule is defined as:

$$r_t = \rho r_{t-1} - \rho\tilde{r} + (1-\rho)\left(r_\pi ln\left(\frac{\pi_t}{\tilde{\pi}}\right) + r_y\left(\frac{y_t}{\tilde{y}_t}\right)\right) + r_{\Delta y} ln\left(\frac{(y_t/\tilde{y}_t)}{(y_{t-1}/\tilde{y}_{t-1})}\right) + ln\left(\varepsilon^r\right) \quad (4.25)$$

where $\tilde{r}$ and $\tilde{\pi}$ are short term interest rate and inflation rate in the steady-state. $\tilde{y}$ is the potential output under full price and wage flexibility. π_t and y_t are inflation rate and output the central bank focuses on in ites decision finding.

$$ln\left(\varepsilon_t^r\right) = \rho_r ln\left(\varepsilon_{t-1}^r\right) + \sigma_r \epsilon_t^r \qquad \epsilon_t^r \sim N(0,1) \quad (4.26)$$

is the log-normally distributed monetary policy shock. The government faces in its fiscal policy decisions the following budget constraint:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \quad (4.27)$$

where the LHS indicates the government's expenditures for general public sector activities and debt redemption, whereas the RHS indicates the government's revenues from taxes and credit. The government expenditures G_t are described by the random process:

$$G_t = \varepsilon_t^g \quad (4.28)$$

with:

$$ln\left(\varepsilon_t^g\right) = \rho_g ln\left(\varepsilon_{t-1}^g\right) + \sigma_g \epsilon_t^g \qquad \epsilon_t^g \sim N(0,1) \quad (4.29)$$

4.2.1.6 Macroeconomic uncertainty

For the seven macroeconomic shock processes

$$\boldsymbol{\varepsilon}_t^T = \left[\varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^g, \varepsilon_t^i, \varepsilon_t^r, \varepsilon_t^p, \varepsilon_t^w\right]$$

instead of using time-invariant volatilities in their respectiv law of montions outlined in the above equations we use now time-varying stochastic volatility terms

$$\boldsymbol{\varepsilon}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t}]$$

which are desctibed in their dynamics by the following log-normal VAR[1] process:

$$ln\left(\boldsymbol{\sigma}_t\right) = \boldsymbol{\mu}_\sigma + \mathbf{P}_\sigma ln\left(\boldsymbol{\sigma}_{t-1}\right) + \boldsymbol{\Sigma}_\sigma \boldsymbol{\omega}_t \quad (4.30)$$

where $\boldsymbol{\mu}_\sigma$ is the 7×1 vector of constants and $\mathbf{P}_\sigma$ and $\boldsymbol{\Sigma}_\sigma$ are the 7×7 diagonal coefficient and covariance matrix with

$$\boldsymbol{p}_\sigma^T = \left[p_\sigma^a, p_\sigma^b, p_\sigma^g, p_\sigma^i, p_\sigma^r, p_\sigma^p, p_\sigma^w\right]$$

and

$$\boldsymbol{\sigma}_\sigma^T = \left[\sigma_\sigma^a, \sigma_\sigma^b, \sigma_\sigma^g, \sigma_\sigma^i, \sigma_\sigma^r, \sigma_\sigma^p, \sigma_\sigma^w, \right]$$

where $diag\left(\mathbf{P}_\sigma\right) = \boldsymbol{p}_\sigma$ and $diag\left(\boldsymbol{\Sigma}_\sigma\right) = \boldsymbol{\sigma}_\sigma$ respectively. $\boldsymbol{\omega}_t$ is the 7×1 vector of Gaussian uncertainty shocks:

$$\boldsymbol{\omega}_t \sim N\left(\mathbf{0}, \mathbf{I}_{7\times7}\right) \tag{4.31}$$

We specify the VAR[1] in such a way that the (log) stochastic volatilities evolve independently to each other. The process is specified in logs to guarantee only nonnegative standard deviations at every t.

4.2.2 Building rational expectations in an uncertain environment

As described in Herbst and Schorfheide [2016] or Dejong and Dave [2011] to determine the agent's expectations in a first step the log-linearized equations for the 14 endogenous macroeconomic variables of our used DSGE model under sticky and flexible price-wage setting are transferred into the canonical linear rational expectations form:

$$\boldsymbol{\Gamma}_0 \boldsymbol{s}_t = \boldsymbol{\Gamma}_1 \boldsymbol{s}_{t-1} + \boldsymbol{\Psi}(\boldsymbol{\sigma}_t)\boldsymbol{\epsilon}_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t \tag{4.32}$$

where

$$\begin{aligned}\boldsymbol{s}_t^T = [&y_t, c_t, i_t, q_t, k_t^s, z_t, k_t, \mu_t^P, \pi_t, r_t^k, \mu_t^w, w_t, r_t, l_t, \varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^g, \varepsilon_t^i, \varepsilon_t^r, \varepsilon_t^p, \varepsilon_t^w, \mathbb{E}_t\left[c_{t+1}\right], \mathbb{E}_t\left[i_{t+1}\right],\\ &\mathbb{E}_t\left[l_{t+1}\right], \mathbb{E}_t\left[\pi_{t+1}\right], \mathbb{E}_t\left[q_{t+1}\right], \mathbb{E}_t\left[r_{t+1}^k\right], \mathbb{E}_t\left[w_{t+1}\right], \tilde{y}_t, \tilde{c}_t, \tilde{i}_t, \tilde{q}_t, \tilde{k}_t^s, \tilde{z}_t, \tilde{k}_t, \tilde{r}_t^k, \tilde{w}_t, \tilde{r}_t, \tilde{l}_t,\\ &\mathbb{E}_t\left[\tilde{c}_{t+1}\right], \mathbb{E}_t\left[\tilde{i}_{t+1}\right], \mathbb{E}_t\left[\tilde{l}_{t+1}\right], \mathbb{E}_t\left[\tilde{q}_{t+1}\right], \mathbb{E}_t\left[\tilde{r}_{t+1}^k\right], y_{t-1}, c_{t-1}, i_{t-1}, w_{t-1}, \tilde{y}_{t-1}]\end{aligned}$$

defines the 49×1 state vector.

$$\boldsymbol{\epsilon}_t^T = \left[\epsilon_t^a, \epsilon_t^b, \epsilon_t^g, \epsilon_t^i, \epsilon_t^r, \epsilon_t^p, \epsilon_t^w\right]$$

is the 7×1 vector of stochastic innovations and

$$\begin{aligned}\boldsymbol{\eta}_t^T = [&\pi_t - \mathbb{E}_{t-1}\left[\pi_t\right], c_t - \mathbb{E}_{t-1}\left[c_t\right], l_t - \mathbb{E}_{t-1}\left[l_t\right], q_t - \mathbb{E}_{t-1}\left[q_t\right], r_t^k - \mathbb{E}_{t-1}\left[r_t^k\right], i_t - \mathbb{E}_{t-1}\left[i_t\right],\\ &w_t - \mathbb{E}_{t-1}\left[w_t\right], \tilde{c}_t - \mathbb{E}_{t-1}\left[\tilde{c}_t\right], \tilde{l}_t - \mathbb{E}_{t-1}\left[\tilde{l}_t\right], \tilde{q}_t - \mathbb{E}_{t-1}\left[\tilde{q}_t\right], \tilde{r}_t^k - \mathbb{E}_{t-1}\left[\tilde{r}_t^k\right], \tilde{i}_t - \mathbb{E}_{t-1}\left[\tilde{i}_t\right]]\end{aligned}$$

is the 12×1 vector of expectation errors. $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_1$ are 49×49 matrices. $\boldsymbol{\Psi}(\boldsymbol{\sigma}_t)$ and $\boldsymbol{\Pi}$ are 49×7 and 49×12 matrices respectively, relating the vectors of innovations and expectation errors $\boldsymbol{\epsilon}_t$ and $\boldsymbol{\eta}_t$ to the dynamics of the state variables $\boldsymbol{s}_t$. Different to the expression of the canonical linear rational expectations form of the DSGE with constant volatility is the time-varying matrix $\mathbf{\Psi}(\boldsymbol{\sigma}_t)$ dependent on the stochastic volatility factors $\boldsymbol{\sigma}_t$ of $\boldsymbol{\epsilon}_t$. We specify the stochastic matrix $\mathbf{\Psi}(\boldsymbol{\sigma}_t)$ in the above canonical form as follows:

$$\mathbf{\Psi}(\boldsymbol{\sigma}_t)^T = \sum_{i=1}^{7} \left[\mathbf{0}_{14\times 7}, \mathbf{\Delta}_{i,i}\boldsymbol{\sigma}_t\boldsymbol{\delta}_i^T, \mathbf{0}_{28\times 7}\right] \tag{4.33}$$

where $\delta_{i,i}$ is a 7×7 matrix of zeros except the element at position (i,i) which is set to 1 and $\boldsymbol{\delta}_i$ is an indicator vector of zeros except at position i where it is set to 1. The sum in the expression of $\mathbf{\Psi}(\boldsymbol{\sigma}_t)$ leads to a diagonal matrix with the vector $\boldsymbol{\sigma}_t$ of volatilities on its diagonal. To solve the model we use Sim's QZ algorithm using the generalized Schur decomposition briefly outlined in 2.2.2.2 at every time step t to get the solution:

$$\boldsymbol{s}_t = \mathbf{\Theta}_0\boldsymbol{s}_{t-1} + \mathbf{\Theta}(\boldsymbol{\sigma}_t)\boldsymbol{\epsilon}_t \tag{4.34}$$

where the covariance matrix of the state variables $\mathbf{\Theta}(\boldsymbol{\sigma}_t)$ becomes stochastic.

4.2.3 Term structure of interest rates and its uncertainty

4.2.3.1 Implied factor dynamics of the term structure of interest rates

For modeling the term structure of interest rates in an uncertain economic environment we use an arbitrage free affine term structure model with unspanned stochasitc volatility (USV-ATSM). As in the baseline ATSM proposed by Ang and Piazzesi [2003] and more recently discussed by Hamilton and Wu [2012, 2014], where the term structure does not react on changes in the uncertainty of the economic environment or in more recent spanned and unspanned stochastic volatility ATSM's proposed by Cieslak and Povala [2016] and Creal and Wu [2015, 2017], where the models directly react on changes in the economic uncertainty, the factors which determine the dynamical behavior of the term structure of interest rates are described by the following VAR[1] process:

$$\boldsymbol{f}_t = \boldsymbol{\mu}_f + \mathbf{\Psi}_f\boldsymbol{f}_{t-1} + \mathbf{\Sigma}_{f,t}\boldsymbol{\varepsilon}_t \tag{4.35}$$

The factors $\boldsymbol{f}_t$ of the system in 4.35 are $\boldsymbol{f}_t^T = [\mathbf{s}_t, \boldsymbol{\sigma}_t, \mathbf{g}_t, \mathbf{h}_t]$, where $\mathbf{s}_t$ and $\boldsymbol{\sigma}_t$ are the 49 state variables from 4.32 and the 7 DSGE stochastic volatility factors determined in 4.30. The conditional mean $\boldsymbol{\mu}_f$ is specified as $\boldsymbol{\mu}_f^T = \left[\boldsymbol{\mu}_m, \boldsymbol{\mu}_\sigma, \boldsymbol{\mu}_g, \boldsymbol{\mu}_h\right]$, whereas we define the unconditional mean $\bar{\boldsymbol{\mu}}_f$ as $\bar{\boldsymbol{\mu}}_f = \left[\mathbf{I}_{N\times N} - \mathbf{\Psi}_f\right]^{-1}\boldsymbol{\mu}_f$. From the rational equilibrium solution expressed in 4.34 we set $\boldsymbol{\mu}_m = \mathbf{0}$. The coefficient matrix $\mathbf{\Psi}_f$ and time-varying covariance $\mathbf{\Sigma}_{f,t}$ can be expressed in general block matrix form as:

$$\mathbf{\Psi}_f = \begin{bmatrix} \mathbf{\Psi}_m & \mathbf{\Psi}_{m,\sigma} & \mathbf{\Psi}_{m,g} & \mathbf{\Psi}_{m,h} \\ \mathbf{\Psi}_{\sigma,m} & \mathbf{\Psi}_\sigma & \mathbf{\Psi}_{\sigma,g} & \mathbf{\Psi}_{\sigma,h} \\ \mathbf{\Psi}_{g,m} & \mathbf{\Psi}_{g,\sigma} & \mathbf{\Psi}_g & \mathbf{\Psi}_{g,h} \\ \mathbf{\Psi}_{h,m} & \mathbf{\Psi}_{h,\sigma} & \mathbf{\Psi}_{h,g} & \mathbf{\Psi}_h \end{bmatrix} \quad \mathbf{\Sigma}_{f,t} = \begin{bmatrix} \mathbf{\Sigma}_{m,t} & \mathbf{0}_{M\times S} & \mathbf{0}_{M\times G} & \mathbf{0}_{M\times H} \\ \mathbf{\Sigma}_{\sigma,m} & \mathbf{\Sigma}_\sigma & \mathbf{0}_{S\times G} & \mathbf{0}_{S\times H} \\ \mathbf{\Sigma}_{g,m} & \mathbf{\Sigma}_{g,\sigma} & \mathbf{\Sigma}_{g,t} & \mathbf{0}_{G\times H} \\ \mathbf{\Sigma}_{h,m} & \mathbf{\Sigma}_{h,\sigma} & \mathbf{\Sigma}_{h,g} & \mathbf{\Sigma}_h \end{bmatrix} \tag{4.36}$$

To keep the model and the estimation problem as simple as possible, we set all off-diagonal matrices in $\boldsymbol{\Psi}_f$ and $\boldsymbol{\Sigma}_{f,t}$ to zero and define the diagonal elements $\boldsymbol{\Psi}_m, \boldsymbol{\Psi}_\sigma, \boldsymbol{\Psi}_g$ and $\boldsymbol{\Psi}_h$ of $\boldsymbol{\Psi}_f$ with $\boldsymbol{\Psi}_m = \boldsymbol{\Theta}_0$ and $\boldsymbol{\Psi}_\sigma = \mathbf{P}_\sigma$ from 4.34 and 4.30. $\boldsymbol{\Psi}_g$ and $\boldsymbol{\Psi}_h$ are $G \times G$ and $H \times H$ matrices. Both matrices are restricted such that there eigenvalues are modulus less than one. The diagonal elements $\boldsymbol{\Sigma}_{m,t}, \boldsymbol{\Sigma}_\sigma, \boldsymbol{\Sigma}_{g,t}$ are defined as $\boldsymbol{\Sigma}_{m,t} = \boldsymbol{\Theta}(\boldsymbol{\sigma}_t)$ from 4.32, $\boldsymbol{\Sigma}_\sigma$ from 4.30 $\boldsymbol{\Sigma}_{g,t} = \boldsymbol{\Sigma}_g \boldsymbol{\Lambda}_{g,t}$ and $\boldsymbol{\Sigma}_h$ as lower triangular. $\boldsymbol{\Sigma}_g$ is also specified as lower triangular. $\boldsymbol{\Lambda}_{g,t}$ is a diagonal matrix, where the diagonal elements of $\boldsymbol{\Lambda}_{g,t}$ express the impact of the ATSM stochastic volatility factors $\mathbf{h}_t$ on the yield factors' volatilities. In our implementation we choose $G = 2$ as the number of pure latent yield factors and $H = 1$ as the number of stochastic ATSM volatility factors. M is the number of macroeconomic factors, that is equal to 49. With $H = 1$ we specify $\boldsymbol{\Lambda}_{g,t}$ with $diag\left(\boldsymbol{\Lambda}_{g,t}\right)[h_t, h_t]$. Choosing $G = 2$ and $H = 1$ the Gaussian term $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{I})$ in 4.34

$$\boldsymbol{\varepsilon}_t^T = \left[\varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^g, \varepsilon_t^i, \varepsilon_t^r, \varepsilon_t^p, \varepsilon_t^w, \omega_t^a, \omega_t^b, \omega_t^g, \omega_t^i, \omega_t^r, \omega_t^p, \omega_t^w, \varepsilon_{g,1,t}, \varepsilon_{g,2,t}, \varepsilon_{h,1,t}\right]$$

contains the idiosyncratic shocks coming from the macroeconomic environment and its sources of uncertainty as well as from the bond market and its specific source of uncertainty. The important aspect of the outlined model specification of the process in 4.35 is the orthogonality especially between the macroeconomic state variables $\mathbf{s}_t$ and the latent yield factors $\mathbf{g}_t$ expressed in the diagonal block matrices $\boldsymbol{\Psi}_f$ and $\boldsymbol{\Sigma}_{f,t}$. As in the two-step estimation procedure applied by Ang and Piazzesi [2003] for their macro-finance ATSM with this specification it becomes possible to decouple the estimation of the stochastic volatility DSGE model component from the estimation of the stochastic volatility ATSM component.

To check the robustness of our model implications and for further insights we apply an alternative model specification proposed by Creal and Wu [2017]. Here the macroeconomic sector only consists of two macroeconomic variables following a VAR process and there are multi causal relationships between the model's macroeconomic and yield factors as well as between their volatility factors. We use the rotation of the three dimensional vector of yield factors $\mathbf{g}_t$ outlined by Creal and Wu for interpreting $\mathbf{g}_t$ as the current and future short rate r_t and $\mathbb{E}_t\left[r_{t+n^*}\right]$, where r_{t+n^*} is the n^* periods ahead expected short rate and the term premium $TP(t, \bar{\tau})$ implied by the $\bar{\tau}$ maturity yield. The maturity $\bar{\tau}$ is defined as $\bar{\tau} = t + n^*$. In this interpretation the three latent factors, which are commonly - due to there pure statistical character - abstract now get strong monetary policy implications. The alternative specification of $\mathbf{f}_t, \boldsymbol{\mu}_f, \boldsymbol{\Psi}_f$ and $\boldsymbol{\Sigma}_f$ as well as of the parameters $\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}_g^Q, \boldsymbol{\Psi}_g^Q$ and $\boldsymbol{\Sigma}_g^Q$ of the pricing scheme described in the next section and the estimation procedure applied to this alternative model specification, are outlined in detail in Appendix C.1.1.

4.2.3.2 Arbitrage-free bond pricing scheme in the USV-ATSM

As in the yields only ATSM by Ang and Piazzesi [2003] the short rate r_t is a linear affine function of the yield factors $\boldsymbol{g}_t$:

$$r_t = \delta_0 + \boldsymbol{\delta}_1^T \boldsymbol{g}_t \tag{4.37}$$

where δ_0 is a constant and the $G \times 1$ parameter vector $\boldsymbol{\delta}_1$ describes the impact of the M monetary policy factors on the short rate r_t. Arbitrage-free bond prices $P(t,\tau)$ at time t with time to maturity $\tau \geq 0$ are described by a martingale process:

$$P(t,\tau) = \mathbb{E}_t^Q \left[exp(-r_t)P(t+1,\tau-1)\right] \tag{4.38}$$

under the probability measure Q which implies the risk premiums of the risk averse investors for bond markets stated in equilibrium. The monetary policy factors under the measure Q follow the VAR[1] process:

$$\boldsymbol{g}_t = \boldsymbol{\mu}_g^Q + \boldsymbol{\Psi}_g^Q \boldsymbol{g}_{t-1} + \boldsymbol{\Sigma}_g^Q \boldsymbol{\varepsilon}_{g,1}^Q \tag{4.39}$$

with the Gaussian error term $\boldsymbol{\varepsilon}_{g,t} \sim N\left(\mathbf{0}, I_{G\times G}\right)$ under Q. As derived in Appendix C.1.1 the bond price in the USV-ATSM is an exponential affine function of the monetary factors $\boldsymbol{g}_t$:

$$P(t,\tau) = exp\left(A_\tau + \mathbf{B}_\tau^T \boldsymbol{g}_t\right) \tag{4.40}$$

where A_τ and $\mathbf{B}_\tau$ are recursively determined by the following system of difference equations:

$$A_\tau = -\delta_0 + A_{\tau-1} + \boldsymbol{\mu}_g^Q \mathbf{B}_{\tau-1} + \frac{1}{2}\mathbf{B}_{\tau-1}\boldsymbol{\Sigma}_g^Q \left(\boldsymbol{\Sigma}_g^Q\right)^T \mathbf{B}_{\tau-1} \tag{4.41}$$

and

$$\mathbf{B}_\tau = -\boldsymbol{\delta}_1 + \left(\boldsymbol{\Psi}_g^Q\right)^T \mathbf{B}_{\tau-1} \tag{4.42}$$

The recursions in 4.41 and 4.42 are initialized with $A_1 = -\delta_0$ and $\mathbf{B}_1 = -\boldsymbol{\delta}_1$ respectively. Using the definition:

$$y(t,\tau) = -\frac{1}{\tau} ln\left(P(t,\tau)\right) \tag{4.43}$$

of the spot rate $y(t,\tau)$ at time t with time to maturity τ the spot rates for various maturities can be expressed as:

$$y(t,\tau) = a_\tau + \boldsymbol{b}_\tau^T \boldsymbol{g}_t \tag{4.44}$$

with $a_\tau = -\frac{1}{\tau}A_\tau$ and $\boldsymbol{b}_\tau = -\frac{1}{\tau}\mathbf{B}_\tau$.

4.3 Two block Bayesian model estimation

For estimating the model we use a Markov-Chain-Monte-Carlo (MCMC) procedure in form of a Gibbs block sampler. Due to the stochastic volatilities our estimation problem becomes non-linear, so that we have to extend our MCMC procedure beyond the usage of the Kalman filter for the iterative drawing of the model parameters. For extracting the stochastic volatility factors of the macroeconomic variables and the latent yield factors we combine the forward-backward Kalman filter drawing algorithm proposed by Carter and Kohn [1994] with the forward-backward (Gibbs) particle filter and drawing scheme developed by Andrieu, Doucet and Holenstein [2010] and Whiteley [2010]. The forward-backward (Gibbs) particle filter is used for extracting the stochastic volatility factors. The forward filtering of this type of particle filter is done by the Gibbs particle filter with conditional resampling proposed by Andrieu, Doucet and Holenstein [2010], whereas the backward drawing of the stochastic volatility factors is done by applying the algorithm proposed by Whiteley [2010]. In Appendices C.6 and C.7 we describe the used algorithms in more detail. A general description of particle filters can be found in Creal [2012] and Saerkkae [2013].

To handle such a large number of parameters we separate our estimation problem into two larger blocks. Because of its orthogonal formulation in a first step we separately estimate our macroeconomic DSGE model component. The stochastic volatility DSGE model component defines the first estimation block. This kind of separation is in line with the estimation procedure of the macro-finance ATSM outlined in Ang and Piazzesi [2003]. Beside the first block which implies the drawing of the parameters, state variables and volatility factors of the macroeconomic stochastic volatility DSGE, the second block implies the drawing of the parameters, state variables and volatility factors of the USV-ATSM component conditional on the draws in the first block. The drawing of the stochastic volatility factors of the DSGE and the USV-ATSM components makes it necessary that in every block our MCMC alternates between two state space models, such that in total in every iteration our MCMC runs through four different state space models. We outline the implementation of our MCMC procedure in the next two sections.

4.3.1 Structural macroeconomic DSGE model block

Estimation of our stochastic volatility macroeconomic model block is very similar to the estimation procedure Justiano and Primiceri [2008] apply. Here too, the solution of the model is done by log-linearization of the DSGE model's implied laws of motion of the economic state variables. In our macroeconomic stochastic volatility DSGE block every iteration of our MCMC procedure alternates between two state-space models. In the next two subsections we describe the specification of these two macroeconomic state-space models.

4.3.1.1 First DSGE model state-space form conditional on $\boldsymbol{\sigma}_{t=1,2,\ldots,T}$

The first state-space model is used for drawing the structural DSGE model parameters and state-variables $\boldsymbol{s}_t$ conditional on the series of the stochastic macroeconomic volatility factors $\{\boldsymbol{\sigma}_t\}_{t=1,2,\ldots,T}$. The first state-space model is specified as:

$$\tilde{\boldsymbol{y}}_t = \boldsymbol{c} + \boldsymbol{M}\tilde{\boldsymbol{s}}_t + \boldsymbol{\vartheta}_t \tag{4.45}$$

with the Gaussian measurement error $\boldsymbol{\vartheta}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma})$. The 14×1 vector of measurements of the first state-space model

$$\begin{aligned}\tilde{\boldsymbol{y}}_t^T = [&ln(\Delta GDP_t), ln(\Delta CONS_t), ln(\Delta INV_t), ln(\Delta WAGE_t), ln(LABOR_t), ln(INF_t), ECB_t,\\ &ln(\sigma_{a,t}), ln(\sigma_{b,t}), ln(\sigma_{g,t}), ln(\sigma_{i,t}), ln(\sigma_{r,t}), ln(\sigma_{p,t}), ln(\sigma_{w,t})]\end{aligned}$$

contains the measurements of GDP, consumption, investment, wage, labor (measured in working hours), GDP deflator based inflation, the monetary policy rate set by the ECB (approximated by the EONIA swap rate) and the 7 volatility factors

$$\boldsymbol{\sigma}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t}]$$

$\tilde{\boldsymbol{s}}_t^T = [\boldsymbol{s}_t, \boldsymbol{\sigma}_t]$ is the vector of the DSGE model state variables $\boldsymbol{s}_t$ extended by the stochastic macroeconomic volatility factors $\boldsymbol{\sigma}_t$. The 14×1 constant vector $\boldsymbol{c}$ is specified as:

$$\boldsymbol{c}^T = \left[\bar{\gamma}, \bar{\gamma}, \bar{\gamma}, \bar{\gamma}, \bar{l}, \bar{\pi}, \bar{r}, \mathbf{0}_{7\times 1}\right]$$

$\bar{\gamma}, \bar{l}, \bar{\pi}$ and $\bar{r}$ are the quarterly trend growth rate, the quarterly working hours and the quarterly inflation and nominal short term interest rate in the steady-state of our used DSGE model. The 14×56 matrix $\boldsymbol{M}$ and the 14×14 covariance $\boldsymbol{\Sigma}$ are specified as:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}^* & \mathbf{0}_{49\times 7} \\ \mathbf{0}_{7\times 49} & \mathbf{I}_{7\times 7} \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}^* & \mathbf{0}_{7\times 7} \\ \mathbf{0}_{7\times 7} & \mathbf{0}_{7\times 7} \end{bmatrix} \tag{4.46}$$

where $\mathbf{M}^*$ is a 7×49 matrix and $\boldsymbol{\Sigma}^*$ is a 7×7 diagonal matrix. The transition of the state variables of the first state-space model is defined as:

$$\tilde{\boldsymbol{s}}_t = \mathbf{T}\tilde{\boldsymbol{s}}_{t-1} + \mathbf{R}_t \tilde{\boldsymbol{\epsilon}}_t \tag{4.47}$$

with the 14×1 Gaussian errors $\tilde{\boldsymbol{\epsilon}}_t \sim N(\mathbf{0}, \mathbf{I}_{14\times 14})$, where $\tilde{\boldsymbol{\epsilon}}_t^T\, [\boldsymbol{\epsilon}_t, \boldsymbol{\omega}_t]$ includes the 7×1 vector $\boldsymbol{\epsilon}_t$ of structural macroeconomic shocks of our used DSGE and the 7×1 vector $\boldsymbol{\omega}_t$ of uncertainty shocks affecting the volatility of $\boldsymbol{\epsilon}_t$. The 56×56 coefficient matrix $\boldsymbol{T}$ and the 56×14 time varying covariance matrix $\boldsymbol{R}_t$ of the transition equation are specified as:

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\Theta}_0 & \mathbf{0}_{49\times 7} \\ \mathbf{0}_{7\times 49} & \mathbf{P} \end{bmatrix} \quad \mathbf{R}_t = \begin{bmatrix} \boldsymbol{\Theta}_1(\boldsymbol{\sigma}_t) & \mathbf{0}_{49\times 7} \\ \mathbf{0}_{7\times 49} & \boldsymbol{\Sigma}_\sigma \end{bmatrix} \tag{4.48}$$

Drawing the structural DSGE model parameters

$$\begin{aligned} \boldsymbol{\theta}_{DSGE}^T = [&\bar{\gamma}, \alpha, \lambda, \sigma_c, \beta, \varphi, \psi, \iota_p, \Phi, \xi_p, \iota_w, \xi_w, \sigma_l, \rho, r_\pi, r_y, r_{\Delta y}, \rho_g, \rho_b, \rho_i, \rho_a, \rho_p, \rho_w, \rho_r, \bar{\pi}, \bar{l} \\ &\sigma_{\Delta GDP}, \sigma_{\Delta CONS}, \sigma_{\Delta INV}, \sigma_{\Delta WAGE}, \sigma_{LABOUR}, \sigma_{INF}, \sigma_{ECB}, \mu_\sigma^a, \mu_\sigma^b, \mu_\sigma^g, \mu_\sigma^i, \mu_\sigma^r, \mu_\sigma^p, \mu_\sigma^w, \\ &\rho_\sigma^a, \rho_\sigma^b, \rho_\sigma^g, \rho_\sigma^i, \rho_\sigma^r, \rho_\sigma^p, \rho_\sigma^w, \sigma_\sigma^a, \sigma_\sigma^b, \sigma_\sigma^g, \sigma_\sigma^i, \sigma_\sigma^r, \sigma_\sigma^p, \sigma_\sigma^w] \end{aligned}$$

is done by the Random-Block-Random-Walk-Metropolis-Hastings (RB-RW-MH) algorithm where the likelihoods are evaluated by applying the Kalman filter on the state-space model defined in 4.45 and 4.47. For drawing the series of the DSGE model implied state variables $\{\boldsymbol{\sigma}\}_{t=1,2,\ldots,T}$ we apply the mentioned forward-backward Kalman filter and drawing algorithm proposed by Carter and Kohn [1994] on the state-space model in 4.45 and 4.47.

4.3.1.2 Second DSGE model state-space form conditional on $\boldsymbol{s}_{t=1,2,\ldots,T}$

Drawing the stochastic macroeconomic volatility factors $\{\boldsymbol{\sigma}_t\}_{t=1,2,\ldots,T}$ is done by using the second state-space model conditional on the DSGE model implied state variables $\{\boldsymbol{s}_t\}_{t=1,2,\ldots,T}$ drawn by using the first state-space model. Our second state-space model of the macroeconomic DSGE block by Sim's rational equilibrium solution outlined in 4.34 as the second state-space model's measurement and the (log) VAR[1] process for $\boldsymbol{\sigma}_t$ expressed in 4.30 as its transition equation. Filtering and drawing the series of volatility factors $\{\boldsymbol{\sigma}_t\}_{t=1,2,\ldots,T}$ is done by applying our second state-space model to the Gibbs particle filter with conditional resampling proposed by Andrieu, Doucet and Holenstein [2010] and the backward drawing algorithm proposed by Whiteley [2010].

4.3.2 Structural ATSM block

As in the first block described in the foregoing section, for the estimation of our second block in which the structural parameters as well as the yield and volatility factors of our stochastic volatility ATSM are drawn, the MCMC procedure alternates between two statespace models conditional on $\{\boldsymbol{g}_t\}_{t=1,2,\ldots,T}$ and $\{\boldsymbol{h}_t\}_{t=1,2,\ldots,T}$ respectively. The alternating Gibbs MCMC procedure for estimating the parameters and factors of the stochastic volatility ATSM outlined in this section is similar to the estimation scheme of our used alternative implementation of the USV-ATSM worked out by Creal and Wu [2017] outlined in detail in Appendix C.1.1.

4.3.2.1 First ATSM state-space form conditional on $\boldsymbol{h}_{t=1,2,\ldots,T}$

Starting point of the estimation of the structural parameters

$$\begin{aligned}\boldsymbol{\theta}_{ATSM}^T = [&\boldsymbol{\mu}_g^T, \boldsymbol{\mu}_h^T, vec(\boldsymbol{\Psi}_g)^T, vec(\boldsymbol{\Psi}_h)^T, vech(\boldsymbol{\Sigma}_g)^T, vech(\Sigma_h)^T, \mu_1^Q, \boldsymbol{\mu}_g^{QT},\\ &\psi_{1,1}^Q, vec(\boldsymbol{\Psi}_g^Q), \boldsymbol{\delta}_1^T, diag(\boldsymbol{\Sigma})^T]\end{aligned}$$

of our stochastic volatility ATSM is the (modified) state space model proposed by Chen and Scott [1993] which relates the observed yields with the ATSM's yield factors $\boldsymbol{g}_t$. For our purposes we modify this state-space model in regarding our DSGE model component of the first block in the estimation of our stochastic ATSM. We specify the first state-space model of the second block as:

$$\mathbf{Y}_t = \boldsymbol{c} + \mathbf{C}\boldsymbol{f}_t + \boldsymbol{\eta}_t \tag{4.49}$$

with the 65×1 and 56×1 vectors of measurement $\mathbf{Y}_t^T = \left[\boldsymbol{s}_t^T, \boldsymbol{y}_t^T, \boldsymbol{\sigma}_t^T, \boldsymbol{h}_t^T\right]$ and state variables $\boldsymbol{f}_t^T = \left[\boldsymbol{s}_t^T, \boldsymbol{\sigma}_t^T, \boldsymbol{g}_t^T, \boldsymbol{h}_t^T\right]$ as outlined in section 4.2.3.1 $\boldsymbol{c}$ and $\mathbf{C}$ in the measurement equation are specified as:

$$\mathbf{c} = \left[\mathbf{0}_{49\times1}^T, \boldsymbol{a}^T, \mathbf{0}_{7\times1}^T, \mathbf{0}_{1\times1}^T\right] \quad \mathbf{C} = \begin{bmatrix} \mathbf{I}_{49\times49} & \mathbf{0}_{49\times7} & \mathbf{0}_{49\times2} & \mathbf{0}_{49\times1} \\ \tilde{\mathbf{C}} & \mathbf{0}_{6\times7} & \tilde{\mathbf{B}} & \mathbf{0}_{6\times1} \\ \mathbf{0}_{7\times49} & \mathbf{I}_{7\times7} & \mathbf{0}_{7\times2} & \mathbf{0}_{7\times1} \\ \mathbf{0}_{1\times49} & \mathbf{0}_{1\times7} & \mathbf{0}_{1\times2} & \mathbf{I}_{1\times1} \end{bmatrix} \tag{4.50}$$

where $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{C}}$ are specified as $\tilde{\mathbf{B}} = \left[\mathbf{B}\tilde{\mathbf{L}}\right]$ and $\tilde{\mathbf{C}}_r = \left[\mathbf{0}_{6\times12}, \mathbf{B}\tilde{\boldsymbol{\delta}}_1, \mathbf{0}_{6\times36}\right]$ respectively.
$\mathbf{B}^T = \left[\boldsymbol{b}_6^T, \boldsymbol{b}_{12}^T, \boldsymbol{b}_{24}^T, \boldsymbol{b}_{36}^T, \boldsymbol{b}_{48}^T, \boldsymbol{b}_{60}^T,\right]$ contains the bond loadings from the arbitragefree recursive pricing scheme outlined in 4.42. $\tilde{L}^T = [\mathbf{0}_{2\times1}, \mathbf{I}_{2\times2}]$ is a selection matrix, selecting the loadings of the two yield factors $\boldsymbol{g}_t$, whereas $\tilde{\boldsymbol{\delta}}_1$ is a 3×1 indicator vector with 1 at its first position and zero else for selecting the bond loading for the ECB controlled short rate r_t implied by our DSGE model. $\boldsymbol{a}^T = [a_6, a_{12}, a_{24}, a_{36}, a_{60}]$ contains the maturity dependent constants expressed in 4.41. The 6×1 vector $\boldsymbol{y}_t$ of measurements contains the observed zero-coupon spot rates $y_t^T = [y(t,6), y(t,12), y(t,24), y(t,36), y(t,60)]$ of the six maturities $\tau = 6, 12, 24, 36, 60$ month. $\boldsymbol{\eta}_t \sim N(0, \mathbf{E})$ is the Gaussian error where $\mathbf{E}$ is specified as:

$$\mathbf{E} = \begin{bmatrix} \mathbf{0}_{49\times49} & \mathbf{0}_{49\times6} & \mathbf{0}_{49\times7} & \mathbf{0}_{49\times1} \\ \mathbf{0}_{6\times49} & \boldsymbol{\Omega} & \mathbf{0}_{6\times7} & \mathbf{0}_{6\times1} \\ \mathbf{0}_{7\times49} & \mathbf{0}_{7\times6} & \mathbf{0}_{7\times7} & \mathbf{0}_{7\times1} \\ \mathbf{0}_{7\times49} & \mathbf{0}_{7\times6} & \mathbf{0}_{7\times7} & \mathbf{0}_{7\times1} \end{bmatrix} \tag{4.51}$$

$\boldsymbol{\Omega}$ is the 6×6 diagonal containing the (squared) bond pricing errors. As distinguished from the formulation originated by Chen and Scott [1993] all spot rates in y_t are observed with measurement errors. The transition equation of this first state-space model in the ATSM block is equal to 4.35. Conditional on the series of stochastic ATSM volatility factors $\{\boldsymbol{h}_t\}_{t=1,2,\ldots,T}$, the first state space model in this estimation block is Gaussian and linear in $\boldsymbol{f}_t$ so that we can apply the conventional Kalman filter for drawing the structural ATSM parameters $\boldsymbol{\theta}_{ATSM}$ and the series of latent yield factors $\{\boldsymbol{g}_t\}_{t=1,2,\ldots,T}$ to this state-space model.

4.3.2.2 Second ATSM state-space form conditional on $\boldsymbol{g}_{t=1,2,\ldots,T}$

We derive the second state space model of the stochastic volatility ATSM block which is conditional on the series of latent yield factors $\{\boldsymbol{g}_t\}_{t=1,2,\ldots,T}$ from the VAR[1] process that drives the latent yield factors. Therefore the measurement equation of the second state space model of the ATSM block is:

$$\boldsymbol{g}_t = \boldsymbol{\mu}_g + \boldsymbol{\Psi}_g \boldsymbol{g}_{t-1} + \boldsymbol{\varepsilon}_{g,t} \tag{4.52}$$

where the yield factor shocks $\boldsymbol{\varepsilon}_{g,t} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_g \boldsymbol{\Lambda}_{g,t}^2 \boldsymbol{\Sigma}_g^T\right)$ are Gaussian. The transition equation of this system is given by the process of the ATSM's stochastic volatility factor:

$$\boldsymbol{h}_t = \boldsymbol{\mu}_h + \boldsymbol{\Psi}_h \boldsymbol{h}_{t-1} + \boldsymbol{\varepsilon}_{h,t} \tag{4.53}$$

with the Gaussian $\boldsymbol{\varepsilon}_{h,t} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_h \boldsymbol{\Sigma}_h^T\right)$. Analogue to the filtering and drawing of the DSGE stochastic volatility factors $\{\boldsymbol{\sigma}_t\}_{t=1,2,\ldots,T}$ we apply for the filtering and drawing of the series of stochastic ATSM volatility factors $\{\boldsymbol{h}_t\}_{t=1,2,\ldots,T}$ the Gibbs particle filter with conditional resampling to our state-space system in 4.52 and 4.53.

4.4 Empirical implications

4.4.1 Model estimation results

4.4.1.1 Structural DSGE component's parameter estimates

In Appendix C.8 we list our used prior distributions of both the structural DSGE as well as the USV-ATSM component of our combined model. In Appendix C.9 we list the estimated parameters of our model for Germany and Italy. In C.9.2 and C.9.4 in the Appendix we further outline MCMC diagnostics in showing the histograms of the structural parameters of our DSGE component for both countries. In Figure 4.1 we show the observed and DSGE-USV-ATSM implied dynamics of the macroeconomic state variables GDP, consumption, investment, wage, labour (measured in working hours), GDP deflator based inflation for Germany and Italy between Q1/2005 and Q1/2014. Details related to our used data and their preparation for using them in our stochastic volatility DSGE estimation are given in Appendix A.1. From Figure 4.1 it becomes clear that for both EMU countries our DSGE component fits the observed data remarkable well.

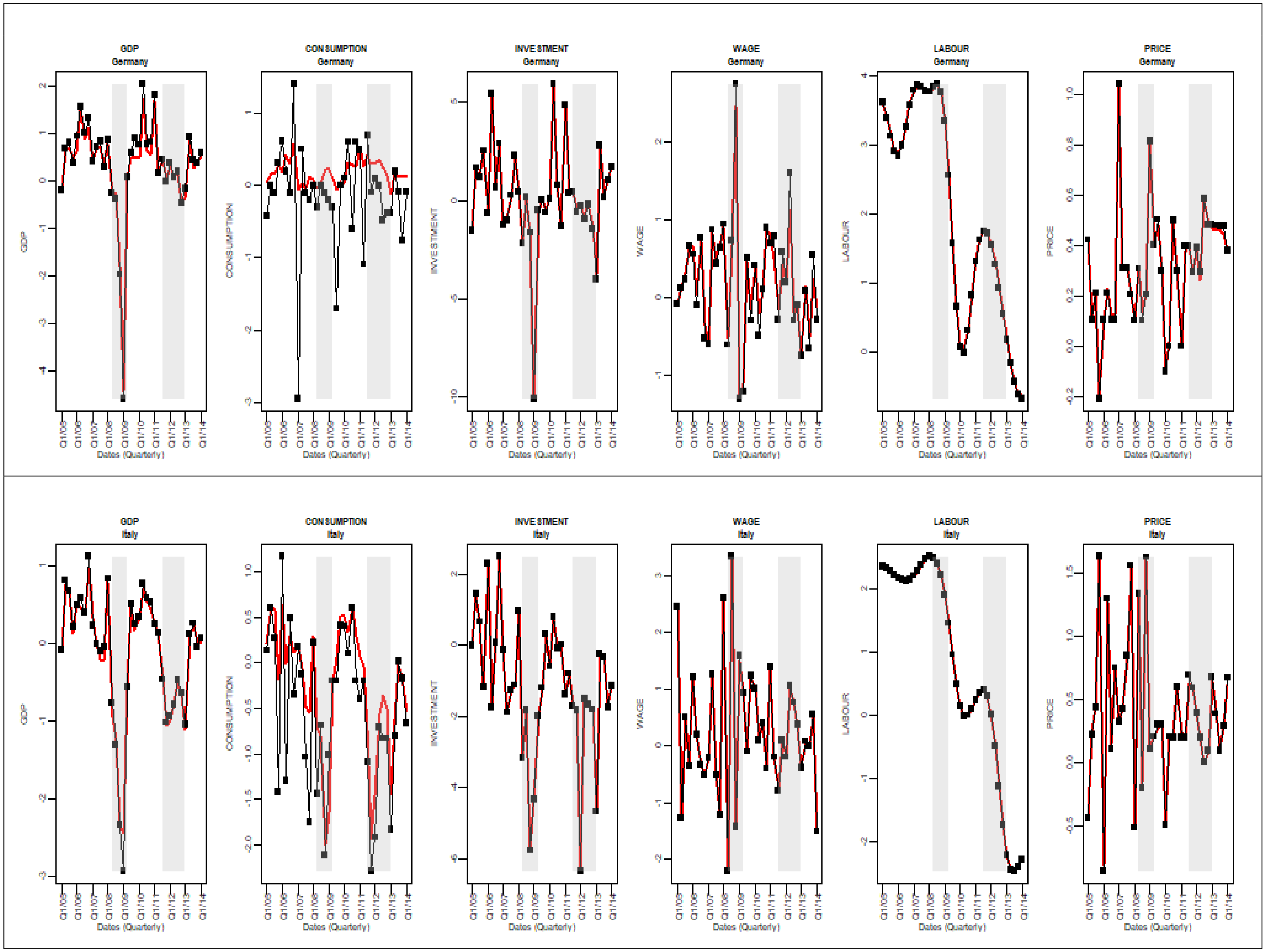


Table 4.1: Observed and model implied data of the stochastic volatility DSGE model for Germany evaluated at the mode of the models posterior.

4.4.1.2 Structural USV-ATSM component's parameter estimates

For the parameter estimation of the USV-ATSM part of our DSGE-USV-ATSM as well as for our alternative term structure of interest rate model estimates implemented in this chapter, we use zero-coupon yields estimated from German and Italian government bond prices. For the estimation we use the parametric Nelson-Siegel-Svenson (NSS) approach proposed by Nelson and Siegel [1987] and Svensson [1995]. We use quarterly bond data between Q1/2005 and Q1/2014. The composition of the German and Italian government bond data are listed in more detail in Appendix A.2. In Appendix A.3 we outline in short the zero-coupon rate extraction by the NSS approach. In Appendix C.9 we report the MCMC estimates of the structural USV-ATSM parameters of our DSGE-USV-ATSM and its standard deviations at the mode of the model's posterior. To get an impression about the fitting quality of our DSGE-USV-ATSM we implemented and estimated a larger number of alternative term structure of interest rate models. These models represent a broad range of different term-structure of interest rate modeling frameworks discussed in the term structure literature. The Vasicek-model implemented for this paper in a more general three factor version is originally proposed by Vasicek [1977]. The dynamical Nelson-Siegel (DNS) models in their independent and correlated form as well as the more recent arbitrage-free formulation of the DNS (AF-DNS) are proposed by Diebold and Li [2006] and Christensen, Diebold and Rudebusch [2011] respectively. We further implement the MF-DNS introduced by Diebold, Rudebusch and Aruoba [2006]. The MF-DNS extends the pure (latent) term structure factor formulation of the DNS by regarding additional macroeconomic factors in the term structure modeling. With the USV-Latent-DNS and USV-MF-DNS models we further extend the latent DNS and MF-DNS models by endogenously regarding an (unspanned) stochastic volatility structure for the modeled interest rates. In Appendix C.2 we shortly outline the USV-Latent-DNS and USV-MF-DNS and the estimation and filtering routines applied to these two models. The Latent-ATSM and the MF-ATSM are the yields-only and macro-finance ATSM with pure (latent) term structure factors (yields-only) and additional macroeconomic factors (macro-finance) proposed by Ang and Piazzesi [2003], whereas the USV-MF-ATSM is the macro-finance ATSM proposed by Creal and Wu [2017] already mentioned in section 4.3.2 and described in more detail in Appendix C.1.1. The SW-DSGE-ATSM combines the (medium- to large-scale) DSGE model proposed by Smets and Wouters [2003, 2007] with an ATSM similar to Ang and Piazzesi [2003]. The BCM-DSGE proposed by Beakert, Moreno and Cho [2010] combines a small-scale DSGE model with an ATSM. So to summarize, our model implementations include seven term structure models which combine pure term structure related factors with macroeconomic factors (MF-DNS, MF-ATSM, USV-MF-ATSM, USVMF-DNS, SW-DSGE-ATSM, DSGE-USV-ATSM and BCM-DSGE). Three of these seven macro-finance models model their macroeconomic components by an implied (middle- to large-scale as well as a small-scale) DSGE modeling framework (SW-DSGE-ATSM, DSGE-USV-ATSM and BCM-DSGE). Seven of our implementations include term structure models with pure (latent) term structure factors (Latent-ATSM, independent and correlated DNS and AFDNS, USV-Latent-DNS and Vasicek). From our overall set of implemented term structure models, four models are (unspanned) stochastic volatility models (DSGE-USV-ATSM, USV-MF-ATSM, USV-Latent-DNS, USV-MF-DNS). Table 4.3 reports the in-sample RMSYE of the implemented term structure models and Figure 4.2 shows the model implied interest rates compared to the observed zero-coupon rates. Both the Table and the Figure compare the interest rates with maturities of 12,24,48 and 60 month.

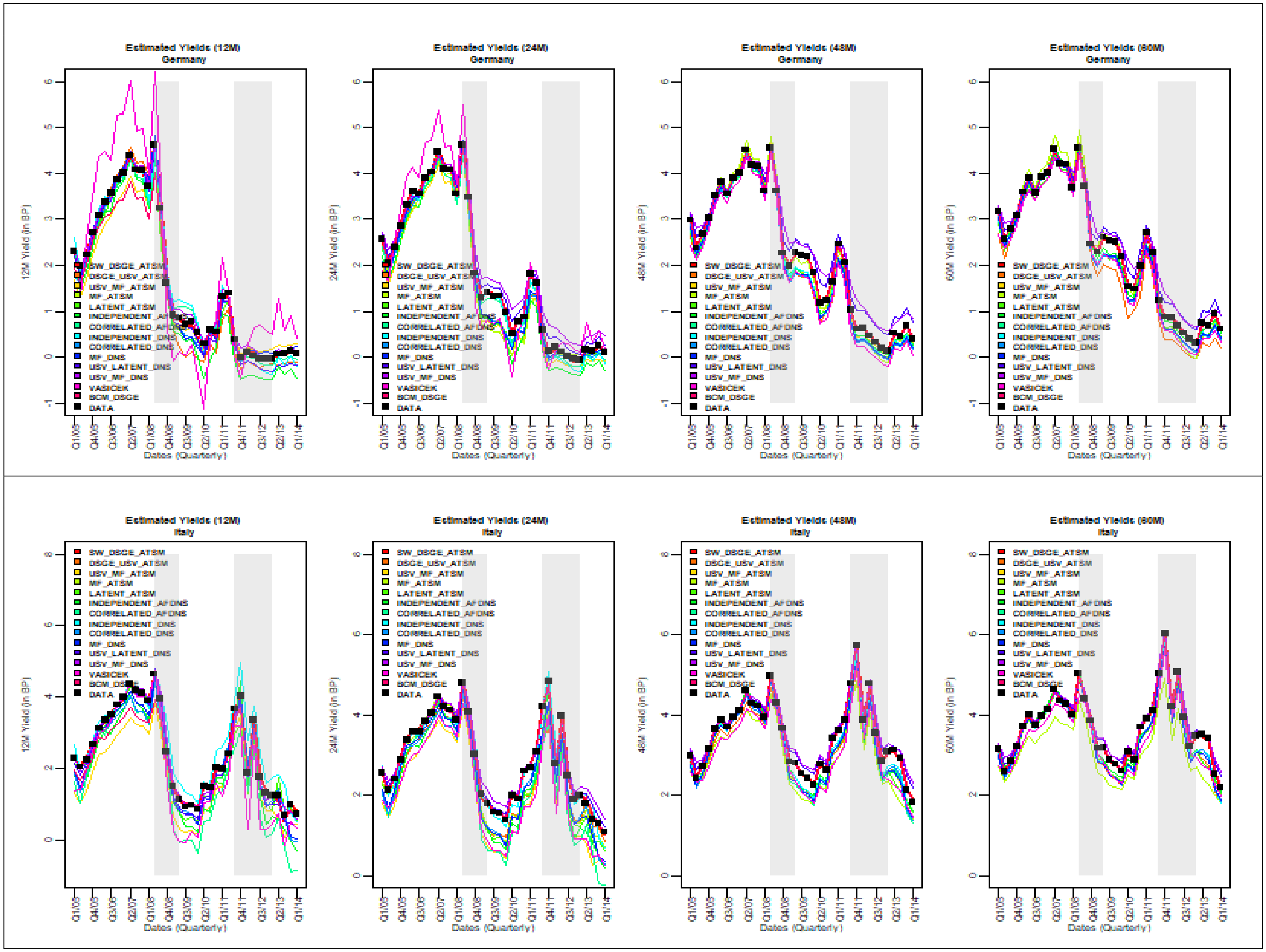


Table 4.2: Comparison of the in-sample-fit for the maturities 12,24,48 and 60 month of 14 term structure of interest rate model implementations for Germany and Italy over the time horizon Q1/2005 and Q1/2014. Ten of these term structure models are constant volatility models and four models are (unspanned) stochastic volatility models.

Germany

	y(12M)	y(24M)	y(36M)	y(48M)	y(60M)		y(12M)	y(24M)	y(36M)	y(48M)	y(60M)
SW-DSGE-ATSM	6.142	2.979	1.570	0.517	0.565	Corr. AFDNS	29.958	19.993	11.661	5.584	0.038
DSGE-USV-ATSM	26.939	2.648	5.152	4.773	19.228	MF-DNS	23.216	7.715	1.649	1.561	1.727
USV-MF-ATSM	30.300	19.346	0.188	7.368	0.258	USV-Latent-DNS	18.477	19.231	28.878	27.348	20.425
Latent-ATSM	0.000	4.854	0.000	4.680	7.030	USV-MF-DNS	7.532	34.040	39.325	35.150	27.732
MF-ATSM	0.020	13.548	0.009	17.245	25.975	Vasicek	91.264	47.510	14.613	14.962	11.728
Ind. DNS	36.346	23.543	11.730	5.238	1.882	BCM-DSGE	30.910	3.917	0.964	1.122	2.558
Ind. AFDNS	37.771	23.302	8.198	7.957	11.548						
Corr. DNS	18.111	5.843	6.106	5.642	4.113						

Italy

	y(12M)	y(24M)	y(36M)	y(48M)	y(60M)		y(12M)	y(24M)	y(36M)	y(48M)	y(60M)
SW-DSGE-ATSM	5.091	4.310	0.003	1.770	1.726	Corr. AFDNS	68.193	39.474	18.601	6.369	0.123
DSGE-USV-ATSM	0.205	20.561	22.192	12.129	5.282	MF-DNS	34.966	21.759	10.197	1.530	3.081
USV-MF-ATSM	59.179	40.410	11.631	1.134	0.009	USV-Latent-DNS	18.641	8.793	16.175	16.291	12.391
Latent-ATSM	0.000	6.744	0.000	7.203	10.015	USV-MF-DNS	8.400	31.855	37.728	33.697	25.266
MF-ATSM	0.022	12.016	0.065	27.825	42.737	Vasicek	49.843	36.081	30.450	24.352	20.497
Ind. DNS	67.777	51.201	27.364	11.155	3.522	BCM-DSGE	36.149	5.417	0.457	1.662	2.642
Ind. AFDNS	45.641	24.955	10.896	11.908	14.130						
Corr. DNS	40.857	30.495	17.097	6.008	0.000						

Table 4.3: RMSYE (in BP) of the DSGE-USV-ATSM and 13 alternative term structure model implementations for Germany and Italy.

In summary Table 4.3 makes clear that for Germany our DSGE-USV-ATSM has good fitting qualities especially in the maturity range between 24 and 48 month. Here our model partially shows a better fitting than the Latent-ATSM which has a very good in-sample fitting quality over the whole maturity range and the USV-MF-ATSM which has good fitting qualities especially for maturities ranging from 36 to 60 month. For the maturities 12 and 60 month our model is comparable to the constant and time-varying volatility term structure models of the (dynamical) Nelson-Siegel class. For Italy our DSGE-USV-ATSM shows especially for the maturities 12 and 60 month good fitting qualities. For the maturities 24,36 and 48 month the fitting quality of our stochastic DSGE term structure model is comparable to the fitting quality of the term structure models of the DNS class.

4.4.2 Volatility of economic fluctuations

4.4.2.1 Macroeconomic volatility

In Figure 4.4 we plot the estimated series of conditional stochastic macroeconomic volatility factors $\boldsymbol{\sigma}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t},]$ at the posterior's mode for Germany and Italy. For Germany Figure 4.4 shows that all plotted volatilities show a peak in or near the first recession phase between Q2/2008 and Q2/2009. The longer lasting second recession phase between Q3/2011 and Q1/2013 reveals a similar pattern. There is a volatility increase in productivity, in the risk premium component, in government spending activities and in the German economy's price component. The Italian economy shows a similar pattern of macroeconomic uncertainty. As for Germany risk premium induced uncertainty shows a peak in the first recession phase, where different to Germany risk premium uncertainty affecting the Italian economy peaks here earlier in Q4/2008 immediately after the bankruptcy of Lehman Brothers in September 2008. Comparing Germany and Italy a further interesting aspect is that the Italian economy too is affected by high uncertainty related to risk premiums and Italian government spending activities since 2011. Different are the high peaks in the Italian risk premium and government spending related uncertainties in Q4/2011 and Q1/2011 respectively. Monetary policy decisions derived from a Taylor-rule like heuristic applied to the German and Italian economy are also effected by increased uncertainty. A first phase of German induced uncertainty on monetary policy decisions lies between Q1/2007 and Q1/2008, whereas increased uncertainty affecting ECB's monetary policy decisions induced by the Italian economy is observed between Q2/2006 and Q3/2007. Both countries induce peaked uncertainties on monetary policy decisions following the collapse of Lehman Brothers. Here the German induced peak is in Q1/2009 and Q2/2009 whereas the Italian economy reacts more sensitive inducing peaked uncertainty already in Q4/2008. A second phase of increased uncertainty related to ECB's monetary policy decisions is observed in Q2/2012 and Q3/2012 and therefore in temporal proximity to Mario Draghi's London speech in July 2012 and the discussions about more far-reaching ECB measures followed to this speech.

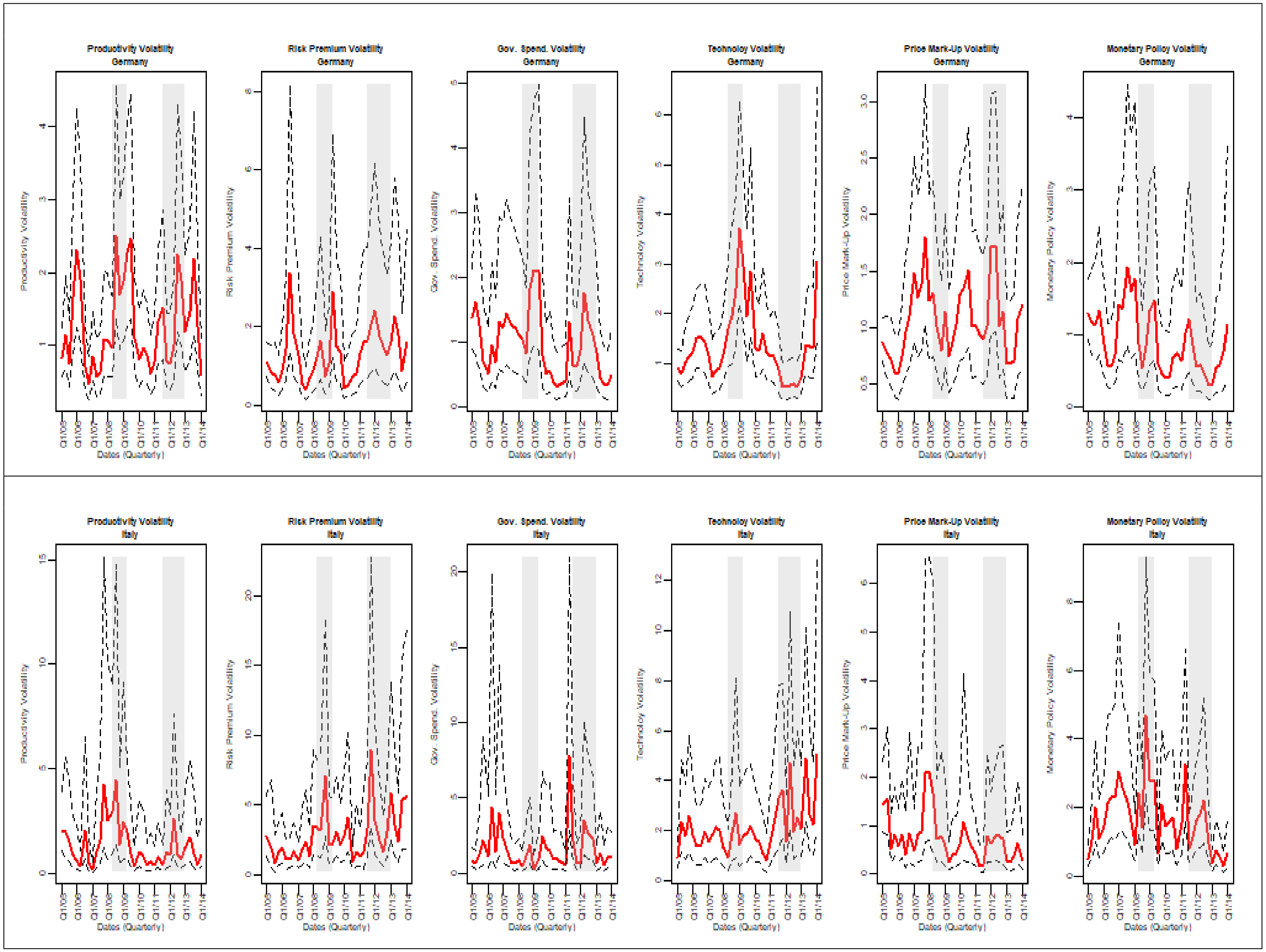


Table 4.4: DSGE model implied stochastic volatilities for Germany and Italy evaluated at the mode of the models posteriors (dashed lines show the 2σ band).

4.4.2.2 Term structure of interest rates volatility

Figure 4.5 shows the interest rate volatilities for Germany and Italy with respect to the six maturities 6,12, 24,36, 48 and 60 month from the DSGE-USV-ATSM, the alternative USV-Macro-Finance-ATSM as well as from the restricted Student's t-GAS[1,1] model. The restricted Student's t-GAS[1,1] is based on the work by Creal, Koopman and Lucas [2011] and its implementation is outlined in short in Appendix C.3. Figure 4.5 shows the GAS implied volatility structure of interest rates where the zero-coupon rates are adjusted by their conditional mean estimated by a conventional VAR[1]. For all six maturities our DSGE-USV-ATSM in Figure 4.5 reveals for Germany and Italy a volatility peak in Q1/2009 and Q4/2008 and therefore immediately after the bankruptcy of Lehman brothers in September 2008. For Germany the interest rate volatilities estimated by the restricted Student's t GAS[1,1] model show a similar pattern, with the difference that the maturities 24 to 60 month show a first peak at the beginning of the recession phase Q2/2008. Compared to the GAS[1,1] the volatilites implied by the DSGE-USV-ATSM are more homogeneous across the various maturities. Further the volatilities of the DSGE-USVATSM are less fluctuating than in the volatilities of the GAS. For Italy the DSGE-USVATSM and the GAS[1,1] show a similar pattern in the first recession phase with a peak in Q4/2008. Different to the DSGE-USV-ATSM volatilities the GAS reveals an increase in the Italian interest rate volatilities in the sample's second recession phase beginning in Q3/2011, similar to the volatility pattern of the short term interest rate implied by the macroeconomic DSGE model component for Italy shown in Figure 4.4. The DSGE-USVATSM shows a sharp peak a bit early in Q2/2011. For both EMU countries the USV-MFATSM implied term structure of volatilities reveal a longer lasting increase since the beginning of 2007 with a volatility peak at the beginning of the sample's first economic recession phase in Q2/2008.

4.4.3 Uncertainty shocks and their historical contributions

4.4.3.1 Macroeconomic uncertainty shocks

To get an understanding about the uncertainty shock pattern the German and Italian economy faces in our sample between Q1/2005 and Q1/2014 in Figure 4.6 we plot the empirical macroeconomic uncertainty shocks implied by our DSGE-USV-ATSM at its posterior mode. From Figure 4.6 it becomes clear that for both countries there is a phase between Q2/2006 and Q2/2009 with larger empirically measured macroeconomic uncertainty shocks related to inflation, monetary policy as well as to government spending activities and the risk premium. Compared to Germany the uncertainty shocks mainly induced by Italian government spending activities, ECB's monetary policy and risk premiums demanded by investors financing the Italian economy show two sharp peaks centered around Q2/Q3 2006 and Q1/Q2 2007 respectively. On the contrary the German economy is affected by increased uncertainties over the whole range between Q2/2006 and Q2/2009, where the first half of this phase is dominated by uncertainty shocks induced by monetary policy and government spending activities

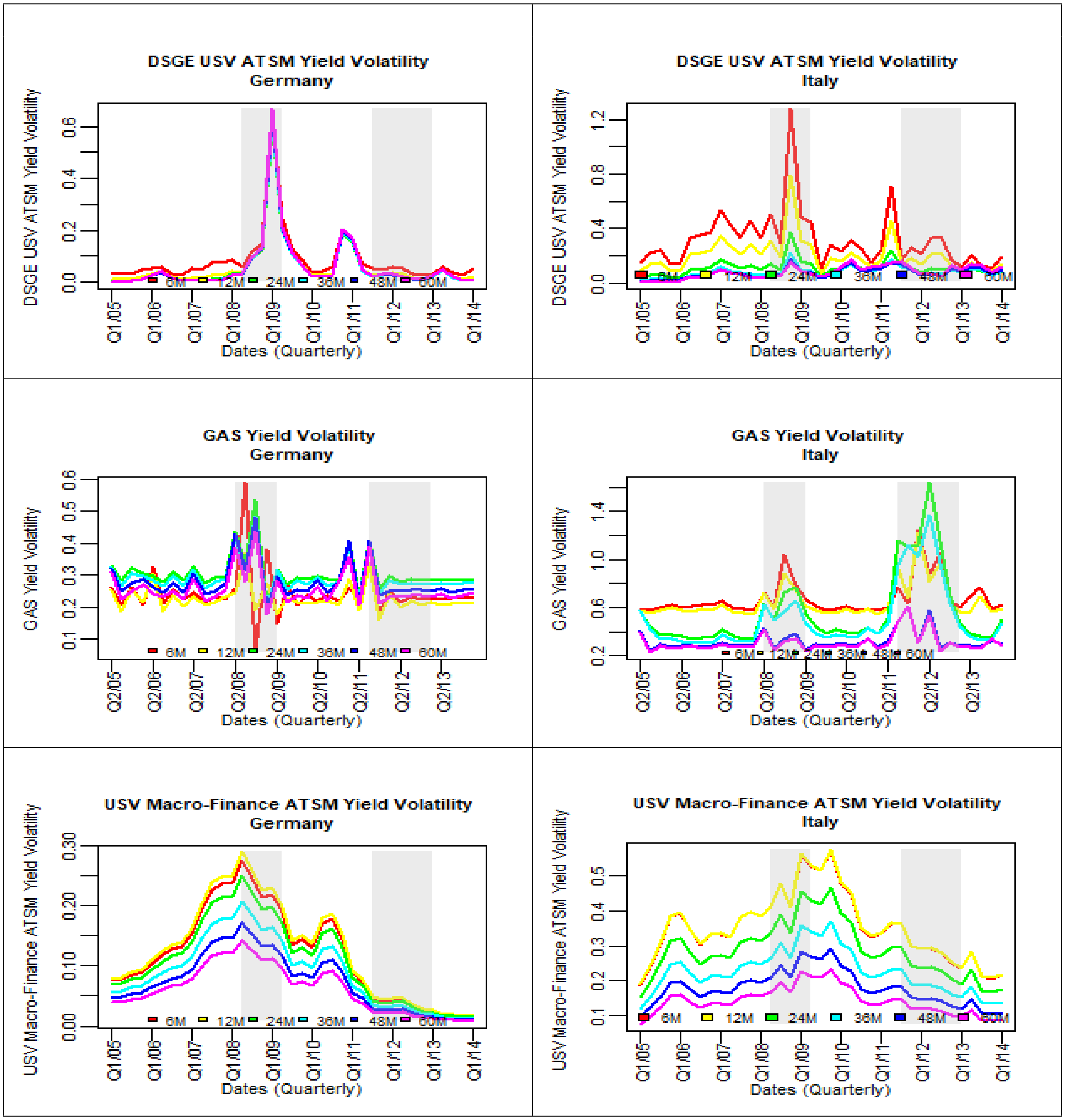


Table 4.5: Stochastic volatilities for the maturities 6,12, 24,36, 48 and 60 month implied by the DSGE-USV-ATSM and USV-ATSM evaluated at the mode of the models posterior and the restricted Student's t GAS[1,1] model for Germany and Italy between Q1/2005 and Q1/2014.

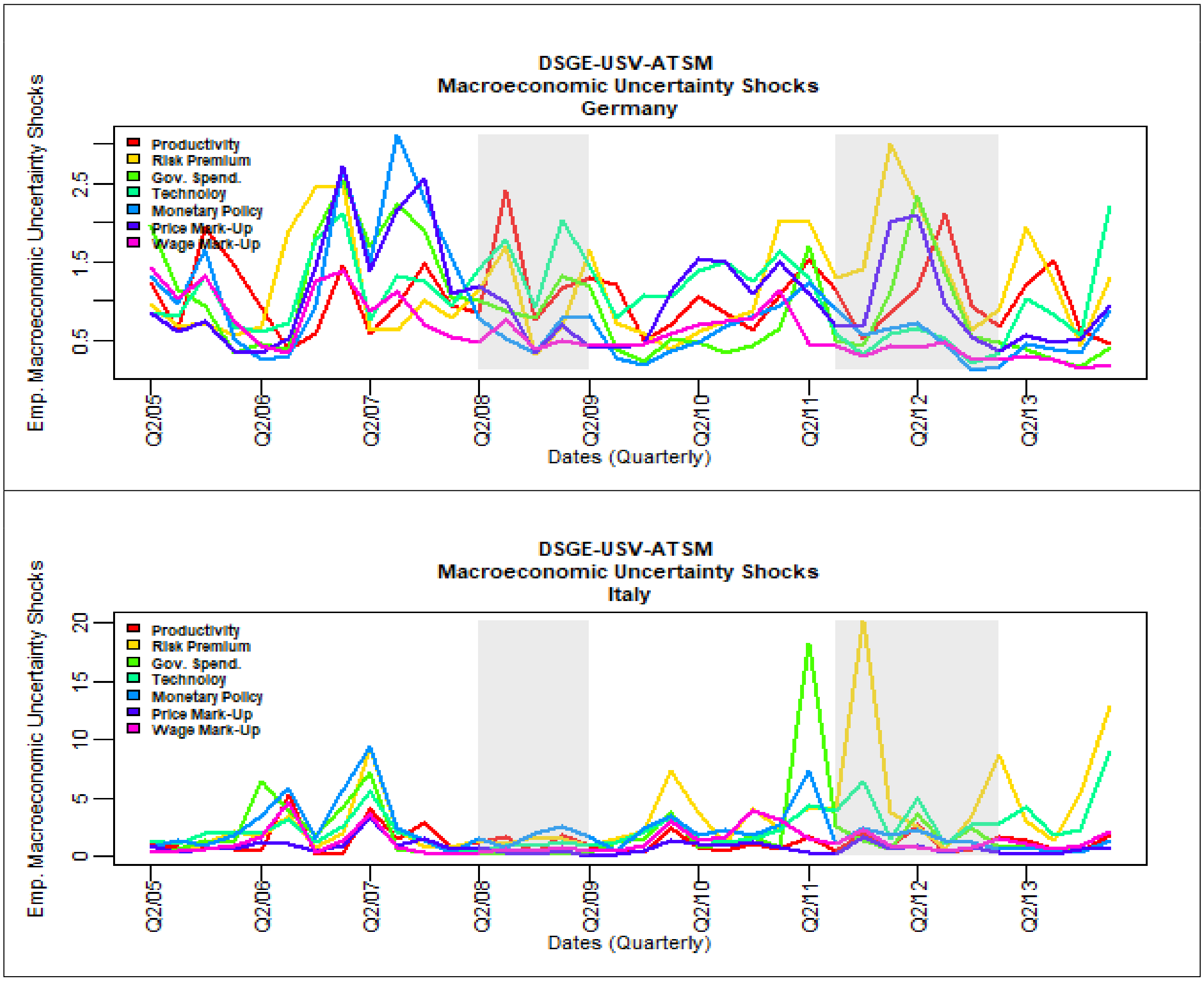


Table 4.6: Empirical macroeconomic uncertainty shocks $\hat{\boldsymbol{\omega}}_t^T = \left[\hat{\omega}_t^a, \hat{\omega}_t^b, \hat{\omega}_t^g, \hat{\omega}_t^i, \hat{\omega}_t^r, \hat{\omega}_t^p, \hat{\omega}_t^w\right]$ implied by our DSGE-USV-ATSM at the posteriors mode for Germany and Italy between Q1/2005 and Q1/2014.

and the risk premium. For Germany a second phase with larger macroeconomic uncertainty shocks lies between Q1/2011 and Q3/2012. In this phase the main sources of uncertainty are related to the risk premium and inflation. Beside these two uncertainty components, there are higher measured uncertainty shocks related to government spending activities as well as to the German real economy's productivity. A similar pattern is revealed for the Italian economy. Here there are large shocks induced by Italian government spending in Q2/2011 and from issues related to investor's risk premiums in Q4/2011. In their magnitude uncertainty shocks affecting the Italian economy are larger than uncertainty shocks effecting the German economy.

4.4.3.2 Uncertainty contribution to the macroeconomic development

In Figure 4.7 we show the DSGE-USV-ATSM implied historical contributions of volatility shocks from the seven sources of macroeconomic uncertainty to the developments of the three macroeconomic state variables GDP y_t , firm's investment activities i_t and the ECB controlled short term rate r_t for Germany and Italy between Q1/2005 and Q1/2014. The historical decomposition of the observed macroeconomic variables with respect to the empirical uncertainty shocks and their contributions to the macroeconomic development. Between Q4/2006 and Q2/2008 the German GDP is positively effected by uncertainty related to spending activities of the German government - a result which we will see again in 4.4.5 where we discuss the economies' responses to various uncertainty shocks. Negative effects to the German GDP at that time are mainly related to uncertainties induced by the German inflation dynamics. The Italian GDP is also mainly effected by government spending activities. These effects are not as continuous as observed for Germany. Here earlier in Q2/2006 and in Q2/Q3 2007 uncertainties induced by Italian government spending activities reveal stronger effects on the Italian GDP. Again both countries GDP is largely effected by uncertainties from government spending in Q2/2011 where Italy shows a large peak and in the phase ranging between Q1/2012 and Q3/2012.

Investment activities in both countries are mainly effected by uncertainties related to the economies productivity and technology components - results also confirmed in our response analysis in 4.4.5. Analogue to GDP negative effects on the firm's investment decisions are mainly related to uncertainties of the economies price dynamics. In Italy there is an interesting point with respect to the government spending shock observed in Q2/2011. On contrary to the Italian GDP the effect's sign here becomes negative. The Taylor-rule like monetary policy heuristic applied to the German economy reveals that ECB's monetary policy decision is mainly influenced by uncertainties related to the German inflation dynamics. Between Q1/2007 and Q1/2008 when ECB's main refinancing operations rate reached 4.00% ECB's decisions with respect to the German economic environment are also driven by uncertainties related to their own monetary policy measures and decisions. The monetary policy heuristic applied to the Italian economy reveals a similar pattern. Here ECB's monetary policy induced uncertainties related to the Italian economy on ECB's monetary policy decisions itself reveal larger effects as observed for Germany. Also different from Germany is the large uncertainty effect related to the ECB's monetary policy in Q2/2011 observed for the Italian data.

4.4.3.3 Term structure of interest rate uncertainty shocks

Similar to the empirically measured macroeconomic uncertainty shocks $\hat{\boldsymbol{\omega}}_t^T = \left[\hat{\omega}_t^a, \hat{\omega}_t^b, \hat{\omega}_t^g, \hat{\omega}_t^i, \hat{\omega}_t^r, \hat{\omega}_t^p, \hat{\omega}_t^w\right]$ shown in Figure 4.5, in Figure 4.8 we show the empirical term structure of interest rate uncertainty shock $\hat{\varepsilon}_{h,t}$. Obviously for both countries in the first recession phase between Q2/2008 and Q2/2009 we measure a high uncertainty affecting

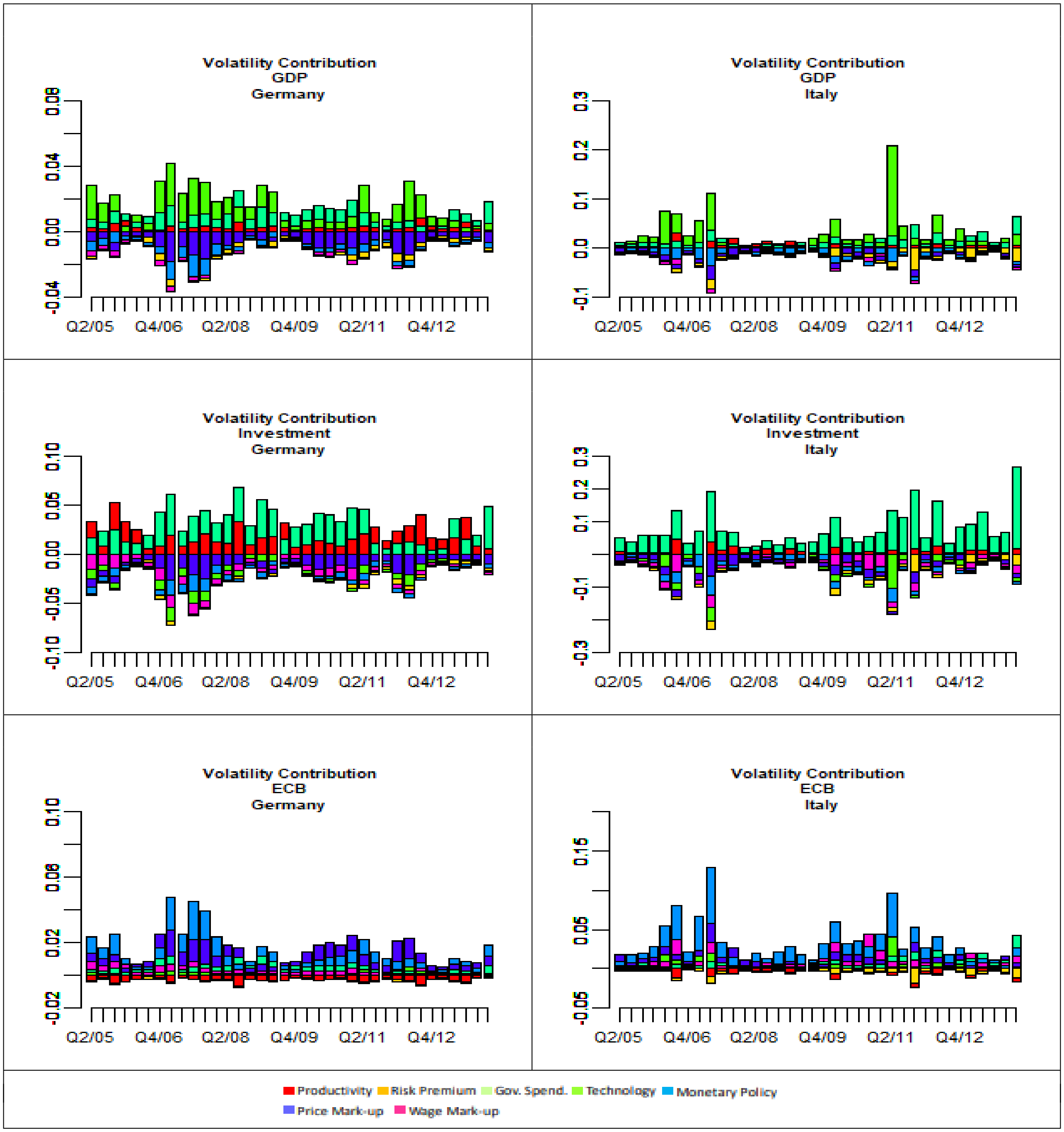


Table 4.7: Historical contributions of macroeconomic uncertainty shocks in the DSGE-USV-ATSM to the observed macroeconomic variables GDP y_t, firm's investment activities i_t and the ECB controlled short term interest rate r_t at the mode of the posterior for Germany between Q1/2005 and Q1/2014.

the term structure of interest rate volatilities. For Germany Figure 4.8 further points out that there is a second phase of high interest rate uncertainty with its peak in Q4/2010, in which the term structure of interest rates starts to increase after a two years lasting period of decreasing interest rates and a few month before the ECB's official decision to increase its short term monetary policy rate for the EMU in April 2011. Italy shows a sharp uncertainty shock in Q4/2012 with the announcement of the resigning of Mario Monti, who headed a non-elected cabinet of economic experts since 2011 that launches the Italian austerity reform programme and the dissolution of the Italian parliament that followed the resignation. In Figure 4.8 we additionally plot the Economic Policy Uncertainty (EPU) index proposed by Baker, Bloom and Davis [2016] measured for Germany and Italy. For Germany the EPU index indicates a phase of higher economic policy uncertainty in the first recession phase between Q2/2008 and Q2/2009 and the three quarters before this phase which is in line with the macroeconomic uncertainty pattern shown in Figure 4.6 and the term structure related uncertainty shock pattern in Figure 4.8. The peak of the German EPU index at the beginning of the second recession phase in Q3/2011 reflects the political discussions related to the 50% writedown of the value of Greek government debt held by private investors announced in October 2011. The uncertainty pattern revealed for the German term structure of interest rates developments shows an increased uncertainty one to two periods before the German EPU index peaks. For Italy the interest rates uncertainty shock $\hat{\varepsilon}_{h,t}$ peaks at the end of 2012 whereas the Italian EPU index peaks at the beginning of 2013 with the Italian election in February 2013.

4.4.3.4 Historical uncertainty contribution to term structure development

To get an understanding about the impact of the various sources of economic uncertainty on the term structure of interest rates, in Figure 4.9 we plot the historical decomposition of the term structure of volatility. The decomposition shows the historical contribution of empirically determined volatility shocks - interpreted in our context as uncertainty shocks - to the historical dynamics of yield volatilities with different maturities. Our methodology for calculating the historical decomposition of the term structure of volatility into the various historical volatility shock contributions is derived in Appendix C.4. Figure 4.9 shows that the bond market specific uncertainty shocks coming from the two latent term structure factors have the largest historical contributions to the term structure of volatilities. Here the first peak lies in the recession phase between Q2/2008 and Q2/2009 with the sharp decrease of the ECB's main refinancing operations rate from 3.25% to 1.00% between Q4/2008 and Q2/2009. The second peak of bond market uncertainty lies in the period Q4/2010 and Q1/2011 immediately before the ECB's short term restrictive/expansive monetary policy decisions, with the increase in the main refinancing operations rate from 1.00% in Q1/2011 to 1.50% in Q3/2011 and the decrease back to 1.00% at the end of Q4/2011. Macroeconomic uncertainty shocks only have an impact on the short term yield volatilities with maturities ranging from 6 to 12 month. Here we can see that there is a phase between Q4/2007 and Q2/2008, immediately before the first recession phase in our sample, in which monetary pol-

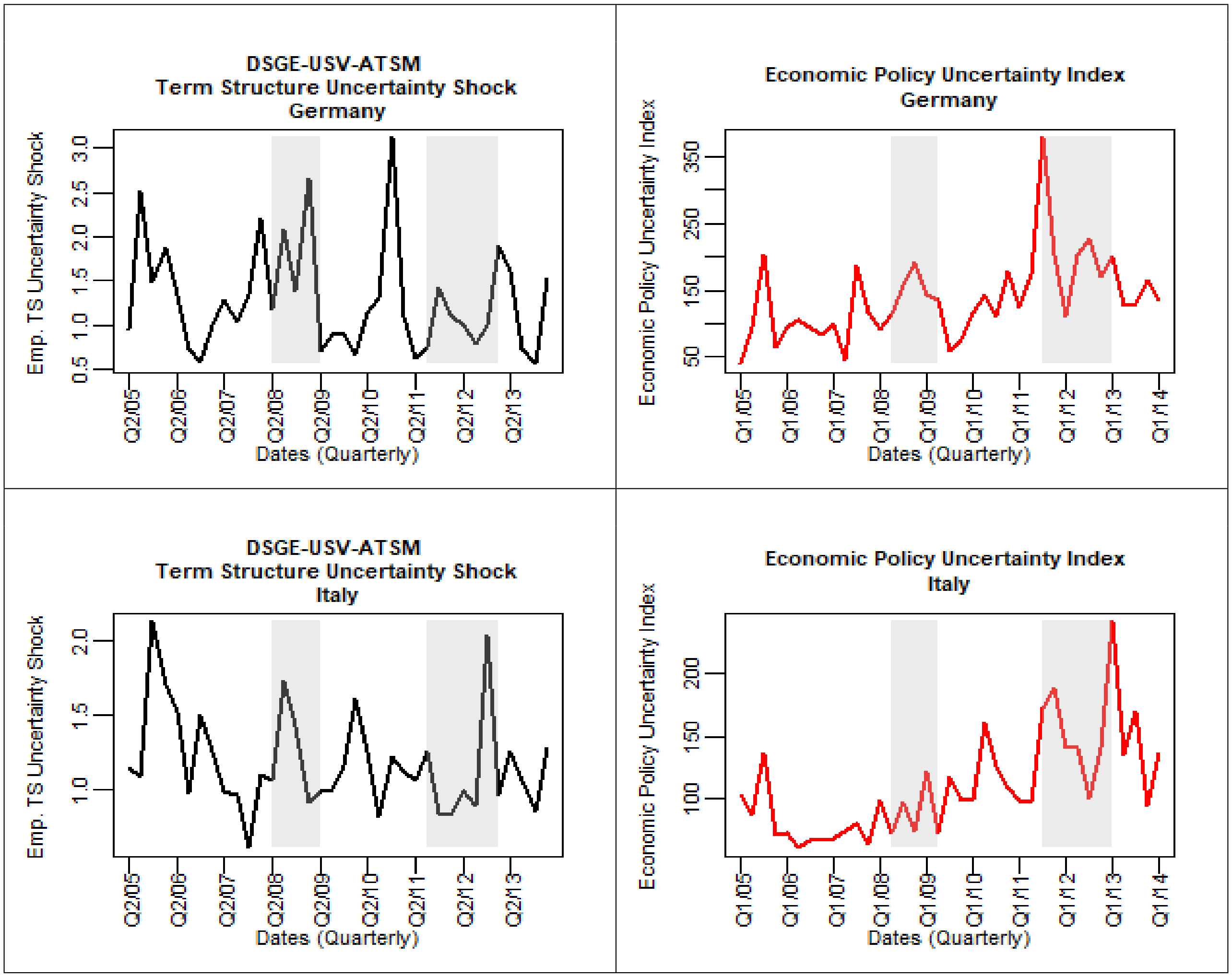


Table 4.8: Empirical term structure of interest rates uncertainty shock $\hat{\varepsilon}_{h,t}$ implied by the DSGE-USV-ATSM evaluated at the posteriors mode and the Economic Policy Uncertainty Index between Q1/2005 and Q1/2014 for Germany and Italy.

icy uncertainty contributes significantly to the composition of 6 and 12 month short term yield volatilities. In Figure 4.10 we plot the historical contributions of volatility shocks from the seven sources of macroeconomic uncertainty and the bond market uncertainty shocks to the term structure of interest rate volatilities for Italy. Compared to the German volatility decomposition in Figure 4.9 the Italian decomposition reveals an interesting and different pattern. There is a very singular uncertainty shock contribution to the volatilities especially of Italian short term interest rates induced from sources of monetary policy in Q2/2007. The beginning of 2007 denotes a crucial point in the U.S. subprime crisis, where in Q1/Q2 2007 U.S. home prices starts sharply to decline. In reaction the FED decreases its federal funds rate in September 2007 from 5.25% to 4.75% and initiates its interest rates decreasing path

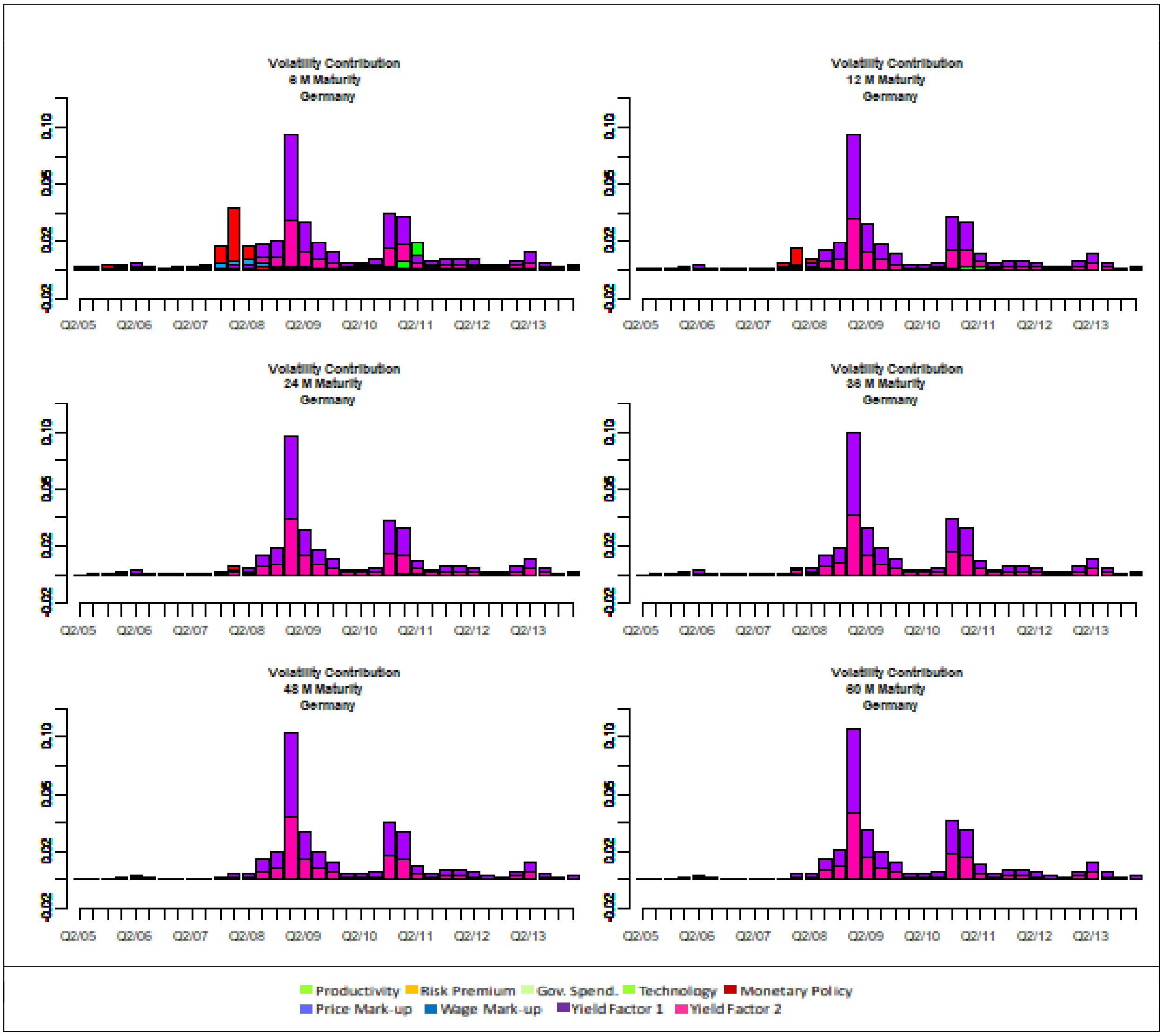


Table 4.9: Historical contributions of volatility shocks from the seven sources of macroeconomic uncertainty and of the bond market uncertainty coming from the two latent yield factor volatilities to the yield volatilities with maturities 6,12, 24,36, 48 and 60 month implied by the DSGE-USV-ATSM evaluated at the mode of the models posterior for Germany.

reaching its historical minimum of 0.25% in December 2008 which lasts until December 2015. The second large contribution to the Italian interest rate volatilities also comes from sources of monetary policy. Similar to Germany the Italian volatilities increasingly react to ECB's Q2/2011 decision to increase its main refinancing operations rate from 1.00% in Q1/2011 to 1.25% in Q2/2011 and finally to 1.50% in Q3/2011.

For validating the robustness of our findings we compare our results with the historical de-

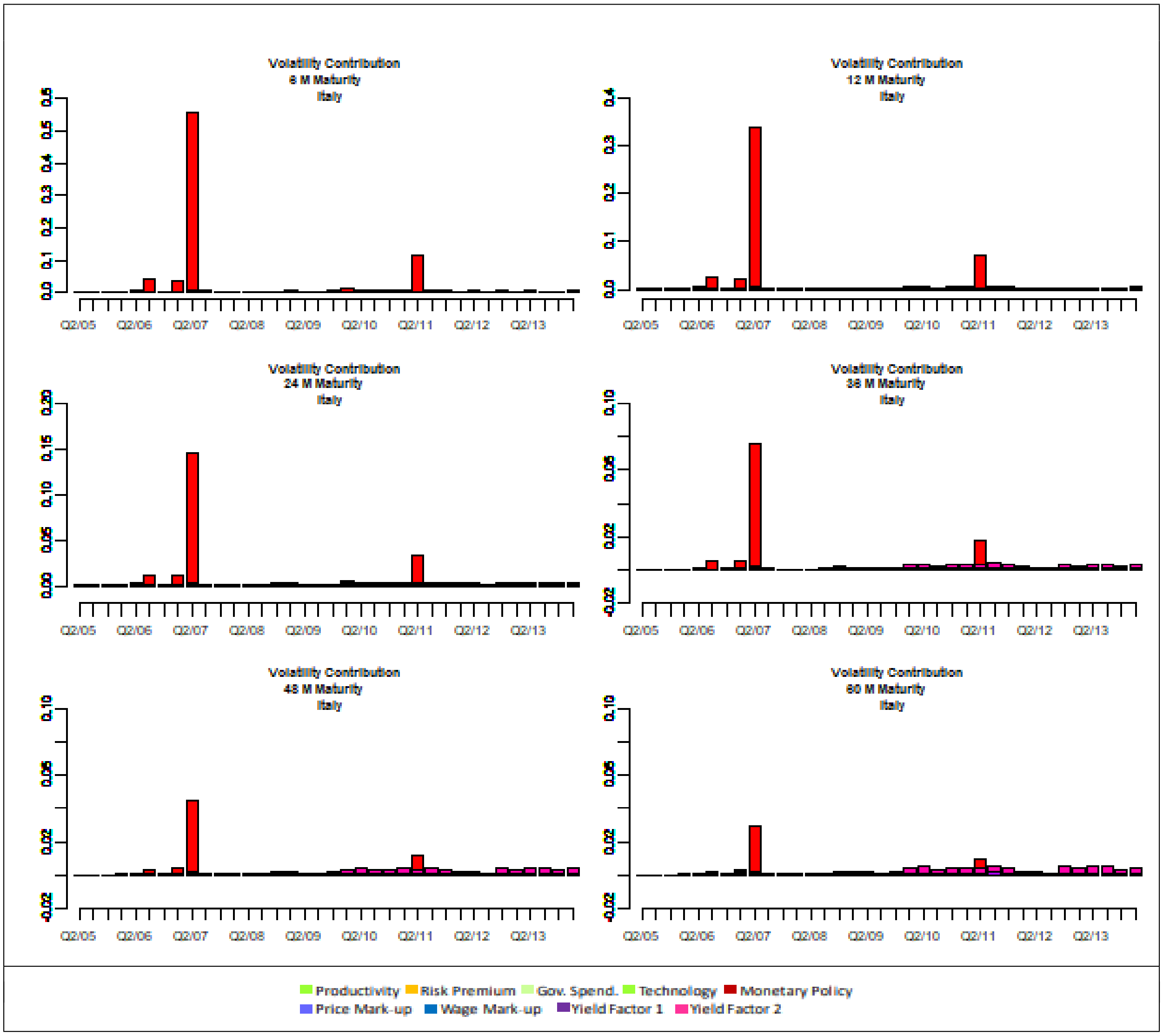


Table 4.10: Historical contributions of volatility shocks from the seven sources of macroeconomic uncertainty and of the bond market uncertainty coming from the two latent yield factor volatilities to the yield volatilities with maturities 6,12,24,36,48 and 60 month implied by the DSGE-USV-ATSM evaluated at the mode of the models posterior for Italy.

composition of the term structure of interest rate volatilities implied by the USV-MF-ATSM proposed by Creal and Wu [2017]. As outlined in detail in Appendix C.2 the latent term structure of interest rate factors $\boldsymbol{g}_t$ of the USV-MF-ATSM are interpreted as the current short term rate r_t, the expected n^* periods ahead short term rate $\mathbb{E}_t[r_{t+n^*}]$ and the term premium implied by the model's n^* period maturity zero-coupon rate $TP(t, t+n^*)$. According to Creal and Wu [2017] we set in our implementation of the USV-MF-ATSM $n^* = 60$ month. Beside $\boldsymbol{g}_t$ the USV-MF-ATSM contains with the GDP growth rate $ln(\Delta GDP_t)$ and the GDP deflator based inflation $ln(INF_t)$ (both in logs) two additional observed macroeconomic factors. In Figure 4.11 we show the historical contributions of the volatility shocks implied by the alternative USV-MF-ATSM to the yield volatilities with maturities 6,12,24,36,48 and 60 month. Obviously uncertainty coming from the latent term structure factor which is interpreted as the expected 60 month ahead short term rate has the largest historical contribution to term structure of interest rate volatilities. This confirms our findings implied by our DSGE-USV-ATSM. In Figure 4.11 the observed macroeconomic factors do not show any significant contribution to the yield volatilities.

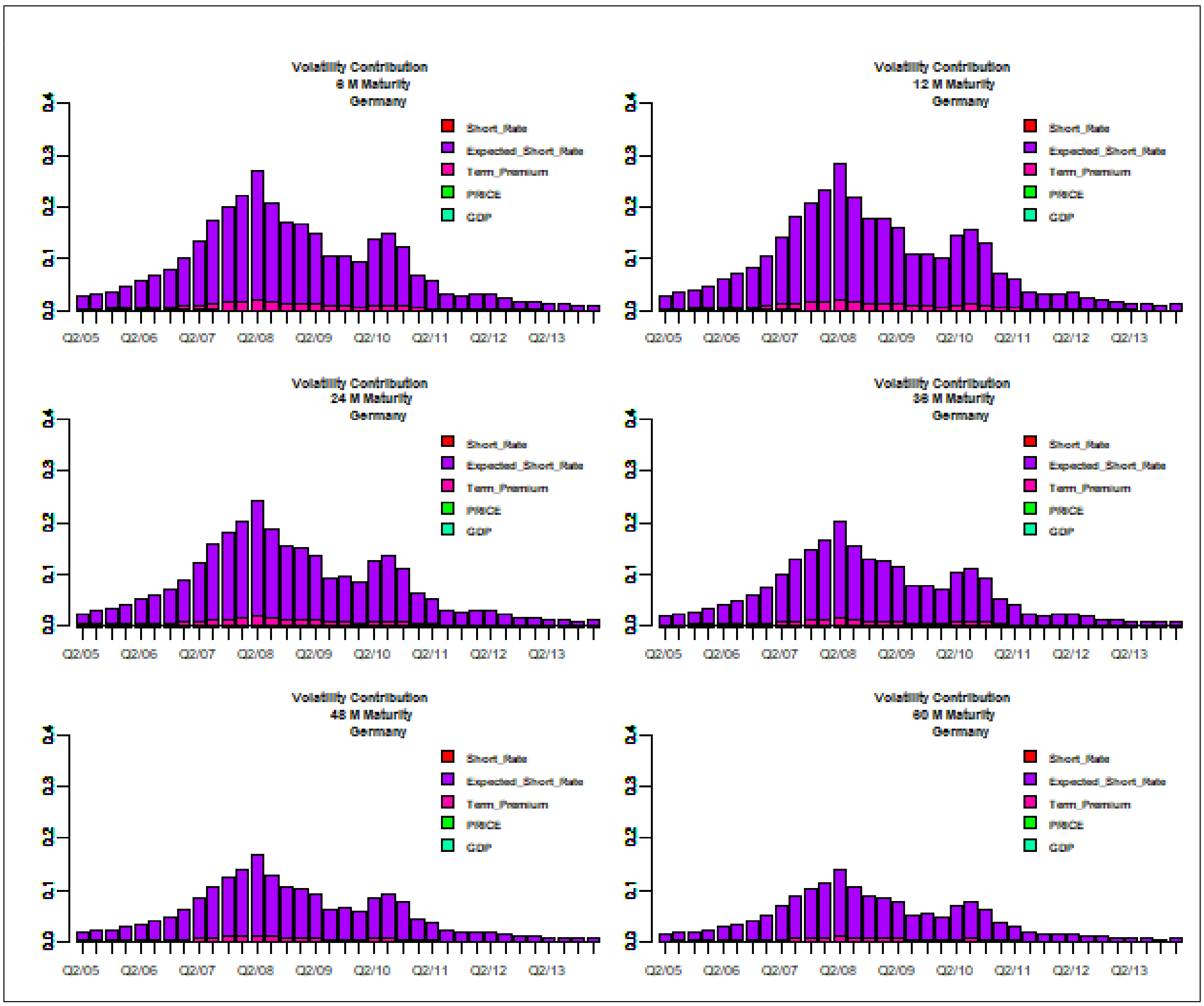


Table 4.11: Historical contributions of volatility shocks from the two sources of macroeconomic uncertainty and of the bond market uncertainty coming from the three latent yield factor volatilities to the yield volatilites with maturities 6,12, 24,36, 48 and 60 month implied by the alternative USV-Macro-Finance-ATSM evaluated at the mode of the models posterior for Germany.

4.4.4 Structural macroeconomic shock transition in the EMU economies

In Figure 4.12 we show the DSGE-USV-ATSM implied innovations of the structural macroeconomic shock variables $\hat{\boldsymbol{\varepsilon}}_t = \left[\hat{\varepsilon}_t^a, \hat{\varepsilon}_t^b, \hat{\varepsilon}_t^g, \hat{\varepsilon}_t^i, \hat{\varepsilon}_t^r, \hat{\varepsilon}_t^p, \hat{\varepsilon}_t^w\right]$ for Germany and Italy. Obviously in our estimated model for Germany the innovations of the technology shock component and the productivity component reveal the largest amplitudes in their fluctuations. Especially in the first recession phase between Q2/2008 and Q2/2009 both shock components show large changes, with a sharp decrease in the second half of the recession phase, leading to negative technology and productivity shocks. Both shocks very rapidly change their signs from Q2/2009 to Q3/2009 becoming positive shocks from the economy's sources of technology and productivity. Figure 4.12 is in line with the extracted patterns of stochastic volatilities implied by the DSGE modeling component's structural shock processes of Germany shown in Figure 4.6, where five of the six figured volatilities show a peak in or near the first recession phase between Q2/2008 and Q2/2009. We further compare the innovations shown in Figure 4.12 with the innovation pattern implied by the SW-DSGE-ATSM (not shown here), which confirms in general the innovations of the structural macroeconomic shocks shown in Figure 4.12. For Italy we find larger structural macroeconomic shocks related to Italian wage and price developments in the first recession phase between Q2/2008 and Q2/2009. After the bankruptcy of Lehman Brothers there is a large risk premium related shock in Q1/2009. Exogenous risk premium shocks disturb the Italian economy again in a phase beginning in Q4/2011 with high fluctuations in the risk premium shock component in before the Italian election in Q1/2013 after the resignation of Mario Monti and its cabinet of economic experts in Q4/2012 and the dissolution of the Italian parliament following the resignment.

4.4.5 Macroeconomic responses to uncertainty shocks

To become a deeper understanding about the economic uncertainty transmission, in this section we look at uncertainty shocks affecting the vector of macroeconomic volatilities $\boldsymbol{\sigma}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t}]$ and the corresponding responses of the macroeconomic variables to these uncertainty shocks. In Figure 4.13 we plot the responses of GDP y_t, investment i_t and the ECB's short term rate r_t to a one standard deviation uncertainty shock to each of the 7 volatility terms $\boldsymbol{\sigma}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t}]$. Therefore we simulate 1000 impulse responses from the parameters of the posterior of our MCMC. Beside the median (black) Figure 4.13 shows the mean (green) as well as the 40% and 80% confidence intervals for Germany. In Appendix C.9.7 we further show the simulated macroeconomic responses to various economic uncertainty shocks for the Italian economy. Looking at the German responses of GDP in Figure 4.13 shows that uncertainty shocks in government spending activities, technology and productivity have a positive impact on GDP. A negative impact from an increase in the macroeconomic environment comes from price and wage mark-up shocks as well as from uncertainty shocks to the term-premium and monetary policy. Interesting is the strength of the impact. Here an increasing uncertainty about government spending activities,

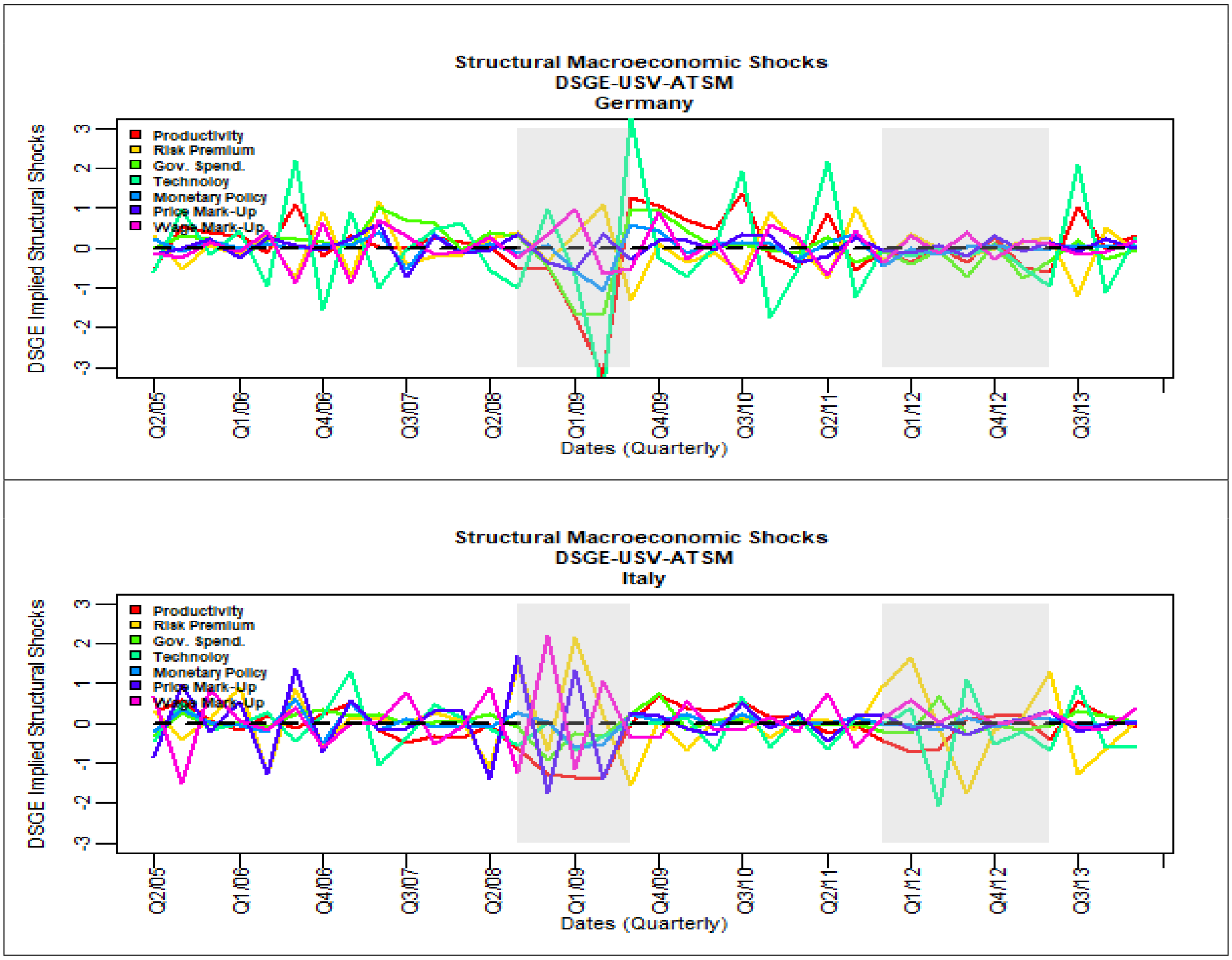


Table 4.12: DSGE-USV-ATSM implied innovations of the structural macroeconomic shock variables $\hat{\boldsymbol{\varepsilon}}_t = \left[\hat{\varepsilon}_t^a, \hat{\varepsilon}_t^b, \hat{\varepsilon}_t^g, \hat{\varepsilon}_t^i, \hat{\varepsilon}_t^r, \hat{\varepsilon}_t^p, \hat{\varepsilon}_t^w\right]$ for Germany and Italy at the mode of the models posteriors.

the technological development and the development of the economy's productivity reveal the largest impacts. A similar pattern is revealed by the responses in the firm's investment activities. Here the negative impact of an increase in the uncertainty about government spending activities is interesting. Uncertainty about prices and wages also leads to decreasing investment activities in the firms sector. Uncertainty in the price- and wage-components leads to an increase of the ECB's controlled short term interest rate. Further interesting is the reaction of the ECB to an increase of the uncertainty about German government spending activities. Here the ECB's response shown in Figure 4.5 reveals a more restrictive monetary policy reaction. The response patterns of the Italian economy shown in the Appendix C.9.7 are very similar to the responses of the German economy. As shown in Figure 4.4 the uncertainty about German and Italian government spending activities shows larger peaks in the two recessive phases in which governments in the EMU increased their spending activities for stimulating their economies and stabilizing their banking and financial sectors. Obviously an increase in the uncertainty about government spending activities in these phases seems to be related with the increased spending activities of the governments. This argumentation is in line with the decrease in the firm's investments. Here an increase in the uncertainty of government spending activities is related to an uncertain outlook of the economy's future path. In such a situation firms decrease (in real terms) their investment activities. With respect to the current and future price stability uncertainty about government spending activities weakens the stability of the economies prices. Here there is a more indirect transition path leading from an increase of the uncertainty about government spending activities to the revealed restrictive monetary policy reaction pattern observed for the ECB in response to an increase of the current and expected future price level π_t and $\mathbb{E}_t[\pi_{t+1}]$. The current and expected price responses on an increase in the uncertainty about government spending activities for Germany and Italy are shown in the Figure of Appendix C.9.8.

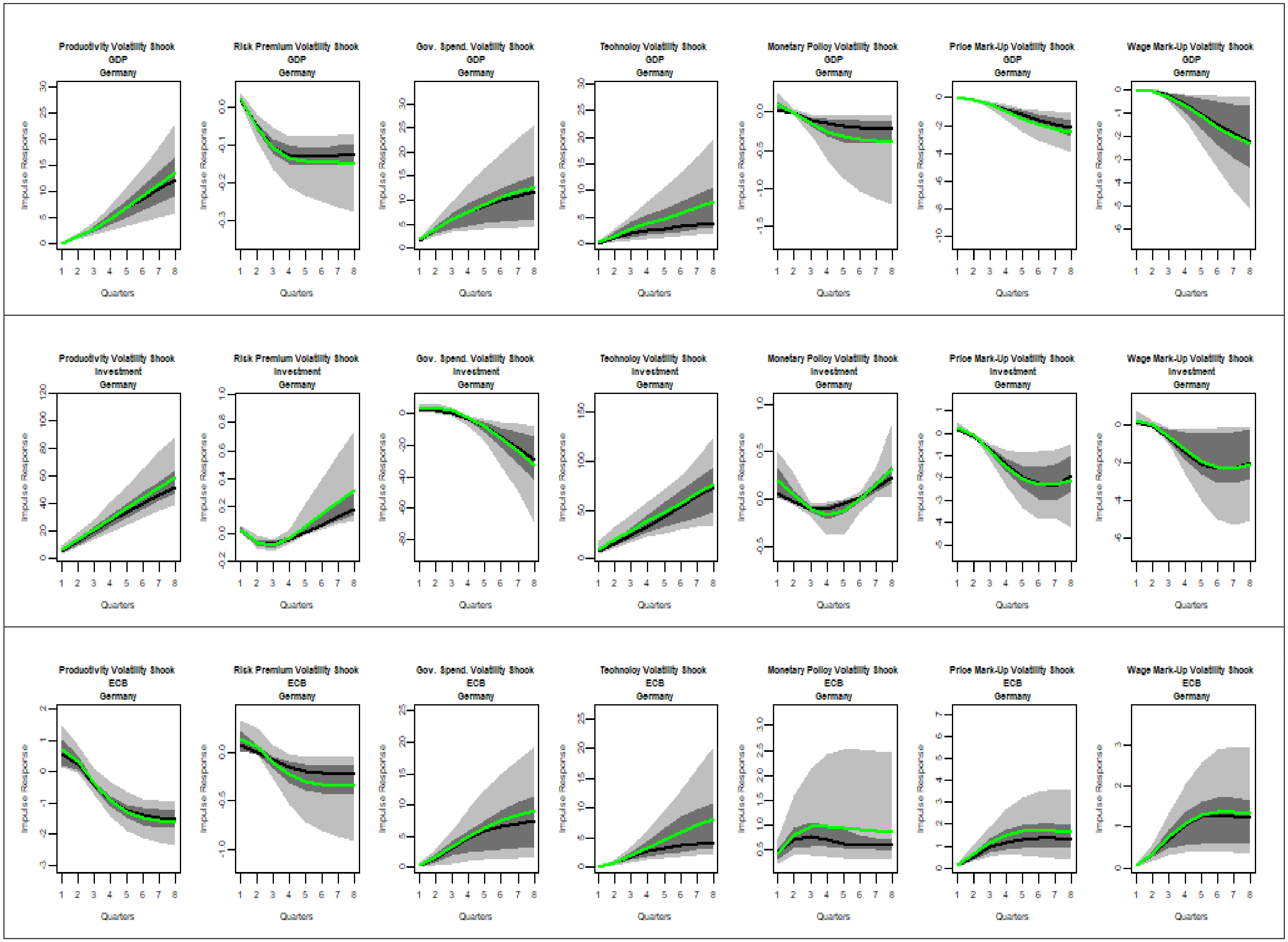


Table 4.13: Macroeconomic responses to one standard deviation uncertainty shocks to $\boldsymbol{\sigma}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t}]$. We show the responses of GDP y_t, investment i_t and ECB's short term rate r_t for Germany. Based on the models posterior distribution we calculate 1000 responses and show the median (black), the mean (green) and the 40% and 80% confidence intervals.

4.5 Conclusion

In combining two newer strands of economic research in this chapter we have introduced and applied a new medium- to large-scale modeling framework that integrates the medium- to large-scale structural modeling of the macroeconomy affected by stochastic economic uncertainty on the one hand with the term structure of interest rate dynamics which itself is effected by the macroeconomy and therefore by the macroeconomy's uncertainty and by its specific bond market related uncertainty on the other hand. In our period between Q1/2005 and Q1/2014 our model estimations for Germany and Italy reveal a pattern of high macroeconomic and term structure of interest rates volatilities in the first recession phase between Q2/2008 and Q2/2009 with the Lehman bankruptcy at the beginning of this economic recession. The macroeconomic state variables further show high volatilities in the second recession phase of our data sample between Q3/2011 and Q1/2013. To deepen our understanding we extract and decompose the macroeconomic uncertainty shock components driving the idiosyncratic part of the economic volatilities. Here we find that for both countries macroeconomic uncertainty shocks are very high in the second half of 2006 and in 2007. Especially the Italian economy strongly reacts to the singular event of the sharp decline in U.S. housing prices in Q1/Q2 2007. For both countries we further reveal an increase in the uncertainties related to issues concerning ECB's monetary policy at the time of the decline in U.S. housing prices and the FED's reaction in September 2007 following this decline. Between Q1/2011 and Q2/2012 both countries show high uncertainties related to economic shocks induced by government spending and risk premium issues. Looking at the term structure volatilities beside the high volatilities in the recession phase between Q2/2008 and Q2/2009 both EMU countries show high interest rate volatilities in the first half of 2011 when the ECB increases its main refinancing operations rate from 1.00% to 1.25%. More interesting are the bond market induced uncertainty shocks to the term structure of interest rate volatilities. For Germany there is a sharp peak in the middle of 2011 close to a sharp peak in the Economic Policy Uncertainty index in Q3/2011 related to the discussions about the haircut of Greece's government debt held by private investors announced in October 2011. In Italy the term structure is effected by a sharp uncertainty shock at the end of 2012 reflecting the increased political uncertainty in Italy in December 2012 when Mario Monti resigned and the Italian parliament was dissolved. Because of the outstanding economic environment reflected by our data sample, beside our large-scale DSGE-USV-ATSM we implement a broad range of alternative pure and macro-finance term-structure of interest rate models for evaluating the quality and robustness of our estimation. Here for both countries we find, that our model shows a good fit and is comparable to the recently introduced stochastic ATSM with implied macroeconomic modeling proposed by Creal and Wu [2017] and to the latent- and MF-ATSM by Ang and Piazzesi [2003] one of the benchmark models in the term structure of interest rates literature.

5. Economic uncertainty in the EMU

5.1 Introduction

What are the sources of economic uncertainty of the European Monetary Union (EMU)? And when was the EMU affected by higher uncertainties? These crucial questions are the key questions of this chapter. To answer these two questions, we have to differentiate our view on the economy of the EMU and on the economic processes driving the EMU's economy in particular. Therefore at first we keep the EMU's economy as a whole and divide the economy into various processes for which we identify the sources of uncertainty lying behind these processes. Our result is the extraction of uncertainty patterns induced by 21 sources of macroeconomic uncertainty. For an observation sample ranging from the Q1/1987 to Q1/2014 we find that between Q4/2005 and Q3/2008 the uncertainty related to EMU's monetary policy sharply increased. This phase of high monetary policy uncertainty is dominated by the overheating of the U.S. housing market and the upcoming of the U.S. subprime crisis starting with the sharp decline in U.S. housing prices in the beginning of 2007 and the reaction of the FED in decreasing its federal funds rate since September 2007. We further find that the ECB's short term interest rates decrease initialized after the bankruptcy of Lehman Brothers in September 2008 where the ECB's main refinancing operations rate decreased from 4.25% to 1.00% between September 2008 and May 2009 reduced the monetary policy related uncertainty in the EMU. Nevertheless we find that monetary policy uncertainty in the EMU still remains historically high. From a more global perspective on monetary policy activities we find that uncertainty more or less continuously increases since the default of Lehman Brothers in September 2008. With respect to the Euro and its introduction - first as accounting currency in 1999 and later as the day-to-day operating currency in 2002 - we find that beside the economic convergence - especially the convergence of interest rates - among the EMU countries until 2006 the introduction of the Euro lead to historically low uncertainties related to financial risk premiums demanded by investors in the EMU. In 2006 the risk premium uncertainty in the EMU began gradually to increase until peaking in Q1/2012, with the agreement of the EMU's finance ministers on the second rescue program for Greece in February 2012 implying the 50% "haircut" of Greece's government debt. In a global context there was a phase of a high global risk premium uncertainty between Q2/1989 and Q2/1992 with the fall of the Berlin wall in November 1989, the German reunification in 1990 and the dissolution of the Soviet Union in December 1991. After that the uncertainty related to the global risk premium declines until the beginning of 2006, when the premium's volatility starts again to increase. A further interesting finding is that since the introduction of the Euro the uncertainty related to the EMU countries' government spending activities steadily increased until reaching a plateau of high uncertainty in the beginning of 2005. To make our perspective a bit broader we extend our perspective in also regarding uncertainties related to the EMU's financial markets. We extract from more than one hundred companies listed in the stock indices DAX 30, CAC 40, AEX 25, FTSE MIB and in the IBEX 35 of Germany, France, Netherlands, Italy and Spain the volatility patterns of their respective stock price movements. Using a data period between Q1/2005 and Q1/2014 - a phase where the Euro and the EMU institutions become more settled and before ECB initializes its unconventional expanded asset purchase program (EAPP) in Q4/2014 and the public sector purchase program (PSPP) in Q1/2015, we find that the EMU based company stocks especially in the recession phase ranging for the five EMU countries between Q1/2008 and Q2/2009 show high volatilities. Here especially the stock markets of Germany and France reveal exceptionally high volatilities. There is a second phase of high financial markets uncertainty at the beginning of the recession phase around October 2011 when the state representatives of the 17 EMU countries announced the 50% "haircut" for Greece on October 27th 2011. Here, Germany too shows the largest reaction with a sharp increase in its stock markets volatility dominated by an increase of the volatilities of stock price movements of German insurance and banking companies.

Because of the international financial crisis becoming a sovereign debt crisis in the EMU since 2010 in this chapter we show the uncertainty patterns in the market for government bonds of the EMU countries Germany and Italy - as two EMU country representatives - between Q1/2005 and Q1/2014 revealed in the foregoing chapter. Here the volatilities of the term structure of interest rates sharply increase after the bankruptcy of Lehman Brothers and ECB's interest rate decreasing reaction. The German term structure also reveals higher uncertainty in the phase around Q4/2010 in which the ECB after two years of decreasing interest rates tried to increase again its short term interest rates. For Italy we further find that the resignation of Mario Monti in Q4/2012 who implemented the Italian austerity policy lead to a larger uncertainty shock.

From a methodological point of view in the first part of this chapter we implement and estimate a large-scale second generation New-Keynesian open economy DSGE model with stochastic volatilities for modeling the economy and the economic uncertainties of the EMU as a whole. For our macroeconomic modeling framework we use the New Area Wide Model (NAWM) proposed by Christoffel, Coenen and Warne [2008]. The modeling of the domestic decision problems of the EMU's households and firms as well as of the ECB's decision finding in the NAWM are close to the economic modeling by Smets and Wouters [2003, 2007] and Christiano, Eichenbaum and Evans [2005] with monopolistic competition in the intermediate and final goods sectors as well as the non-neutrality of money through price and wage stickiness. Monetary policy measures are implemented by a Taylor rule like monetary policy decision rule. Due to its open economy formulation the NAWM internalizes global state variables like the oil price or the global income into its modeling of the EMU's economy. As in ECB [2016] the NAWM is of high practical relevance for ECB's monetary policy decisions and is in direct line with the large-scale open economy models GEM (Global Economy Model by the IMF, cf. Bayoumi, Laxton and Pesenti [2004]) and the Federal Reserve Board's SIGMA model (cf. Erceg, Guerrieri and Gust [2006]). In implementing and estimating our large-scale open economy stochastic volatility DSGE model we are the first who reveal insights about a broad range of different sources of economic uncertainty and the temporal occurrence of phases of higher uncertainty in the EMU. Our work is directly related to the work concerning economic uncertainty in more general by Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry [2018], Baker, Bloom and Davis [2016] and Bloom [2009] and to more specific non-linear macroeconomic DSGE models implying time-varying stochastic volatilities. Here our approach is related to the work by Justiano and Primiceri [2008], Fernandez-Villaverde Guerron-Quintana, RubioRamirez and Uribe [2011], Fernandez-Villaverde and Rubio-Ramirez [2013] and Curdia, Del Negro and Greenwald [2014], Born and Pfeifer [2014], Diebold, Schorfheide and Shin [2017] and Basu and Bundick [2017]. Estimation of our stochastic volatility area wide DSGE model is very similar to the estimation procedure applied by Justiano and Primiceri [2008]. We apply a Markov-Chain-Monte-Carlo Gibbs sampling procedure where we alternate between two large modeling blocks described by two state-space models. Because of the non-linear character of our open economy DSGE we apply a forward Gibbs particle filter with conditional resampling and backward drawing for extracting the volatility patterns induced by the 21 sources of economic uncertainty of our EMU wide macroeconomic model.

For revealing the EMU's financial markets uncertainties in focusing on the EMU's major stock markets we use the theoretical background of the intertemporal capital asset pricing model (iCAPM) proposed by Merton [1973] and more recently empirically investigated by Bali and Engle [2010], where Bali and Engle [2010] use the mean-reverting dynamical conditional correlation model by Engle [2002] for estimating time-varying conditional covariances between company stock's excess returns and the market portfolio. In this paper we use the generalized autoregressive score model recently proposed by Creal, Koopman and Lucas [2011] for estimating the time-varying covariances where the stock's excess returns and the market risk factor are described by a multivariate Student's t-distribution - taking into account more extreme financial risks in the fat tails of the multivariate t-distribution.

This chapter is organized as follows: In section 5.2 we outline in detail the economic decision problems the households, firms, the government and the monetary authority in the EMU and abroad face. We further outline the stochastic processes of the time varying macroeconomic volatilities as well as the rational expectations building in this modeling framework. In section 5.3 we outline in detail our estimation procedure for both the estimation of the model's parameters as well as the extraction of the 21 dynamical volatility patterns. In section 5.4 we discuss the estimation results of our stochastic volatility NAWM. First we focus on the model's in-sample-fitting performance. After that we discuss the extracted EMU uncertainty patterns. In section 5.5 we draw our attention to the uncertainty patterns extracted for the EMU's major stock markets. Beside stock market uncertainties in 5.5 we further discuss term structure of interest rates uncertainty patterns. The conclusion in section 5.6 closes the chapter.

5.2 The macroeconomy of the EMU

To model the EMU wide aggregated macroeconomic development we use the large-scale open economy New Area-Wide Model (NAWM) proposed by Christoffel, Coenen and Warne [2008]. In the next sub-section 5.2.1 we outline in detail the agents intertemporal decision problems implied by the NAWM. In 5.2.2 we describe the processes driving the stochastic volatility terms of our NAWM extension. 5.2.3 outlines the calibration and the rational expectations form of the NAWM.

5.2.1 EMU's economic decision problems

5.2.1.1 Households

The continuum of households indexed by $h \in [0,1]$ maximizes its lifetime expected utility where household's h lifetime utility function is defined as:

$$U_h\left(C_{h,t}, C_{h,t+1}, ..., N_{h,t}, N_{h,t+1}, ...\right) = \mathbb{E}_t\left[\sum_{k=0}^{\infty} \beta^k \left(\epsilon^c_{t+k} ln\left(C_{h,t+k} - \kappa C_{h,t+k-1}\right) - \frac{\epsilon^N_{t+k}}{(1+\zeta)} N^{(1+\zeta)}_{h,t+k}\right)\right] \tag{5.1}$$

Depending on the household's h future consumption and labor path $C_{h,t}, C_{h,t+1}, ...$ and $N_{h,t}, N_{h,t+1}, ...$ where $C_{h,t}$ and $N_{h,t}$ denotes household's h time t purchases of consumption goods and hours worked respectively. β is the household's discount factor for discounting future flows of income and expenditures. ζ denotes the inverse of the Frisch elasticity of labor supply. κ expresses the degree of external habit formation in consumption. ϵ^c_t and ϵ^N_t express consumption preference and labor supply side shocks effecting the households lifetime utility.

At every t household h faces the following budget constraint:

$$
\begin{aligned}
&(1+\tau_t^C)P_{C,t}C_{h,t} + P_{I,t}I_{h,t} + \left(\epsilon_t^{RP}R_t\right)^{-1}B_{h,t+1} + \left[\left(1-\Gamma_{B^*}\left(s_{B^*,t+1},\epsilon_t^{RP^*}\right)\right)R_t^*\right]^{-1}S_tB_{h,t+1} \\
&+\Gamma_{h,t}^{SC} + \Xi_t \\
&= \left(1-\tau_t^N-\tau_t^{W_h}\right)W_{h,t}N_{h,t} + \left(1-\tau_t^K\right)\left[R_{K,t}u_{h,t} - \Gamma_u\left(u_{h,t}\right)P_{I,t}\right]K_{h,t} \\
&+\tau_t^K\delta P_{I,t}K_{h,t} + \left(1-\tau_t^D\right)D_{h,t} - T_t + B_{h,t} + S_tB_{h,t}^*
\end{aligned}
\tag{5.2}
$$

where the household's consumption expenditures, saving and investment activities are listed on the LHS and the (after-tax) income composed of labor income, capital and dividend income as well as income from holding domestic and foreign bonds on the RHS. In 5.2 $P_{C,t}$ and $P_{I,t}$ are the prices for consumption and investment goods respectively. $W_{h,t}$ denotes household's h wage rate. $R_{K,t}$ is the rental rate for the effective capital service households rented to firms. $u_{h,t}R_{K,t}$ and $D_{h,t}$ are the dividends households receive from their owned firms. R_t and R_t^* respectively indicate the risk-less returns on domestic and internationally traded government bonds households receive on their time $t-1$ held bond volumes $B_{h,t-1}$ and $S_{t-1}B_{h,t-1}^*$ respectively. S_t is the nominal exchange rate (where the domestic currency is expressed in units of the foreign currency). On the income side of 5.2 the NAWM's fiscal authority absorbs parts of the households incomes by levying tax rates τ_t^N, τ_t^K and τ_t^D on households wage, captial and dividend income $W_{h,t}, N_{h,t}, R_{K,t}, K_{h,t}$ and $D_{h,t}$. $\tau_t^{W_h}$ denotes the household's wage income contribution to social security programs. $\delta P_{I,t}K_{h,t}$ expresses the cost of physical capital depreciation, where δ denotes the economy's depreciation rate households are facing in t. Multiplying $\delta P_{I,t}K_{h,t}$ with the capital income tax rate τ_t^K reduces households capital income tax payments by the amount of depreciation costs. T_t is an additional lump-sum tax households face. The Households budget restriction is effected by the risk premium shocks ϵ_t^{RP} and $\epsilon_t^{RP^*}$ investor's demand for their holdings of domestic and foreign government bonds. The risk premium $\epsilon_t^{RP^*}$ on internationally traded foreign bonds effects the return of these bonds by:

$$
\Gamma_{B^*}\left(s_{B^*,t+1},\epsilon_t^{RP^*}\right) = \gamma_{B^*}\left[\left(\epsilon_t^{RP^*}\right)^{\frac{1}{\gamma_{B^*}}} exp\left(s_{B_t^*+1}\right) - 1\right] \tag{5.3}
$$

where $(s_{B_t^*+1} = S_tB_{t+1}^*/P_{Y,t}Y_t$ are the EMU's net holdings of internationally traded foreign bonds $S_tB_{t+1}^*$ in domestic currency relative to domestic nominal output, where Y_t and $P_{Y,t}$ denote EMU's real output and output's price index respectively. $\gamma_{B^*} > 0$ denotes the sensitivity of the rate of return on foreign bond with respect to risk premium shocks. From 5.2 and 5.3 it follows, that for the EMU as a net debtor a positive risk premium shock demanded by investors outside the EMU leads to an increase in the rate of return on internationally traded government bonds issued by EMU countries. Ξ_t is a lump-sum intermediation premium encouraging investors for holding foreign bonds in the non-stochastic steady-state. $\Gamma_{h,t}^{SC}$ is a state-contingent security household h holds as an insurance against household-specific wage income risk. These securities are traded among the households and technically guarantees, that the marginal utility of consumption out of wage income is identical across the households so that in equilibrium households will choose identical allocations. Capital utilization $u_{h,t}$ leads to utilization costs described by the following cost function:

$$\Gamma_u\left(u_{h,t}\right) = \gamma_{u,1}\left(u_{h,t} - 1\right) + \frac{\gamma_{u,2}}{2}\left(u_{h,t} - 1\right)^2 \tag{5.4}$$

with $\gamma_{u,1}, \gamma_{u,2} > 0$. Beside the budget constraint households face in their consumption and working decisions the capital sock's law of motion given by:

$$K_{h,t+1} = (1-\delta) K_{h,t} + \epsilon_{t,I}\left[1 - \Gamma_I\left(I_{h,t}/I_{h,t-1}\right) I_{h,t}\right] \tag{5.5}$$

which defines the capital accumulation of the capital stock owned by household h. $\Gamma_I\left(I_{h,t}/I_{h,t-1}\right)$ is the adjustment cost function depending on investment's gross growth rate $I_{h,t}/I_{h,t-1}$ and is defined as:

$$\Gamma_I\left(I_{h,t}/I_{h,t-1}\right) = \frac{\gamma_I}{2}\left(\frac{I_{h,t}}{I_{h,t-1}} - g_z\right) \tag{5.6}$$

with $\gamma_I > 0$ and g_z denoting the economy's trend growth rate in the non-stochastic steady state.

5.2.1.2 Labor market

Similar to the economy proposed by Smets and Wouters [2003, 2007] wage setting in the NAWM is done by applying the Calvo scheme proposed by Calvo [1983] where only a fraction $(1-\xi_w)$ with $0 \leq \xi_w \leq 1$ of households set there required wages actively in renegotiating their wage contracts in requiring wage markups above the general development of productivity and inflation. For all actively setting households these wages are equal:

$$\tilde{W}_t = \tilde{W}_{h,t} \tag{5.7}$$

The fraction of passively wage adjusting households ξ_w lags behind the actively setting households by adjusting their wages only with respect to the economy's productivity and inflation development:

$$W_{h,t} = g_{z,t}\Pi_{C,t}W_{h,t-1} \tag{5.8}$$

where $g_{z,t}$ defines the gross rate of labor productivity growth and $\tilde{\Pi}_{C,t} = \Pi_{C,t-1}^{\chi_w}\bar{\Pi}_t^{(1-\chi_w)}$ is the geometric mean of past consumer price inflation $\Pi_{C,t-1} = P_{C,t-1}/P_{C,t-2}$ and the inflation target $\bar{\Pi}_t$ communicated by the ECB. χ_w indicates the factor with which past inflation is weighted in the wage adjustments of households fraction ξ_w. In consequence these backward looking passive wage adjustments lead to sticky wages in the NAWM economy. For the active wage setting households the wage setting immediately effects their optimal lifetime utility such that from maximizing the households lifetime expected utility we can derive the following first order condition (FOC) with respect to the wage $\tilde{W}_t$:

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}\xi_w^k\beta^k N_{h,t+k}\left(\Lambda_{t+k}\left(1 - \tau_{t+k}^N - \tau_{t+k}^{W_h}\right) g_{z,t:t+k}\frac{\tilde{\Pi}_{C,t:t+k}}{\Pi_{C,t:t+k}}\frac{\tilde{W}_t}{P_{C,t}} - \varphi_{t+k}^W\epsilon_{t+k}^N\left(N_{h,t+k}\right)^{\zeta}\right)\right] = 0 \tag{5.9}$$

$$g_{z,t:t+k} = \prod_{s=1}^{k} g_{z,t+s}$$

$$\tilde{\Pi}_{C,t:t+k} = \prod_{s=1}^{k} \Pi_{C,t+s-1}^{\chi_w} \bar{\Pi}_{t+s}^{(1-\chi_w)}$$

$$\Pi_{C,t:t+k} = \prod_{s=1}^{k} \Pi_{C,t+s-1}$$

The FOC in 5.9 implies that in optimum the wage renegotiating households choose their wage in a way that the (discounted) expected marginal (after-tax) revenues (in consumption based utility) Λ_{t+k} equal the (discounted) expected marginal disutility of labor $N_{h,t}^{\zeta}$.

In the absence of passive wage adjusting households $\xi_w = 0$ from the FOC in 5.9 it follows that:

$$\varphi_t^W = \left(1 - \tau_t^N - \tau_t^{W_h}\right) \frac{\tilde{W}_t}{P_{C,t}} - \epsilon_t^N \frac{N_{h,t+k}^{\zeta}}{\Lambda_t} \tag{5.10}$$

where φ_t^W expresses the wage-markup households require in addition to the compensation of their marginal costs of labor received in a perfectly competitive labor market, such that the wage markup reflects φ_t^W the household's monopolistic power in the NAWM's labor market. The aggregated wage consisting of the fractions ξ_w and $(1-\xi_w)$ simply is:

$$W_t = \left[\xi_w \left(g_{z,t}\tilde{\Pi}_{C,t} W_{t-1}\right)^{\frac{1}{(1-\varphi_t^w)}} + (1-\xi_w)\, \tilde{W}_t^{\frac{1}{(1-\varphi_t^w)}}\right]^{(1-\varphi_t^W)} \tag{5.11}$$

5.2.1.3 Firms

In the NAWM there are four different types of firms differentiated according to their location and the place the respective firm is located in the EMU's value chain. Differentiation starts in the EMU's intermediate-goods producing sector organized under monopolistic competition where a continuum of EMU settled firms indexed by $f \in [0,1]$ selling their products domestically and abroad compete with a continuum of foreign intermediate goods producing firms indexed by $f^* \in [0,1]$ selling their products in the EMU. The domestically produced intermediate goods are combined with imported intermediate goods produced abroad by three types of representative EMU final goods producing firms, which produce non-tradable private consumption, private investment and public consumption goods respectively. Additionally there is a representative export oriented retail firm that combines domestically produced intermediate goods for selling them abroad.

Domestic intermediate-goods producing firms

EMU's intermediate-goods producing firms f minimize their costs:

$$[K_{f,t}, N_{f,t}] = \arg \max_{[K_{f,t}, N_{f,t}]} R_{K,t} K_{f,t} + \left(1 + \tau_t^{W_f}\right) W_t N_{f,t} \tag{5.12}$$

by using a Cobb-Douglas production technology:

$$Y_{f,t} = max \left(\epsilon_t \left(K_{f,t}^S\right)^{\alpha} (z_t N_{f,t})^{(1-\alpha)} - z_t \psi, 0\right) \tag{5.13}$$

where firm's f used production factors are the homogenous capital services $K_{f,t}^S$ rented from the households under perfect competition and household specific differentiated labor h composed in f's labor demand $N_{f,t}$:

$$N_{f,t} = \int_0^1 \left(\left(N_{f,t}^h\right)^{\frac{1}{\varphi_t^W}}\right)^{\varphi_t^W} dh \tag{5.14}$$

ϵ_t and z_t in the firm's production function indicate the economy's transitory and permanent technology shock effecting the NAWM's implied total factor and labor productivity respectively. ψ are the fixed costs of production equal across all EMU settled intermediate goods producers, where these costs are scaled by the permanent technology shock. $\tau_t^{W_f}$ is the tax rate levied on wage payments reflecting the firm's contribution to social security programs. From the FOC of the firms cost minimization the firms nominal marginal cost $MC_{f,t}$ are:

$$MC_t = MC_{f,t} \tag{5.15}$$

with:

$$MC_t = \frac{1}{\epsilon_t z_t^{(1-\alpha)} \alpha^{\alpha} (1-\alpha)^{(1-\alpha)}} (R_{K,t})^{\alpha} \left[(1 + \tau_t^{W_f}) W_t\right]^{(1-\alpha)} \tag{5.16}$$

where the marginal cost of domestic intermediate goods production are equal across the firms because of the same production technology all firms in the intermediate goods sector use and the same factor prices all firms face. Taking the wages of their demanded specific labor h as given, minimizing their costs with respect to their labor costs the firm's demand for household-specific labor h is:

$$N_{f,t}^h = \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} N_{f,t} \tag{5.17}$$

Integrating over the continuum of domestic intermediate-goods producing firms f , the aggregated demand for the household-specific labor h is:

$$N_t^h = \int_0^1 N_{f,t}^h df = \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} N_t \tag{5.18}$$

where $\varphi_t^W/(\varphi_t^W-1)$ expresses the wage elasticity of the labor demand. Integrating over the continuum of households h leads to the aggregated wage expression:

$$W_t = \left(\int_0^1 W_{h,t}^{\frac{\varphi_t^W}{(1-\varphi_t^W)}} dh\right)^{(1-\varphi_t^W)} \tag{5.19}$$

With respect to their costs the firms f are price takers, whereas in selling their products $Y_{f,t}$ parts of the firms f are active price setters. The firms sell their products on the domestic markets in the EMU and abroad. Similar to the labor markets there are sluggish prices due to staggered price contracts, such that price setting in both the domestic and the foreign intermediate-goods markets is described by applying the Calvo scheme, proposed by Calvo [1983], where the fractions $(1-\xi_H)$ and $(1-\xi_X)$ are active price setting firms in their domestic and foreign markets respectively. The prices of the passive price adjusting firms are $P_{H,f,t}$ and $P_{X,f,t}$ where these firms adjust their prices with respect to the ECB's inflation objective and the intermediate goods inflation the firms face domestically and abroad according to the geometric average:

$$P_{H,f,t} = \Pi_{H,t-1}^{\chi_H} \bar{\Pi}^{(1-\chi_H)} P_{H,f,t-1} \tag{5.20}$$

$$P_{X,f,t} = \Pi_{X,t-1}^{\chi_H} \bar{\Pi}^{(1-\chi_H)} P_{X,f,t-1} \tag{5.21}$$

with the intermediate-goods domestic and abroad inflation $\Pi_{H,t-1} = P_{H,t-1}/P_{H,t-2}$ and $\Pi_{X,t-1} = P_{X,t-1}/P_{X,t-2}$ respectively. χ_H is the weighting factor for the past inflation in the geometric mean the firms uses for their passive price adjustments. Intermediate-goods producing firms actively setting their prices domestically and abroad maximize their expected nominal profits:

$$\mathbb{E}_t \left[\sum_{k=1}^{\infty} \Lambda_{t,t+k} \xi_H^k \left(P_{H,f,t} H_{f,t} - MC_t H_{f,t}\right) + \Lambda_{t,t+k} \xi_X^k \left(P_{X,f,t} X_{f,t} - MC_t X_{f,t}\right)\right] \tag{5.22}$$

where the expected profit is composed of the domestically generated profit and the profit generated abroad. $H_{f,t}$ and $X_{f,t}$ express the domestic and the foreign demand firm f faces in t . $\Lambda_{t,t+k}$ is the stochastic discount factor derived from the consumption Euler equation of the households. The profits firms yielded domestically and abroad are distributed as dividends to the households. From the expected profit maximization the following FOCs for the active price adjusting firm with respect to its optimal price decisions domestically and abroad $P_{H,f,t}$ and $P_{X,f,t}$ can be derived:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \xi_H^k \left(\tilde{\Pi}_{H,t:t+k} \tilde{P}_{H,t} - \varphi_{t+k}^H MC_{t+k}\right) H_{f,t+k}\right] = 0 \tag{5.23}$$

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \xi_X^k \left(\tilde{\Pi}_{X,t:t+k} \tilde{P}_{X,t} - \varphi_{t+k}^X MC_{t+k}\right) X_{f,t+k}\right] = 0 \tag{5.24}$$

where $\tilde{P}_{H,t} = \tilde{P}_{H,f,t}$ and $\tilde{P}_{X,t} = \tilde{P}_{X,f,t}$ are the prices the active price setting fractions of intermediate goods producers $(1-\xi_h)$ and $(1-\xi_X)$ demand for their products with:

$$P_{H,f,t+k} = \tilde{\Pi}_{H,t:t+k}\tilde{P}_{H,t}$$

$$\tilde{\Pi}_{H,t:t+k} = \prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_H} \bar{\Pi}_{t+s}^{(1-\chi_H)}$$

$$P_{X,f,t+k} = \tilde{\Pi}_{X,t:t+k}\tilde{P}_{X,t}$$

$$\tilde{\Pi}_{X,t:t+k} = \prod_{s=1}^{k} \Pi_{X,t+s-1}^{\chi_x} \bar{\Pi}_{t+s}^{(1-\chi_x)}$$

respectively. Similar to the active wage setting outlined 5.2.1.2 optimal prices equate the sum of expected discounted revenues to the sum of expected discounted marginal costs, where φ_t^H and φ_t^X expresses the price-markups the intermediate goods producing firms can demand domestically and abroad above their marginal costs. Analogue to the aggregated wage the aggregated price indices $P_{H,t}$ and $P_{X,t}$ are determined by:

$$P_{H,t} = \left[(1-\xi_H)\left(\tilde{P}_{H,t}\right)^{\frac{1}{(1-\varphi_t^H)}} + \xi_H\left(\Pi_{H,t-1}^{\chi_H}\bar{\Pi}_t^{(1-\chi_H)}P_{H,t-1}\right)^{\frac{1}{(1-\varphi_t^H)}}\right] \quad (5.25)$$

$$P_{X,t} = \left[(1-\xi_X)\left(\tilde{P}_{X,t}\right)^{\frac{1}{(1-\varphi_t^X)}} + \xi_X\left(\Pi_{X,t-1}^{\chi_x}\bar{\Pi}_t^{(1-\chi_x)}P_{X,t-1}\right)^{\frac{1}{(1-\varphi_t^X)}}\right] \quad (5.26)$$

Foreign intermediate-goods producing firms

The exporting foreign intermediate-goods producing firms f^* sell their differentiated products $Y_{f^*,t}^*$ in the monopolistically competitive domestic markets of the EMU, setting their prices in their local currency as described by Betts and Devereux [1996]. Analogue to their EMU competitors the continuum of foreign firms is divided into a fraction $(1-\xi^*)$ of active price setting firms and a fraction of firms ξ^* which only passively adjust their prices $P_{IM,f^*,t}$ according to the acquainted scheme:

$$P_{IM,f^*,t} = \Pi_{IM,t-1}^{\chi^*}\bar{\Pi}_t^{(1-\chi^*)}P_{IM,f^*,t-1} \quad (5.27)$$

with: $P_{IM,f^*,t} = P_{X,f^*,t}^*$ and $\Pi_{IM,t-1} = P_{IM,t-1}/P_{IM,t-2}$ with $P_{IM,t} = P_{X,t}^*$. Here it is assumed that the import price the product is sold in the domestic market of the EMU is equal to the price the exporting foreign firm f^* $P_{X,f^*,t}^*$ demands for its products. The prices here are expressed in terms of the domestic currency. The active price setting firms maximize their expected profits with respect to their decisions concerning their price adjustments. The expected nominal profit the firm f^* maximizes is expressed as:

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\xi^*)^k\Lambda_{t,t+k}^*\left(P_{IM,f^*,t}IM_{f^*,t} - MC_t^*IM_{f^*,t}\right)/S_{t+k}\right] \quad (5.28)$$

with $IM_{f^*,t} = X^*_{f^*,t}$. Firm's f^* marginal cost MC^*_t are:

$$MC^*_t = S_t \left(P_{O,t}\right)^{\omega^*} \left(P^*_{Y,t}\right)^{(1-\omega^*)} \tag{5.29}$$

such that the marginal cost are the geometric mean of the price of oil $P_{O,t}$ and the foreign prices $P^*_{Y,t}$, where ω^* determines the share of oil in EMU imports. Similar to the domestic firms the FOC with respect to the exporting firm's f^* optimal price setting decision can be derived as:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\xi^*\right)^k \Lambda^*_{t,t+k} \left(\tilde{\Pi}_{IM,t:t+k} \tilde{P}_{IM,t} - \varphi^*_{t+k} MC^*_{t+k} \right) IM_{f^*,t+k}/S_{t+k} \right] = 0 \tag{5.30}$$

with $P_{IM,f^*,t+k} = \tilde{\Pi}_{IM,t:t+k} \tilde{P}_{IM,t}$ and $\tilde{\Pi}_{IM,t:t+k} = \prod_{s=1}^{k} \Pi^{\chi^*}_{IM,t+s-1} \bar{\Pi}^{(1-\chi^*)}_{t+s}$. The aggregate price index of foreign intermediate goods imported into the EMU is given by:

$$P_{IM,t} = \left[(1-\xi^*) \left(\tilde{P}_{IM,t} \right)^{\frac{1}{(1-\varphi^*_t)}} + \xi^* \left(\Pi^{\chi^*}_{IM,t-1} \bar{\Pi}^{(1-\chi^*)}_t P_{IM,t-1} \right)^{\frac{1}{(1-\varphi^*_t)}} \right]^{(1-\varphi^*_t)} \tag{5.31}$$

where φ^*_t is the price markup required by foreign firms in the market of imported intermediate goods.

Domestic final-goods producing firms

As mentioned above the NAWM implies three different types of (representative) finalgoods firms combining domestic and foreign imported intermediate goods into final private and public consumption goods and private investment goods Q^C_t, Q^G_t and Q^I_t respectively. The production of the private goods Q^C_t and Q^I_t is very similar. Both representative firms use for their respective productions the returns-to-scale CES production technologies:

$$Q^C_t = \left[v_{c,t}^{\frac{1}{\mu_c}} \left(H^C_t\right)^{\left(1-\frac{1}{\mu_c}\right)} + \left(1-v_{c,t}\right)^{\frac{1}{\mu_c}} \left(1 - \Gamma_{IM^c}\left(IM^C_t/Q^C_t, \epsilon^{IM}_t\right) IM^C_t\right)^{\left(1-\frac{1}{\mu_c}\right)} \right]^{\frac{\mu_c}{(\mu_c-1)}} \tag{5.32}$$

and

$$Q^I_t = \left[v_{I,t}^{\frac{1}{\mu_I}} \left(H^I_t\right)^{\left(1-\frac{1}{\mu_I}\right)} + \left(1-v_{I,t}\right)^{\frac{1}{\mu_I}} \left(1 - \Gamma_{IM^I}\left(IM^I_t/Q^I_t, \epsilon^{IM}_t\right) IM^I_t\right)^{\left(1-\frac{1}{\mu_I}\right)} \right]^{\frac{\mu_I}{(\mu_I-1)}} \tag{5.33}$$

where H^C_t and H^I_t denotes the bundle of domestic intermediate goods the final private consumption and investment goods producing firms use and IM^C_t and IM^I_t represent the foreign intermediate goods they need for their final-goods production. $v_{c,t}$ and $v_{I,t}$ indicate the share of domestic goods (home bias) in the production of the private consumption and investment goods respectively. The CES parameters μ_c and μ_I denote the intertemporal elasticity of substitution between the distinct bundles of intermediate goods produced domestically and abroad. Variations in the share of imported goods IM_t^C/Q_t^C and IM_t^I/Q_t^I lead to costs in the production of the final goods Q_t^C and Q_t^I expressed by the cost functions:

$$\Gamma_{IM^C}\left(IM_t^C/Q_t^C,\epsilon_t^{IM}\right)=\frac{\gamma_{IM^C}}{2}\left[\left(\epsilon_t^{IM}\right)^{-\frac{1}{\gamma_{IM^C}}}\frac{IM_t^C/Q_t^C}{IM_{t-1}^C/Q_{t-1}^C}-1\right]^2 \tag{5.34}$$

and $\Gamma_{IM^I}\left(IM_t^I/Q_t^I,\epsilon_t^{IM}\right)$, where $\Gamma_{IM^I}\left(IM_t^I/Q_t^I,\epsilon_t^{IM}\right)$ is analogue to 5.34. $\gamma_{IM^C},\gamma_{IM^I}>0$ effect the private consumption and investment goods import adjustment costs respectively. ϵ_t^{IM} denotes an import demand shock. The aggregated quantities H_t^C and IM_t^C demanded by the private consumption goods producer from the domestic and foreign firms f and f^* are expressed as:

$$H_t^C=\left[\int_0^1\left(H_{f,t}^C\right)^{\frac{1}{\varphi_t^H}}df\right]^{\varphi_t^H} \tag{5.35}$$

$$IM_t^C=\left[\int_0^1\left(IM_{f^*,t}^C\right)^{\frac{1}{\varphi_t^*}}df^*\right]^{\varphi_t^*} \tag{5.36}$$

H_t^I and IM_t^I are aggregated in an analogous manner. In their profit maximization the final-goods firms take the prices of the domestic and foreign intermediate goods $P_{H,f,t}$ and $P_{H,f^*,t}$ as given and choose their optimal use of the differentiated goods produced by the firms f and f^*, such that the final-goods producers' demand functions with respect to each of the differentiated intermediate-goods f and f^* are:

$$H_{f,t}^C=\left[\frac{P_{H,f,t}}{P_{H,t}}\right]^{-\frac{\varphi_t^H}{(\varphi_t^H-1)}}H_t^C \tag{5.37}$$

$$IM_{f^*,t}^C=\left[\frac{P_{IM,f^*,t}}{P_{IM,t}}\right]^{-\frac{\varphi_t^*}{(\varphi_t^*-1)}}IM_t^C \tag{5.38}$$

where the aggregated price indices for the bundles of domestic and foreign intermediate goods $P_{H,t}$ and $P_{IM,t}$ are:

$$P_{H,t}=\left[\int_0^1\left(P_{H,f,t}\right)^{\frac{1}{(1-\varphi_t^H)}}df\right]^{(1-\varphi_t^H)} \tag{5.39}$$

$$P_{IM,t}=\left[\int_0^1\left(P_{IM,f^*,t}\right)^{\frac{1}{(1-\varphi_t^*)}}df^*\right]^{(1-\varphi_t^*)} \tag{5.40}$$

With respect to $H_{f,t}^I$ and $IM_{f,t}^I$ the respective demand functions have an analogue form. Beside the choice of the optimal use of each differentiated intermediate good f and f^* respectively, the final goods producer has to decide about the optimal combination of domestic and foreign intermediate-good bundles H_t^C and IM_t^C. Here the final-goods producers face the optimization problem:

$$[H_t^C,IM_t^C]=\arg\min_{[H_t^C,IM_t^C]}P_{H,t}H_t^C+P_{IM,t}IM_t^C \tag{5.41}$$

subject to their CES production technology in 5.32 which leads to their demand functions:

$$H_t^C = \upsilon_{c,t}\left(\frac{P_{H,t}}{P_{C,t}}\right)^{-\mu_c} Q_t^C \tag{5.42}$$

$$IM_t^C = (1-\upsilon_{c,t})\left[\frac{P_{IM,t}}{P_{C,t}\tilde{\Gamma}_{IM^C}\left(IM_t^C/Q_t^C,\epsilon_t^{IM}\right)}\right]^{-\mu_c}\frac{Q_t^C}{\left(1-\Gamma_{IM^C}\left(IM_t^C/Q_t^C,\epsilon_t^{IM}\right)\right)} \tag{5.43}$$

with

$$\tilde{\Gamma}_{IM^C}\left(IM_t^C/Q_t^C,\epsilon_t^{IM}\right) = 1-\Gamma_{IM^C}\left(IM_t^C/Q_t^C,\epsilon_t^{IM}\right)-\Gamma'_{IM^C}\left(IM_t^C/Q_t^C,\epsilon_t^{IM}\right)IM_t^C$$

5.42 and 5.43 in the CES technology leads to the price equation:

$$P_{C,t} = \left[\upsilon_{c,t}\left(P_{H,t}\right)^{(1-\mu_c)}+(1-\upsilon_{c,t})\left(\frac{P_{IM,t}}{\tilde{\Gamma}_{IM^C}\left(IM_t^C/Q_t^C,\epsilon_t^{IM}\right)}\right)\right]^{\frac{1}{(1-\mu_c)}} \tag{5.44}$$

The final-goods producers in the market for private investment goods face a similar minimization problem as in 5.41 leading to demand equations for H_t^I and IM_t^I and a price equation for $P_{I,t}$ analogue to the equations in 5.42, 5.43 and 5.44.

Different to the production of the private consumption and investment good the production of the public consumption good only uses a bundle of domestically produced differentiated intermediate goods. Here, the production function simply is:

$$Q_t^G = H_t^G \tag{5.45}$$

The bundle of intermediate goods H_t^G used for the production of Q_t^G is determined by the aggregation rule applied across the differentiated intermediate goods f:

$$H_t^G = \left[\int_0^1 \left(H_{f,t}^G\right)^{\frac{1}{\varphi_t^H}} df\right]^{\varphi_t^H} \tag{5.46}$$

where the public consumption goods producer chooses its optimal quantity of specific intermediate goods f similar to the private consumption and investment goods producing firms according to:

$$H_{f,t}^G = \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{(\varphi_t^H-1)}} \tag{5.47}$$

With respect to the production function in 5.45 the price of the public consumption good $P_{G,t}$ is given by:

$$P_{G,t} = P_{H,t} \tag{5.48}$$

with the optimal quantities $H_{f,t}^{C}, H_{f,t}^{I}$ and $H_{f,t}^{G}$ the aggregated demand for the domestically produced differentiated intermediate goods f is given by:

$$\begin{aligned} H_{f,t} &= H_{f,t}^{C} + H_{f,t}^{I} + H_{f,t}^{G} \\ &= \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\frac{\varphi_t^H}{\left(\varphi_t^H - 1\right)}} H_t \end{aligned} \tag{5.49}$$

whereas with $IM_{f^*,t}^{C}, IM_{f^*,t}^{I}$ and $IM_{f^*,t}^{G}$ the aggregated demand for the differentiated intermediate goods produced by the foreign firms f^* is:

$$\begin{aligned} IM_{f^*,t} &= IM_{f^*,t}^{C} + IM_{f^*,t}^{I} \\ &= \left(\frac{P_{IM,f^*,t}}{P_{IM,t}} \right)^{-\frac{\varphi_t^*}{\left(\varphi_t^* - 1\right)}} IM_t \end{aligned} \tag{5.50}$$

with $H_t = H_t^C + H_t^I + H_t^G$ and $IM_t = IM_t^C + IM_t^I$.

Domestic export oriented firms

In the NAWM economy the fourth type of firm is a retail firm settled abroad that bundles the differentiated intermediate goods $X_{f,t}$ produced by EMU firms f for their markets abroad, using the following CES production technology:

$$X_t = \left[\int_0^1 \left(X_{f,t} \right)^{\frac{1}{\varphi_t^X}} df \right]^{\varphi_t^X} \tag{5.51}$$

The retailer purchases the domestic intermediate goods in a monopolistially competitive market, so that the foreign retail firm takes the prices $P_{X,f,t}/S_t$ as given. Therefore the retailer minimizes the cost in choosing the optimal quantity of each differentiated intermediate good $X_{f,t}$ given $P_{X,f,t}/S_t$ and the aggregation rule 5.51. This cost minimization leads to the retailers demand for the differentiated intermediate good f:

$$X_{f,t} = \left(\frac{P_{X,f,t}}{P_{X,t}} \right)^{-\frac{\varphi_t^X}{\left(\varphi_t^X - 1\right)}} X_t \tag{5.52}$$

with the aggregated price index for the bundle of exported domestic intermediate goods:

$$P_{X,t} = \left[\int_0^1 \left(P_{X,f,t} \right)^{\frac{1}{\left(1-\varphi_t^X\right)}} df \right]^{\left(1-\varphi_t^X\right)} \tag{5.53}$$

which takes the exporting retail firm as given and supplies the quantity X_t satisfying foreign demand, that is determined by a world demand function similar to the domestic demand function for import goods in 5.43:

$$X_t = \upsilon_t^* \left[\frac{P_{X,t}/S_t}{P_{X,t}^{C\,*} \tilde{\Gamma}_X \left(X_t / Y_t^{d\,*}, \epsilon_t^X \right)} \right]^{-\mu^*} \frac{Y_t^{d\,*}}{\left(1 - \Gamma_X \left(X_t / Y_t^{d\,*}, \epsilon_t^X \right)\right)} \tag{5.54}$$

with:

$$\tilde{\Gamma}_X\left(X_t/Y_t^{d\,*},\epsilon_t^X\right)=1-\Gamma_X\left(X_t/Y_t^{d\,*},\epsilon_t^X\right)-\Gamma_X'\left(X_t/Y_t^{d\,*},\epsilon_t^X\right)X_t \tag{5.55}$$

where μ^* indicates the export's price elasticity. v_t^* expresses the export share of domestic intermediate goods, that captures the foreign non-price related preferences for domestic goods. $P_{X,t}^{C\,*}$ denotes the price of foreign firms competing with the EMU firms in their export markets. $Y_t^{d\,*}$ indicates foreign (world) demand and similar to the NAWM's imports, exports also faces adjustment costs in their variation relative to the demand $Y_t^{d\,*}$ expressed by:

$$\Gamma_X\left(X_t/Y_t^{d\,*},\epsilon_t^X\right)=\frac{\gamma^*}{2}\left[\left(\epsilon_t^X\right)^{-\frac{1}{\gamma^*}}\frac{X_t/Y_t^{d\,*}}{X_{t-1}Y_{t-1}^{d\,*}}-1\right]^2 \tag{5.56}$$

5.2.1.4 Fiscal and Monetary Authorities

Fiscal authority and its budged constraint

In the NAWM the fiscal authority purchases the quantity of public consumption goods G_t. The fiscal authority's financial sources for its activities are taxes on wage income, on private consumption spending on capital and dividends income as well as from lumpsum taxes. Deficits in the fiscal budget are financed by issuing sovereign bonds B_t, such that the fiscal authority faces the following budget constraint:

$$\begin{aligned}P_{G,t}G_t+B_t=&\tau_t^C P_{C,t}C_t+\left(\tau_t^N+\tau_t^{W_h}\right)\left(\int_0^1 W_{h,t}N_{h,t}dh\right)+\tau_t^{W_f}W_tN_t\\&+\tau_t^K\left[R_{K,t}u_t-\left(\Gamma_u\left(u_t\right)+\delta\right)P_{I,t}\right]K_t+\tau_t^D D_t+T_t+R_t^{-1}B_{t+1}\end{aligned} \tag{5.57}$$

For financing budget deficits it is assumed that Riccardian equivalence holds such that in the long run there is no difference in financing the budget deficit by issuing new bonds or levying lump-sum taxes. In the NAWM without loss of generality it is assumed that the fiscal authority closes its budget deficits by raising lump-sum taxes.

Monetary authority and its monetary policy decision rule

Monetary policy decisions in the NAWM are based on a Taylor-rule like (log) reaction function:

$$\begin{aligned}ln\left(\frac{R_t}{R}\right)=&\phi_R\,ln\left(\frac{R_{t-1}}{R}\right)+(1-\phi_T)\left[ln\left(\frac{\Pi_t}{\bar{\Pi}}\right)+\phi_\pi\left(ln\left(\frac{\Pi_{C,t-1}}{\bar{\Pi}}\right)-ln\left(\frac{\Pi_t}{\bar{\Pi}}\right)\right)\right]\\&+ln\left(\frac{\tilde{Y}_t}{Y}\right)+\phi_{\Delta\Pi}\left(ln\left(\frac{\bar{\Pi}_{C,t}}{\bar{\Pi}}\right)-ln\left(\frac{\bar{\Pi}_{C,t-1}}{\bar{\Pi}}\right)\right)\\&+\phi_{\Delta Y}\left(ln\left(\frac{\tilde{Y}_t}{Y}\right)-ln\left(\frac{\tilde{Y}_{t-1}}{Y}\right)\right)+ln\left(\eta_t^R\right)\end{aligned} \tag{5.58}$$

where $\tilde{Y}_t = Y_t/z_t$ denotes the aggregated output Y_t scaled by the permanent technology shock lastingly effecting the economy's labor productivity. R, Π and Y are the steady-state values of the nominal short term interest rate, the ECB's long-run inflation target and the (productivity scaled) aggregated output. $ln\left(\bar{\Pi}_t/\bar{\Pi}\right)$ in 5.58 expresses a temporal (log) deviation of the inflation objective from its long-run target $\bar{\Pi}$. In our specification of the NAWM we allow a time-variation in the ECB's inflation target (relative to its long-term target) described by:

$$ln\left(\frac{\bar{\Pi}_t}{\bar{\Pi}}\right) = \rho_\Pi \, ln\left(\frac{\bar{\Pi}_{t-1}}{\bar{\Pi}}\right) + ln\left(\eta_t^\Pi\right) \tag{5.59}$$

such that in total monetary policy is effected by the two shock variables η_t^R directly affecting the monetary policy rate R_t and η_t^Π indirectly affecting R_t.

5.2.1.5 Net foreign Assets, Trade Balance and Terms of Trade

In the NAWM unbalances with the international trade partners are financed by the domestic economy's net holdings of foreign bonds:

$$\frac{B_{t+1}^*}{R_t^*} = B_t^* \frac{TB_t}{S_t} \tag{5.60}$$

where the LHS are the foreign bonds with redemption in $t+1$ financing (possible) trade surpluses in the trade balance:

$$TB_t = P_{X,t}X_t - P_{IM,t}IM_t \tag{5.61}$$

in refinancing maturing bonds B_t^* in t. The economy's terms of trade, expressing the domestic prices of imports $P_{IM,t}$ relative to the prices of exports $P_{X,t}$ payed to firms abroad, are expressed as:

$$ToT_t = \frac{P_{IM,t}}{P_{X,t}} \tag{5.62}$$

In Appendix D.2 we further outline the DSGE model's market clearing and aggregate resource constraints.

5.2.2 Shocks and uncertainty in the EMU wide economy

In our NAWM implementation the 21 structural shocks:

$$\begin{aligned}\hat{\boldsymbol{\vartheta}}_t^T = [&\hat{\epsilon}_t^c, \hat{g}_{z,t}, \hat{\epsilon}_t^i, \hat{\epsilon}_t^{RP}, \hat{\epsilon}_t^{RP^*}, \hat{\varphi}_t^w, \hat{\epsilon}_t^N, \hat{\varphi}_t^h, \hat{\varphi}_t^x, \hat{\varphi}_t^*, \hat{\epsilon}_t^{im}, \hat{\eta}_t^r,\\ &\hat{\eta}_t^\pi, \hat{\epsilon}_t^x, \hat{\epsilon}_t, \hat{\upsilon}_t^*, \hat{\epsilon}_t^{y^*}, \hat{\epsilon}_t^{r^*}, \hat{\epsilon}_t^{P_{OIL}}, \hat{\epsilon}_t^{p_y^*}, \hat{\epsilon}_t^g]\end{aligned}$$

are independently but autocorrelated and evolve according to:

$$\hat{\boldsymbol{\vartheta}}_t = \mathbf{P}\hat{\boldsymbol{\vartheta}}_{t-1} + \boldsymbol{\Sigma}_{\vartheta,t}\boldsymbol{\epsilon}_t \tag{5.63}$$

where:

$$\boldsymbol{\varepsilon}_t^T = \left[\varepsilon_t^c, \varepsilon_t^{g_z}, \varepsilon_t^i, \varepsilon_t^{RP}, \varepsilon_t^{RP^*}, \varepsilon_t^{\varphi^w}, \varepsilon_t^N, \varepsilon_t^{\varphi^h}, \varepsilon_t^{\varphi^x}, \varepsilon_t^{\varphi^*}, \varepsilon_t^{im}, \varepsilon_t^r, \varepsilon_t^{\bar{\pi}}, \varepsilon_t^x, \varepsilon_t, \varepsilon_t^{v^*}, \varepsilon_t^{y^*}, \varepsilon_t^{r^*}, \varepsilon_t^{p_{OIL}}, \varepsilon_t^{p_y^*}, \varepsilon_t^g,\right]$$

defines the vector of economic disturbances, $\mathbf{P}$ is a diagonal 21×21 matrix expressing the first order autoregression and $\mathbf{\Sigma}_{\vartheta,t}$ is a diagonal matrix with the 21 time-varying stochastic volatilities:

$$\begin{aligned}\boldsymbol{\sigma}_t^T = [&\sigma_t^c, \sigma_t^{g_z}, \sigma_t^i, \sigma_t^{RP}, \sigma_t^{RP^*}, \sigma_t^{\varphi^w}, \sigma_t^N, \sigma_t^{\varphi^h}, \sigma_t^{\varphi^x}, \sigma_t^{\varphi^*}, \sigma_t^{im}, \sigma_t^r, \sigma_t^{\bar{\pi}}, \sigma_t^x, \sigma_t, \sigma_t^{v^*}, \\ &\sigma_t^{y^*}, \sigma_t^{r^*}, \sigma_t^{p_{OIL}}, \sigma_t^{p_y^*}, \sigma_t^g]\end{aligned} \tag{5.64}$$

on its diagonal. The dynamics of the second moments in $\boldsymbol{\sigma}_t$ follow the VAR[1] process:

$$ln\left(\boldsymbol{\sigma}_t\right) = \boldsymbol{\mu}_\sigma + \mathbf{P}_\sigma ln\left(\boldsymbol{\sigma}_{t-1}\right) + \mathbf{\Sigma}_\sigma \boldsymbol{\varepsilon}_{\sigma,t} \tag{5.65}$$

where $\boldsymbol{\mu}_\sigma$ is the 21×1 conditional mean of the (log) volatilities, $\mathbf{P}_\sigma$ is the 21×21 autoregressor matrix and the 21×21 covariance matrix $\mathbf{\Sigma}_{\sigma,t}$ is diagonal. The idiosyncratic term of the time-varying volatilities interpreted as uncertainty shocks to the EMU economy is Gaussian with $\boldsymbol{\varepsilon}_{\sigma,t} \sim N(\mathbf{0}, \mathbf{I}_{21\times 21})$.

5.2.3 Calibration and rational expectations form

5.2.3.1 Calibration and steady state

In line to Kydland and Prescott [1982] parts of the NAWM's parameters are calibrated. In the calibration of the NAWM we follow Christoffel, Coenen and Warne [2008]. Similar to Christoffel, Coenen and Warne [2008] we reduce the steady-state version of the NAWM to a system consisting of four equations expressing the equilibrium relations in the labor-, the capital-, and the goods-markets with their relative prices respectively. We simultaneously solve this equation system, receiving the steady-state values of k, c, N and p_i where the last one is the price of the investment good expressed relative to the price of the consumption good. For the dynamics of the foreign global economy state variables $\hat{\boldsymbol{s}}_t^{*\,T} = [\hat{y}_t^*, \hat{r}_t^*, \hat{p}_{OIL,t}, \hat{p}_{y^*,t}]$ and we define:

$$\hat{\boldsymbol{s}}_t^* = \mathbf{A}\hat{\boldsymbol{s}}_{t-1}^* + \hat{\boldsymbol{\varepsilon}}_t^* \tag{5.66}$$

with $\hat{\boldsymbol{\varepsilon}}_t^{*\,T} = \left[\hat{\epsilon}_t^{y^*}, \hat{\epsilon}_t^{r^*}, \hat{\epsilon}_t^{p_{OIL}}, \hat{\epsilon}_t^{p_y^*}\right]$ in $\hat{\boldsymbol{\vartheta}}_t$ where $\hat{\boldsymbol{\vartheta}}_t$ follows the dynamics defined in 5.63. $\mathbf{A}$ is a 4×4 lower triangular matrix. The process of government spending activities is defined by:

$$\hat{g}_t = a\hat{g}_{t-1} + \varepsilon_t^g \tag{5.67}$$

with ε_t^g also in $\hat{\boldsymbol{\vartheta}}_t$ which follows the dynamics in 5.63.

5.2.3.2 Canonical rational expectations form of the implemented NAWM

Following Herbst and Schorfheide [2016] or Dejong and Dave [2011] to determine the agent's rational expectations in a first step the 62 log-linearized equations of 5.2.1 and Appendix D.2 combined with 55 additional necessary (log) linear equations for the 117 endogenous macroeconomic variables of our stochastic volatility NAWM are transferred into the DSGE's canonical linear rational expectations form:

$$\mathbf{\Gamma}_0 \hat{\boldsymbol{s}}_t = \mathbf{\Gamma}_1 \hat{\boldsymbol{s}}_{t-1} + \mathbf{\Psi} \boldsymbol{\varepsilon}_t + \mathbf{\Pi} \boldsymbol{\eta}_t \tag{5.68}$$

where:

$$\begin{aligned}\hat{\boldsymbol{s}}_t^T = [&\hat{\lambda}_t, \hat{c}_t, \hat{Q}_t, \hat{p}_{i,t}, \hat{p}_{k,t}, \hat{p}_{x,t}, \hat{p}_{y,t}, \hat{p}_{im,t}, \hat{p}_{c,t}, \hat{p}_{g,t}, \hat{p}_t^c, , \hat{i}_t, , \hat{r}_{k,t}, \hat{r}_t, \hat{u}_t, \hat{\pi}_{c,t}, \hat{\bar{\pi}}_{c,t}, \hat{\pi}_{k,t}, \hat{\pi}_t, \hat{\pi}_{x,t}, \hat{\pi}_{im,t}, \hat{\pi}_{y,t},\\ &\hat{\pi}_{i,t}, \hat{s}_t, \hat{s}_{g^*,t}, \hat{s}_{g,t}, \hat{s}_{d,t}, \hat{s}_{T,t}, \hat{s}_{TB,t}, \hat{s}_{x,t}, \hat{s}_{im,t}, T\hat{o}T_t, \hat{k}_t, \hat{k}_t^s, \hat{w}_t, \hat{w}_t^\tau, m\hat{r}s_t, \hat{mc}_t, \hat{mc}_t^k, \hat{mc}_t^x, \hat{N}_t, \hat{y}_t, \hat{x}_t,\\ &\hat{q}_t^c, \hat{q}_t^i, \hat{q}_t^g, \hat{h}_t^c, \hat{h}_t^i, \hat{h}_t^g, \hat{h}_t, \hat{im}_t^c, \hat{im}_t^i, \hat{im}_t, \hat{v}_{c,t}, \hat{v}_{i,t}, \hat{\Gamma}_{im^c,t}, \hat{\Gamma}_{im^i,t}, \hat{\Gamma}_{im^*,t}, \hat{\bar{z}}_t, \hat{E}_t, \mathbb{E}_t\left[\hat{\lambda}_{t+1}\right], \mathbb{E}_t\left[\hat{c}_{t+1}\right],\\ &\mathbb{E}_t\left[\hat{Q}_{t+1}\right], \mathbb{E}_t\left[\hat{p}_{i,t+1}\right], \mathbb{E}_t\left[\hat{i}_{t+1}\right], \mathbb{E}_t\left[\hat{r}_{k,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{c,t+1}\right], \mathbb{E}_t\left[\hat{\bar{\pi}}_{c,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{y,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{k,t+1}\right],\\ &\mathbb{E}_t\left[\hat{\pi}_{t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{x,t+1}\right], \mathbb{E}_t\left[\hat{\pi}_{im,t+1}\right], \mathbb{E}_t\left[\hat{w}_{t+1}\right], \mathbb{E}_t\left[\hat{s}_{t+1}\right], \mathbb{E}_t\left[\hat{\epsilon}_{t+1}^c\right], \mathbb{E}_t\left[\hat{g}_{z,t+1}\right], \mathbb{E}_t\left[\hat{E}_{t+1}\right], \hat{mc}_t^*, \hat{b}_t^*,\\ &\hat{\epsilon}_t^c, \hat{g}_{z,t}, \hat{\epsilon}_t^i, \hat{\epsilon}_t^{RP}, \hat{\epsilon}_t^{RP^*}, \hat{\varphi}_t^w, \hat{\epsilon}_t^N, \hat{\varphi}_t^k, \hat{\varphi}_t^x, \hat{\varphi}_t^*, \hat{\epsilon}_t^{im}, \hat{\eta}_t^r, \hat{\eta}_t^{\bar{\pi}}, \hat{\epsilon}_t^x, \hat{\epsilon}_t, \hat{v}_t^*, \mathbb{E}_t\left[\hat{\pi}_{y^*,t+1}\right], \hat{y}_t^*, \hat{r}_t^*, \hat{p}_{o,t}, \hat{p}_{y^*,t},\\ &\hat{g}_t, \hat{y}_{t-1}, \hat{c}_{t-1}, \hat{i}_{t-1}, \hat{x}_{t-1}, \hat{p}_{im,t-1}, \hat{p}_{t-1}^c, \hat{im}_{t-1}, \hat{w}_{t-1}, \hat{y}_{t-1}^*, \hat{p}_{y^*,t-1}, \hat{\epsilon}_t^{y^*}, \hat{\epsilon}_t^{r^*}, \hat{\epsilon}_t^{pOIL}, \hat{\epsilon}_t^{p_y^*}]\end{aligned}$$

defines the 117×1 state vector.

$$\boldsymbol{\varepsilon}_t^T = \left[\varepsilon_t^c, \varepsilon_t^{g_z}, \varepsilon_t^i, \varepsilon_t^{RP}, \varepsilon_t^{RP^*}, \varepsilon_t^{\varphi^w}, \varepsilon_t^N, \varepsilon_t^{\varphi^k}, \varepsilon_t^{\varphi^x}, \varepsilon_t^{\varphi^*}, \varepsilon_t^{im}, \varepsilon_t^r, \varepsilon_t^{\bar{\pi}}, \varepsilon_t^x, \varepsilon_t, \varepsilon_t^{v^*}, \hat{\varepsilon}_t^{y^*}, \hat{\varepsilon}_t^{r^*}, \hat{\varepsilon}_t^{pOIL}, \hat{\varepsilon}_t^{p_y^*}\right]$$

is the 21×1 vector of innovations and

$$\begin{aligned}\boldsymbol{\eta}_t^T = [&\hat{\lambda}_t - \mathbb{E}_{t-1}\left[\hat{\lambda}_t\right], \hat{c}_t - \mathbb{E}_{t-1}\left[\hat{c}_t\right], \hat{Q}_t - \mathbb{E}_{t-1}\left[\hat{Q}_t\right], \hat{p}_{i,t} - \mathbb{E}_{t-1}\left[\hat{p}_{i,t}\right], \hat{i}_t - \mathbb{E}_{t-1}\left[\hat{i}_t\right],\\ &\hat{r}_{k,t} - \mathbb{E}_{t-1}\left[\hat{r}_{k,t}\right], \hat{\pi}_{c,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{c,t}\right], \hat{\bar{\pi}}_{c,t} - \mathbb{E}_{t-1}\left[\hat{\bar{\pi}}_{c,t}\right], \hat{\pi}_{y,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{y,t}\right], \hat{\pi}_{h,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{h,t}\right],\\ &\hat{\bar{\pi}}_t - \mathbb{E}_{t-1}\left[\hat{\bar{\pi}}_t\right], \hat{\pi}_{x,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{x,t}\right], \hat{\pi}_{im,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{im,t}\right], \hat{w}_t - \mathbb{E}_{t-1}\left[\hat{w}_t\right], \hat{s}_t - \mathbb{E}_{t-1}\left[\hat{s}_t\right],\\ &\hat{\epsilon}_t^c - \mathbb{E}_{t-1}\left[\hat{\epsilon}_t^c\right], \hat{g}_{z,t} - \mathbb{E}_{t-1}\left[\hat{g}_{z,t}\right], \hat{E}_t - \mathbb{E}_{t-1}\left[\hat{E}_t\right], \hat{\pi}_{y^*,t} - \mathbb{E}_{t-1}\left[\hat{\pi}_{y^*,t}\right]]\end{aligned}$$

is the 19×1 vector of expectation errors. $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_1$ are 117×117 matrices, and $\mathbf{\Psi}$ and $\mathbf{\Pi}$ are 117×21 and 117×19 matrices relating the vectors of innovations and expectation errors to the dynamics of the state variables.

5.3 Estimation

Estimation of our large-scale stochastic volatility macroeconomic model is very similar to the estimation procedure Justiano and Primiceri [2008] apply. Here too, the solution of the model is done by log-linearizing the DSGE model implied laws of motion of the economic state variables. Referred to Born and Pfeifer [2014] or Basu and Bundick [2017] this procedure is not as accurate as the application of second or third order Taylor series, but the effort related to the model's implementation and computation is much lower than for the alternative procedures. Nevertheless estimation of our large-scale stochastic volatility DSGE model remains computational challenging. Therefore we divide our estimation into two runs - a pre-estimation run where we estimate the constant, non-stochastic volatility version of our DSGE model by applying a Markov Chain Monte Carlo (MCMC) procedure, where we draw the model's parameters with the random walk Metropolis-Hastings algorithm. From this first run we use the posteriors mode to start our second run for estimating and extracting the parameters and volatilities of the stochastic volatility version of our large-scale DSGE model. In more technical terms this means that we use the region around the mode of the constant volatility NAWM in the search space for our estimation of the stochastic volatility NAWM. Estimation and extraction in this second run is based on a Gibbs sampling MCMC procedure alternating between two different state-space models - a first state space model that is conditional on the macroeconomic volatilities $\{\boldsymbol{\sigma}_t\}_{t=1,2,\dots,T}$ from which the model's parameter vector $\boldsymbol{\theta}$ and the state variables $\{\hat{\boldsymbol{s}}_t\}_{t=1,2,\dots,T}$ are drawn. Here we also draw the parameters with the random walk Metropolis-Hastings algorithm, whereas $\{\hat{\boldsymbol{s}}_t\}_{t=1,2,\dots,T}$ is drawn from the Kalman filter as described in Carter and Kohn [1994]. The second state-space model is conditional on $\{\hat{\boldsymbol{s}}_t\}_{t=1,2,\dots,T}$. From this state-space model the time series of macroeconomic volatilities $\{\boldsymbol{\sigma}_t\}_{t=1,2,\dots,T}$ are drawn. Drawing $\{\boldsymbol{\sigma}_t\}_{t=1,2,\dots,T}$ is done by applying the Gibbs particle filter with conditional resampling proposed by Andrieu, Doucet and Holenstein [2010] and the backward drawing algorithm proposed by Whiteley [2010]. We formulate the particle filter in its bootstrap form described in Creal [2012] or Saerkkae [2013].

5.3.1 First DSGE state space model conditional on $\boldsymbol{\sigma}_{t=1,2,\dots,T}$

Our DSGE model's first state-space model conditional on $\{\boldsymbol{\sigma}_t\}_{t=1,2,\dots,T}$ is specified by the measurement equation:

$$\begin{bmatrix} \boldsymbol{y}_t \\ ln\left(\boldsymbol{\sigma}_t\right) \end{bmatrix} = \begin{bmatrix} \boldsymbol{c}_{18\times 1} \\ \mathbf{0}_{21\times 1} \end{bmatrix} + \begin{bmatrix} \mathbf{M}_{18\times 117} & \mathbf{0}_{18\times 21} \\ \mathbf{0}_{21\times 117} & \mathbf{I}_{21\times 21} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{s}}_t \\ ln\left(\boldsymbol{\sigma}_t\right) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\vartheta}_{y,t} \\ \boldsymbol{\vartheta}_{\sigma,t} \end{bmatrix} \tag{5.69}$$

with Gaussian measurement errors:

$$\begin{bmatrix} \boldsymbol{\vartheta}_{y,t} \\ \boldsymbol{\vartheta}_{\sigma,t} \end{bmatrix} \sim N\left(\begin{bmatrix} \mathbf{0}_{18\times 1} \\ \mathbf{0}_{21\times 1} \end{bmatrix}, \begin{bmatrix} \mathbf{\Sigma}_{18\times 18} & \mathbf{0}_{18\times 21} \\ \mathbf{0}_{21\times 1} & \mathbf{0}_{21\times 21} \end{bmatrix} \right) \tag{5.70}$$

where the covariance $\mathbf{\Sigma}$ is diagonal. The 18×1 vector:

$$\begin{aligned} \boldsymbol{y}_t^T = [&\Delta GDP_t^{EMU}, \Delta CONS_t, \Delta INV_t, ln(GOV_t), \Delta EXPORT_t, \Delta IMPORT_t, INF_{Y,t}^{EMU}, \\ &INF_{C,t}, INF_{IM,t}, ln(LABOR_{y,t}), \Delta WAGE_t, ECB_t, FX_t, \Delta GDP_t^{WORLD}, INF_{Y,t}^{WORLD}, \\ &LIBOR_t^{\$}, INF_{EXPORT,t}, PRICE_t^{OIL}] \end{aligned}$$

contains the measurements of (EMU) GDP, consumption, investment, government spending expenditures, exports, imports, GDP-, consumption- and import-deflator based inflation rates, labor (measured in total employment), per head wages, the monetary policy rate set by the ECB (approximated by the one month EONIA swap rate), the effective exchange rate, (world) GDP and (world) GDP-deflator based inflation, the USD-LIBOR, export-deflator based inflation and the oil price (UK-Brent). Data details and details relating the preparation and transformation of the data are outlined in Appendix A.2 and A.3. The constants in $\boldsymbol{c}$ are the state-variables' steady-state values and $\mathbf{M}$ is a specified 18×117 matrix. The rational expectation model expressed in the foregoing section 5.2.3.2 is solved by applying Sim's QZ algorithm, which leads to the form:

$$\hat{\boldsymbol{s}}_t = \boldsymbol{\theta}_c + \boldsymbol{\Theta}_0 \hat{\boldsymbol{s}}_{t-1} + \boldsymbol{\Theta}_1(\boldsymbol{\sigma}_t)\boldsymbol{\varepsilon}_t \tag{5.71}$$

such that we can formulate the system's transition equation for describing the dynamics of the state variables as:

$$\begin{bmatrix} \hat{\boldsymbol{s}}_t \\ ln\,(\boldsymbol{\sigma}_t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_c \\ \boldsymbol{\mu}_\sigma \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Theta}_0 & \mathbf{0}_{117\times 21} \\ \mathbf{0}_{21\times 117} & \mathbf{P}_\sigma \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{s}}_{t-1} \\ ln\,(\boldsymbol{\sigma}_{t-1}) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Theta}_1(\boldsymbol{\sigma}_t) & \mathbf{0}_{117\times 21} \\ \mathbf{0}_{21\times 117} & \boldsymbol{\Sigma}_\sigma \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\varepsilon}_{\sigma,t} \end{bmatrix} \tag{5.72}$$

with $\boldsymbol{\mu}_\sigma, \mathbf{P}_\sigma$ and $\boldsymbol{\Sigma}_\sigma$ from the VAR[1] process of the time varying (log) volatilities in 5.65. The solution routine proposed by Sims [2001] is outlined in more detail in Sim's solution algorithm.

5.3.2 Second DSGE state space model conditional on $\hat{\boldsymbol{s}}_{t=1,2,\ldots,T}$

Our second state-space model conditional on the state variables $\{\hat{\boldsymbol{s}}_t\}_{t=1,2,\ldots,T}$ is specified by 5.65 and 5.71 where 5.71 defines the measurement equation and 5.65 the transition equation.

RMSE													
	ΔGDP	$\Delta CONS$	ΔINV	GOV	ΔEXP	ΔIMP	INV_Y	INF_C	INF_ι	$LABOR$	$\Delta WAGE$	ECB	FX
Const. Vola	0.380	0.150	0.328	1.509	0.023	0.094	0.041	0.186	0.412	0.007	0.962	0.383	0.005
Stoch. Vola	0.317	0.047	0.003	0.023	0.024	0.062	0.065	0.148	0.200	0.019	0.011	0.033	0.007

Table 5.1: RMSE comparison of constant and stochastic volatility NAWM estimations. (The RMSEs are calculated at the mode of the models' posteriors).

Sample Mean (Q1/1987 – Q1/2014)													
	ΔGDP	$\Delta CONS$	ΔINV	GOV	ΔEXP	ΔIMP	INV_Y	INF_C	INF_ι	$LABOR$	$\Delta WAGE$	ECB	FX
Obs.	0.436	0.388	0.397	3.936	1.246	1.220	0.588	0.613	0.322	0.357	0.208	4.865	-0.415
Const. Vola	0.055	0.538	0.725	5.445	1.269	1.127	0.630	0.799	0.734	0.363	1.170	4.482	-0.410
Stoch. Vola	0.118	0.436	0.394	3.913	1.270	1.158	0.653	0.761	0.522	0.337	0.219	4.898	-0.408
Sample Standard Deviation (Q1/1987 – Q1/2014)													
	ΔGDP	$\Delta CONS$	ΔINV	GOV	ΔEXP	ΔIMP	INV_Y	INF_C	INF_ι	$LABOR$	$\Delta WAGE$	ECB	FX
Obs.	0.626	0.502	1.473	1.880	1.951	1.820	0.353	0.392	1.166	2.073	0.334	3.364	26.371
Const. Vola	0.492	0.515	0.914	1.936	1.946	1.731	0.458	0.455	0.540	2.073	0.566	3.144	26.370
Stoch. Vola	0.619	0.466	1.423	1.866	1.942	1.711	0.352	0.569	0.626	2.125	0.349	3.320	26.373

Table 5.2: Observed and constant/ stochastic volatility NAWM implied unconditional first and second moments. (First and second moments of the NAWM are calculated at the mode of the models' posteriors).

5.4 Empirical implications of the EMU macroeconomic framework

5.4.1 In-sample-fit

In Appendix D.3 we outline our used prior distributions and their parametrizations as well as our estimated posterior distributions of our constant and stochastic volatility NAWM estimations. Our stochastic volatility NAWM estimation reveals good in-sample-fitting qualities. In Table 5.1 we list the root mean squared errors (RMSE) of the constant volatility and our stochastic volatility NAWM. In total the RMSEs of our stochastic volatility NAWM are lower than the RMSEs of the constant volatility NAWM. The observed and stochastic volatility NAWM implied macroeconomic state variables are shown in Figure 5.3. In Table 5.2 we additionally compare the unconditional first and second moments of the observed and constant and stochastic volatility NAWM implied macroeconomic variables. Table 5.2 also reveals that in total our stochastic volatility NAWM implied unconditional first and second moments are closer to the observed unconditional first and second moments than moments implied by the constant volatility NAWM.

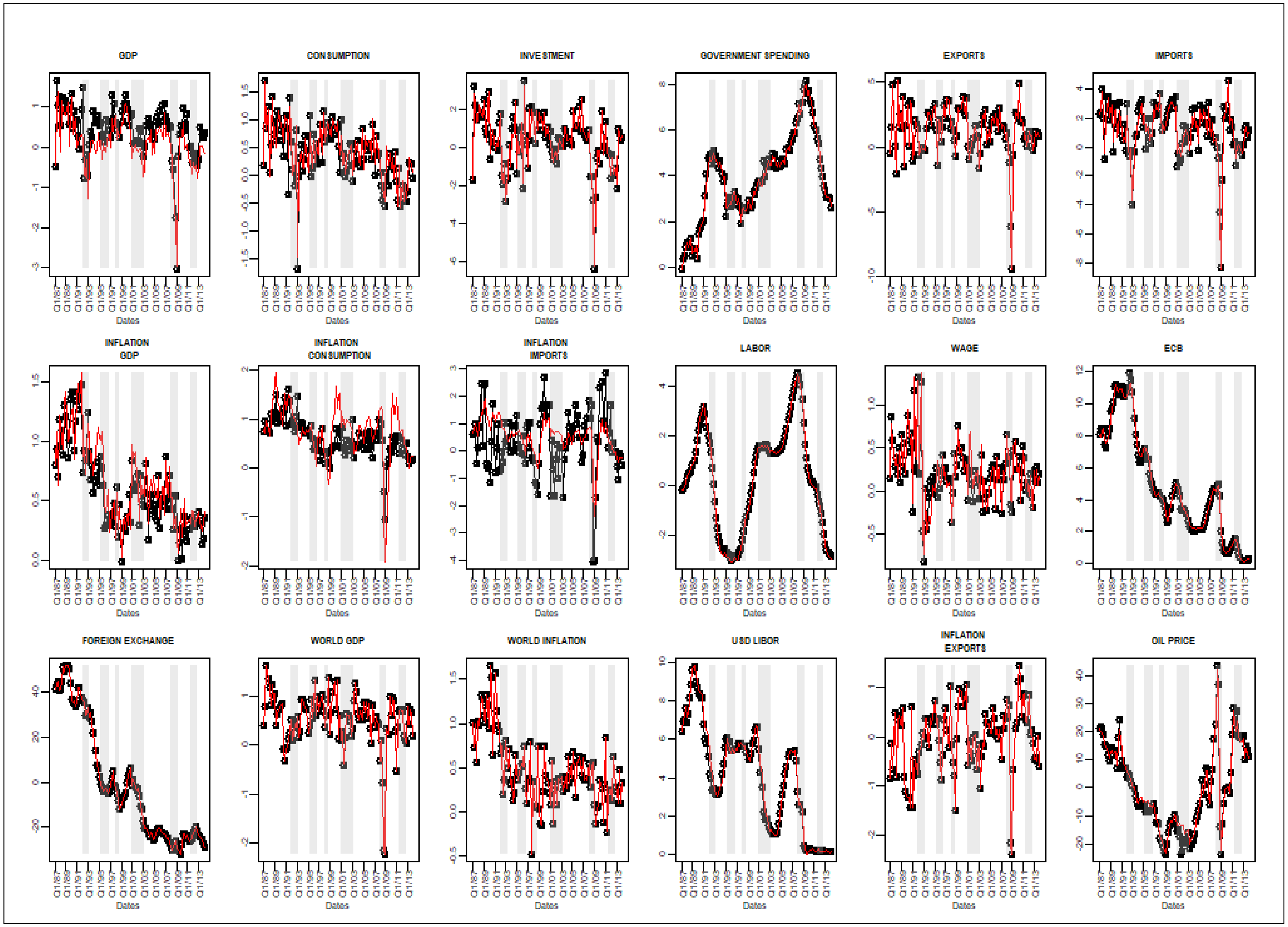


Table 5.3: Observed and stochastic volatility NAWM implied macroeconomic variables between Q1/1987 and Q1/2014 at the posterior's mean.

5.4.2 Macroeconomic uncertainty

In Figure 5.4 we plot the time varying volatilities of the EMU countries' consumption and total employment, world GDP and the oil price. Additionally in Figure 5.5 we plot the time varying volatilities of global and EMU related monetary policy as well as of financial market induced structural shock variables. In Figure 5.4 both EMU's consumption and the world GDP reveal low uncertainty between 1998 and 1999. With the burst of the dot-com bubble starting in the beginning of 2000 reinforced by the terrorist attacks at 09/11 and the following wars lead by the U.S. in Afghanistan in 2001 and in Iraq in 2003 the volatility of EMU's consumption and global income sharply increased. A further interesting aspect is the historically

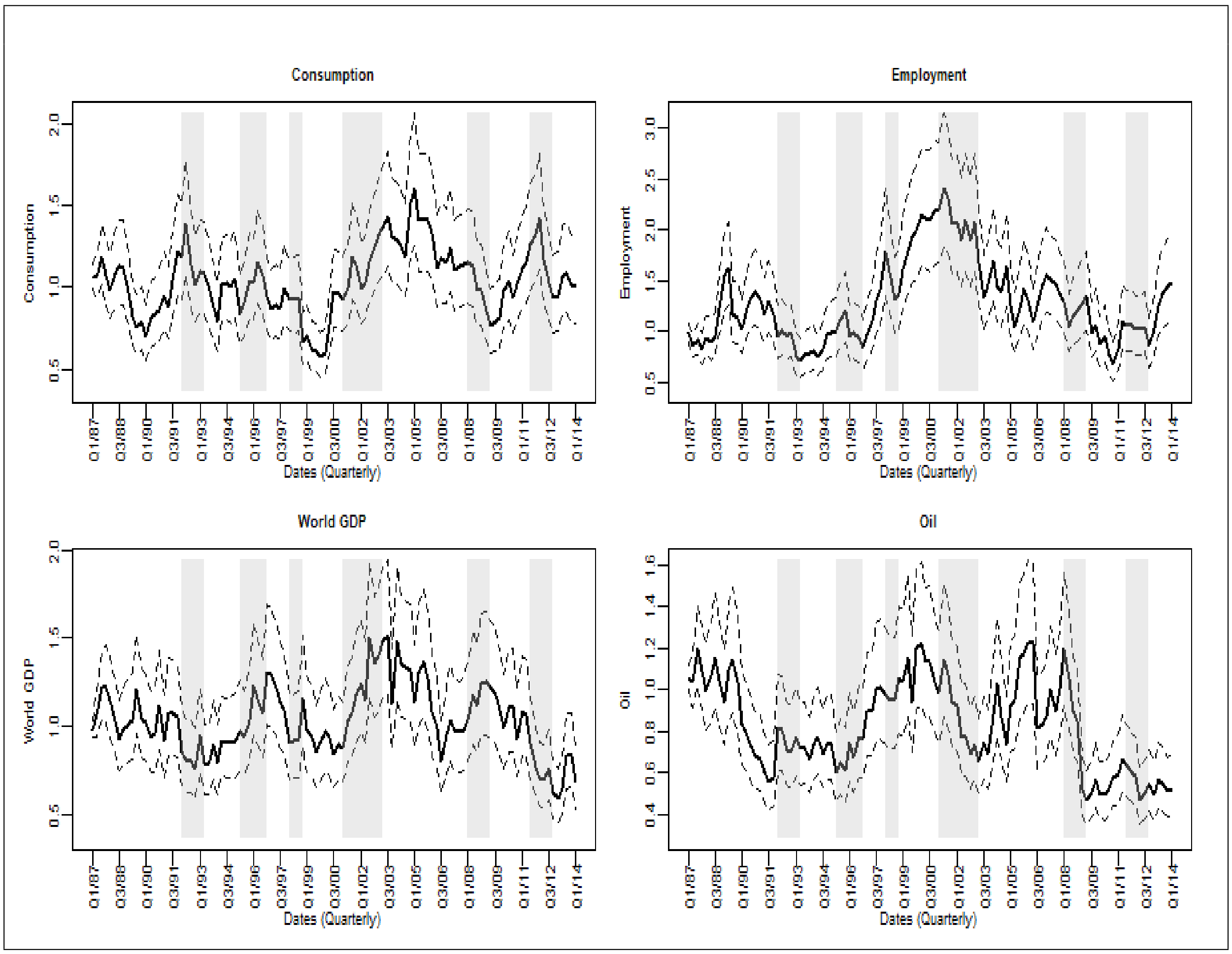


Table 5.4: DSGE model implied stochastic volatilities of consumption, employment, world income as well as oil related structural shock components evaluated at the mode of the models posteriors (dashed lines show the 2σ band).

high uncertainty related to EMU's total employment between 1999 and 2003. At that time especially France and Germany faced low economic growth and high unemployment rates.

Germany at that time called - the sick man of Europe - governed by the Social Democrats and the Green Party under chancellor Gerhard Schroeder reacted to these economic circumstances in introducing the Agenda 2010 - a series of economic and social reforms with the aim to reform the German welfare state and the German labor relations. In Figure 5.5 between Q4/2005 and Q3/2008 the volatility related to EMU's monetary policy sharply increased. This phase of high monetary policy uncertainty is dominated by the overheating of the U.S. housing market and the upcoming of the U.S. subprime crisis starting with the sharp decline in U.S. housing prices in the beginning of 2007 and the reaction of the FED in decreasing its federal funds rate in September 2007 from 5.25% to 4.75%, initiating the FED's interest rates decreasing path reaching its historical minimum of 0.25% in December 2008 lasting until December 2015. With the bankruptcy of Lehman Brothers in September 2008 the ECB decreased its main refinancing operations rate from 4.25% to 1.00% between September 2008 and May 2009 with the result in reducing the monetary policy related uncertainty in the EMU.

Taking into account our overall horizon between Q1/1987 and Q1/2014 uncertainty related to monetary policy in the EMU remains high still after the short term interest rate lowering initialized by the ECB since September 2008. Looking at the volatility of the 1M USD LIBOR uncertainty of this global short term interest rate more or less continuously increased since the second half of 2008. The EMU countries' risk premium uncertainty peaks in the years 1996, 1997 and 1998 - the years of the Asian and Russian economic crisis. With the introduction of the Euro - first as accounting money in 1999 and later in 2002 as day-to-day operating currency - the uncertainty related to the risk premiums of the EMU were historically low until the beginning of 2006. Then EMU's risk premium uncertainty gradually increases until peaking in Q1/2012, with the agreement of the EMU finance ministers on the second rescue program for Greece in February 2012 implying the 50% "haircut" of Greece's government debt already announced in October 2011. There is a phase of a high global risk premium uncertainty between Q2/1989 and Q2/1992 with the fall of the Berlin wall in November 1989, the German reunification in 1990 and the dissolution of the Soviet Union with the Alma-Ata protocol in December 1991. After that the global risk premium uncertainty declines until the beginning of 2006, when the premium's volatility starts again to increase. In section 5.2. we show the estimates of the stochastic volatility DSGE with an integrated term structure of interest rate proposed in the foregoing chapter. This DSGE economy is very similar to the economy proposed by Christiano, Eichenbaum and Evans [2005] and Smets and Wouters [2003, 2007] with disturbing shocks related to monetary policy and the financial risk premium. For both shocks the time varying volatility patterns are similar to the volatility patterns of the EMU's/ ECB's short term interest rate and EMU's risk premiums shown in Figure 5.5. In Figure 5.6 we plot the volatility related to EMU's government spending activities. Figure 5.6 reveals that since the introduction of the Euro as EMU's common accounting currency uncertainty related to the countries' spending activities increases, reaching in the beginning of 2005 a plateau of high uncertainty interrupted by two

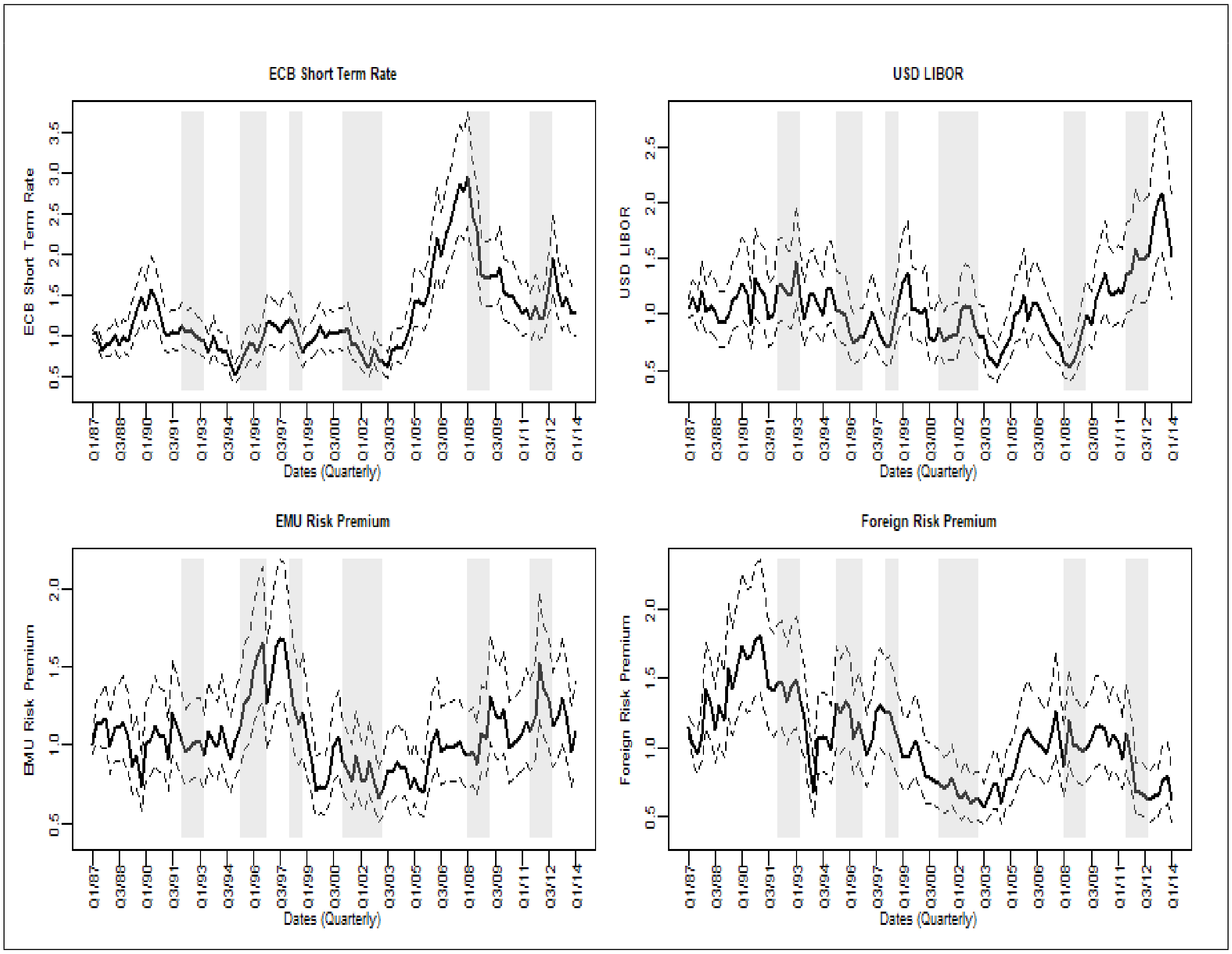


Table 5.5: DSGE model implied stochastic volatilities of EMU countries monetary policy, international interest rate , EMU risk premium as well as international risk premium related structural shock components evaluated at the mode of the models posteriors (dashed lines show the 2σ band).

sharp peaks. The first peak of high uncertainty - historically the highest - is in Q4/2007 after the FED started to decrease its monetary policy rate for stabilizing the U.S. economy. The second sharp peak of uncertainty is in Q4/2008 after the collapse of Lehman Brother and ECB's reaction in decreasing its main refinancing operations rate. In Appendix D.3.3 we show the time varying volatilities for the remaining shock variables of our large-scale DSGE.

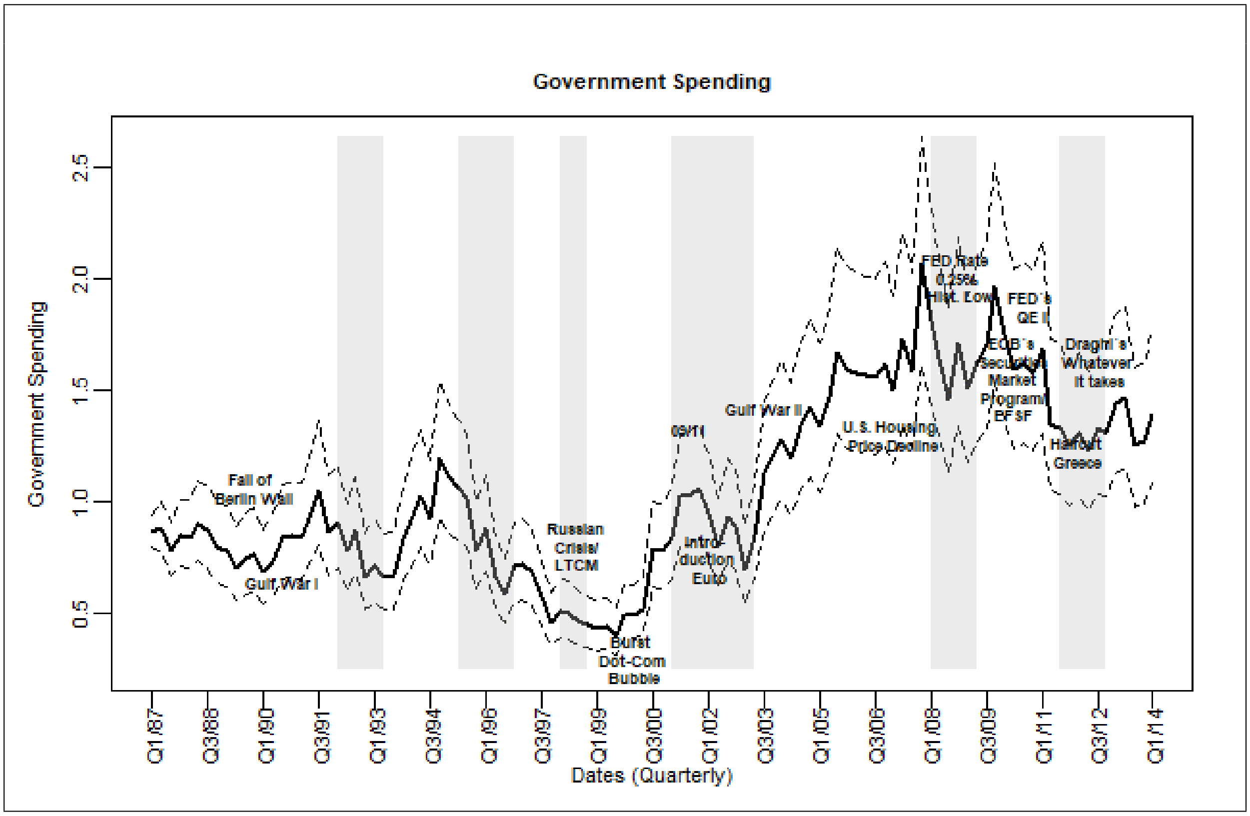


Table 5.6: DSGE model implied stochastic volatility of EMU countries government spending activities related structural shock component evaluated at the mode of the models posteriors (dashed lines show the 2σ band).

5.5 Financial market uncertainties

5.5.1 Stock market uncertainties

Beside the macroeconomic uncertainty we have a look at the uncertainty observed for the financial markets. Therefore we have estimated on a monthly basis between 03/2005 and 02/2014 the time varying volatilities of the stocks listed in the DAX 30, CAC 40, AEX 25, FTSE MIB and in the IBEX 35 for Germany, France, the Netherlands, Italy and Spain respectively. The stock data we used for our volatility estimations are listed in Appendix D.1. For estimating the stock volatilities we have used the generalized autoregressive score (GAS) model proposed by Creal, Koopman and Lucas [2011]. From a theoretical point of view our estimation is based on the intertemporal capital asset pricing model (iCAPM) proposed by Merton [1973] and more recently empirically investigated by Bali and Engle [2010]. We use for our volatility estimations the stock's excess returns:

$$xr_t = (P_t/P_{t-1})\,100 - r_{f,t} \tag{5.73}$$

where P_t and P_{t-1} are the stock's prices at t and $t-1$ and $r_{f,t}$ denotes the risk free rate - in our case the 1M EONIA swap rate. We outline our implementation of the GAS estimation in more detail in Appendix C.3. The GAS estimated volatilities of the excess returns of the stocks for the five EMU countries are shown in Appendix D.4. Because of the high dimensionality of our generated volatility data set we dissolve the implied curse of dimensionality by extracting principal components (PCs) from the countries stock volatilities. Beside the countries stock indices DAX 30, CAC 40, AEX 25, FTSE MIB and in the IBEX 35, the dynamics of the first six PCs are shown in Figure 5.7. For purposes of interpretation in Appendix D.4 we also show for each country the (percentage) contributions of the countries stocks to the six PCs. In absolute terms for all five EMU countries in Figure 5.7 we see large volatility components in the countries first recession phase ranging from 02/2008 to 06/2009. Even earlier since the beginning of 2007 the stock markets of France and the Netherlands show an increase - in absolute terms - in their stock market's volatility components. In the first recession phase Germany and France show the highest volatilities. A second phase of high volatility is at the beginning of the second recession phase of our stock prices sample around October 2011 when the state representatives of the 17 EMU countries announced on October 27th 2011 the 50% "haircut" for Greece. Especially Germany shows a sharp increase - in absolute terms - in temporal proximity to this announcement. The contributions of the countries stocks to their PCs shown in sections D.4.6 to D.4.10 in the Appendix D.4 reveal, that the German insurance company Allianz and Germany's largest bank the Deutsche Bank largely impact the dominant first PC in 09/2011 and 10/2011. Surprisingly the Spanish stock market volatility not directly reacts on the decision of the "haircut" for Greece. The Spanish stock market volatilities sharply increases - in absolute terms - between 05/2012 and 07/2012 when the Spanish government nationalized the Spanish Bankia and the EMU finance ministers agreed about a 100 bn financial support program for stabilizing the Spanish banking sector. From Appendix E it becomes clear that the Spanish Banco Santander has a large impact on the first volatility PC. Later in 09/2012 Banco Sabadell also has a large (30%) impact on the first PC.

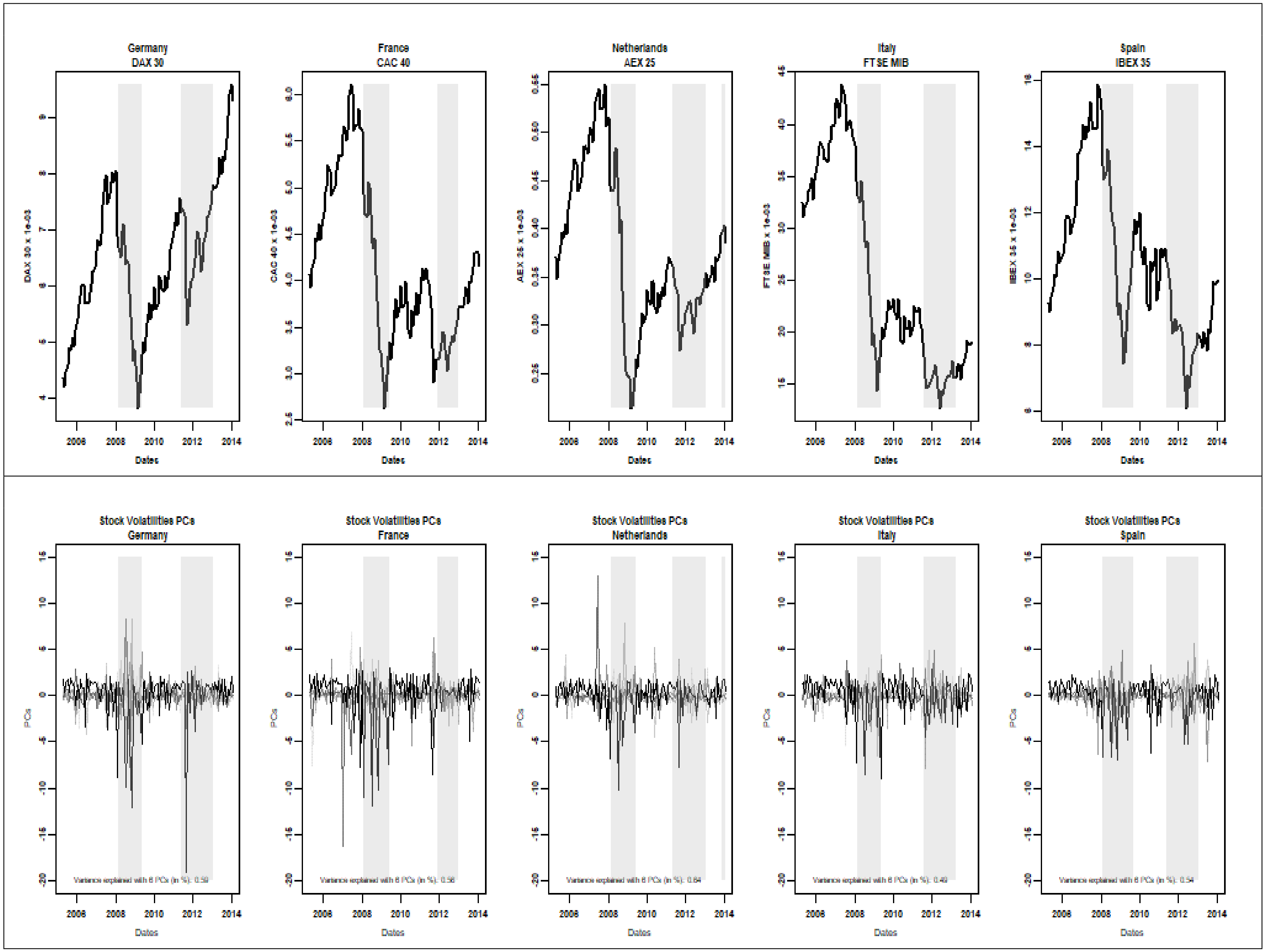


Table 5.7: EMU's stock market indices DAX 30, CAC 40, AEX 25, FTSE MIB and IBEX 35 and the 6 principal components of the time varying GAS volatilities estimated for the countries companies listed in the respective sock market indices between 03/2005 and 02/2014.

5.5.2 Bond market uncertainties

Focusing on the upcoming sovereign debt crisis in the EMU since 2010 in this chapter we use the insights of the medium- to large-scale stochastic volatility New-Keynesian DSGE model combined with an arbitrage-free macro-finance affine term structure of interest rates model with unspanning stochastic volatility factors (USV-ATSM) introduced in the foregoing chapter for discussing the uncertainty patterns of the term structure of interest rates for Germany and Italy - our two representative EMU countries in this section. In the following we label this combined model as DSGE-USV-ATSM. As described in in the foregoing chapter in more detail the DSGE-USV-ATSM is estimated by applying an MCMC procedure, where the procedure alternates between two large modeling blocks, where in each of these blocks the Bayesian estimation routine alternates again between two larger state-space models. In each of these two model blocks we appy the Gibbs particle filter with conditional resampling by Andrieu, Doucet and Holenstein [2010] and backward drawing proposed by Whiteley [2010] for extracting the volatility patterns of the DSGE's as well as of the term structure of interest rates state variables. Because of the complexity of this combined model we have implemented and estimated a larger number of alternative constant and time-varying volatility term structure models for checking the quality and robustness of the combined DSGE-USV-ATSM. For the model estimations we use zero-coupon rates between Q1/2005 and Q1/2014. For extracting the zero-coupon rates from the government bond prices of Germany and Italy we use the parametric Nelson-Siegel-Svensson (NSS) approach proposed by Nelson and Siegel [1987] and Svensson [1995]. Bond data and the NSS approach are listed and outlined in Appendix A.2 and A.3. In Figure 5.8 we show the in-sample-fitting performance of 13 implemented and estimated alternative term structure models. The range of term structure models includes the Vasicek-model proposed by Vasicek [1977], the dynamical Nelson-Siegel (DNS) models in their independent and correlated form as well as the more recent arbitrage-free formulation of the DNS (AFDNS) proposed Christensen, Diebold and Rudebusch [2011]. We further implemented the macro-finance MF-DNS introduced by Diebold, Rudebusch and Aruoba [2006] in extending the latent factor approach of the DNS models by implying observed macroeconomic factors. With the USV-Latent-DNS and USV-MF-DNS models we further extend the latent DNS and MF-DNS models by endogenously regarding an (unspanned) stochastic volatility structure for the model's interest rate factors. Further we implemented the Latent-ATSM and the MF-ATSM with pure (latent) term structure factors and additional macroeconomic factors proposed by Ang and Piazzesi [2003]. With respect to the constant volatility MF-ATSM we implemented the USV-MF-ATSM proposed by Creal and Wu [2017]. With the constant volatility BCM-DSGE proposed by Beakert, Cho and Moreno [2010] we further regard a term structure model that combines a small-scale DSGE model combined with an ATSM. As outlined in the previous chapter our DSGE-USV-ATSM shows a good in-sample fitting comparable to the latent-ATSM which has a very good in-sample fitting quality over the whole maturity spectrum. For Italy our DSGE-USV-ATSM is comparable to the fitting quality of the DNS term structure models.

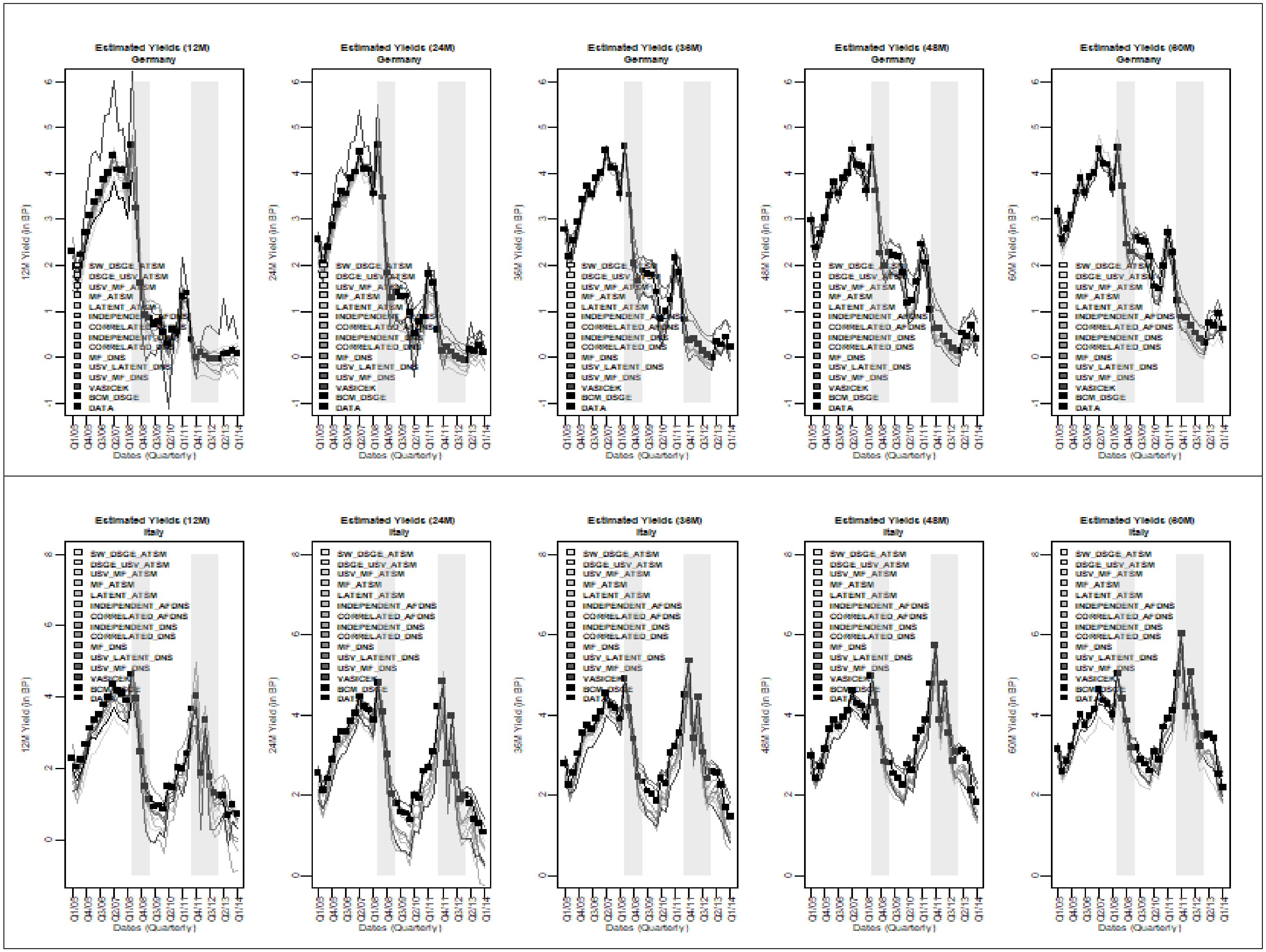


Table 5.8: Comparison of the in-sample-fit for the maturities 12,24,48 and 60 month of 14 term structure of interest rate model implementations for Germany and Italy over the time horizon Q1/2005 and Q1/2014. Ten of these term structure models are constant volatility models and four models are (unspanned) stochastic volatility models.

Figure 5.9 shows the interest rate volatilities for Germany and Italy with respect to the six maturities 6,12, 24,36, 48 and 60 month estimated in our DSGE-USV-ATSM and the GAS model by Creal, Koopman and Lucas [2011] (see previous section 5.5.1 and Appendix C.3). For the GAS we adjusted the interest rates by their conditional mean estimated by a conventional VAR[1]. For our DSGE-USV-ATSM Figure 5.9 reveals a volatility peak for all six maturities in the second half of the first recession phase between Q2/2008 and Q2/2009 after the collapse of Lehman Brothers.

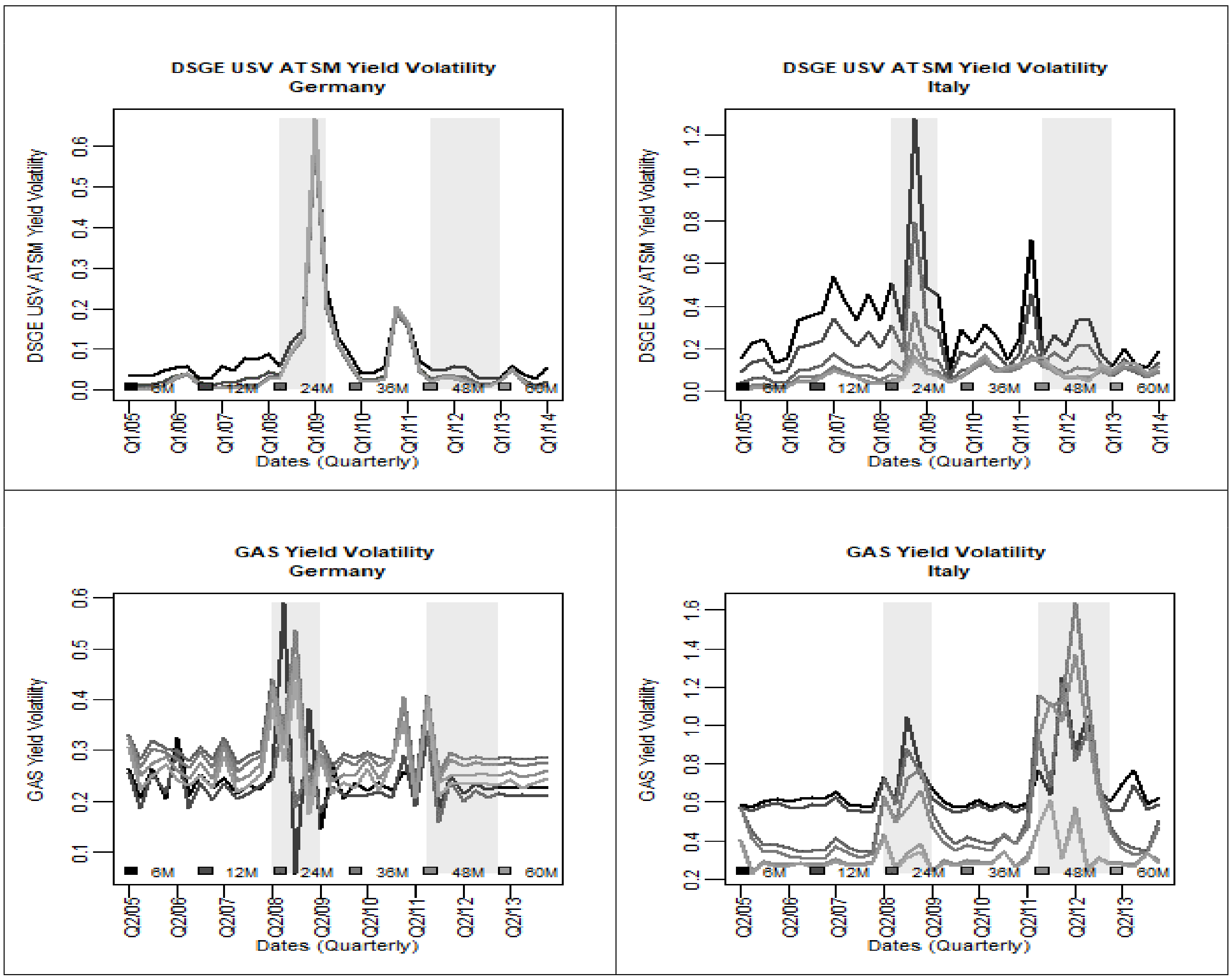


Table 5.9: Stochastic volatilities for the maturities 6,12, 24,36, 48 and 60 month implied by the DSGE-USV-ATSM evaluated at the mode of the models posterior and the restricted Student's t GAS[1,1] model for Germany and Italy between Q1/2005 and Q1/2014.

The interest rate volatilities estimated by the GAS model show a similar pattern, with the difference that the maturities 24 to 60 month show a first peak at the beginning of the recession phase. A further difference between DSGE-USV-ATSM and GAS model is the deviating pattern of the GAS interest rate volatilities for the maturities 12 and 24 month. Here the DSGE-USV-ATSM interest rate volatilities show a more homogeneous pattern. To become an understanding about the structure behind the interest rates volatility patterns shown in Figure 5.9 in Figure 5.10 we show the empirical term structure of interest rates uncertainty shock $\hat{\epsilon}_{h,t}$ disturbing the latent volatility factor that drives the volatilities of the interest rates over the whole maturity spectrum. Obviously for both EMU countries in the first recession phase between Q2/2008 and Q2/2009 we measure a high uncertainty - represented by $\hat{\epsilon}_{h,t}$ - affecting the term structure of interest rate volatilities. For Germany Figure 5.10 further

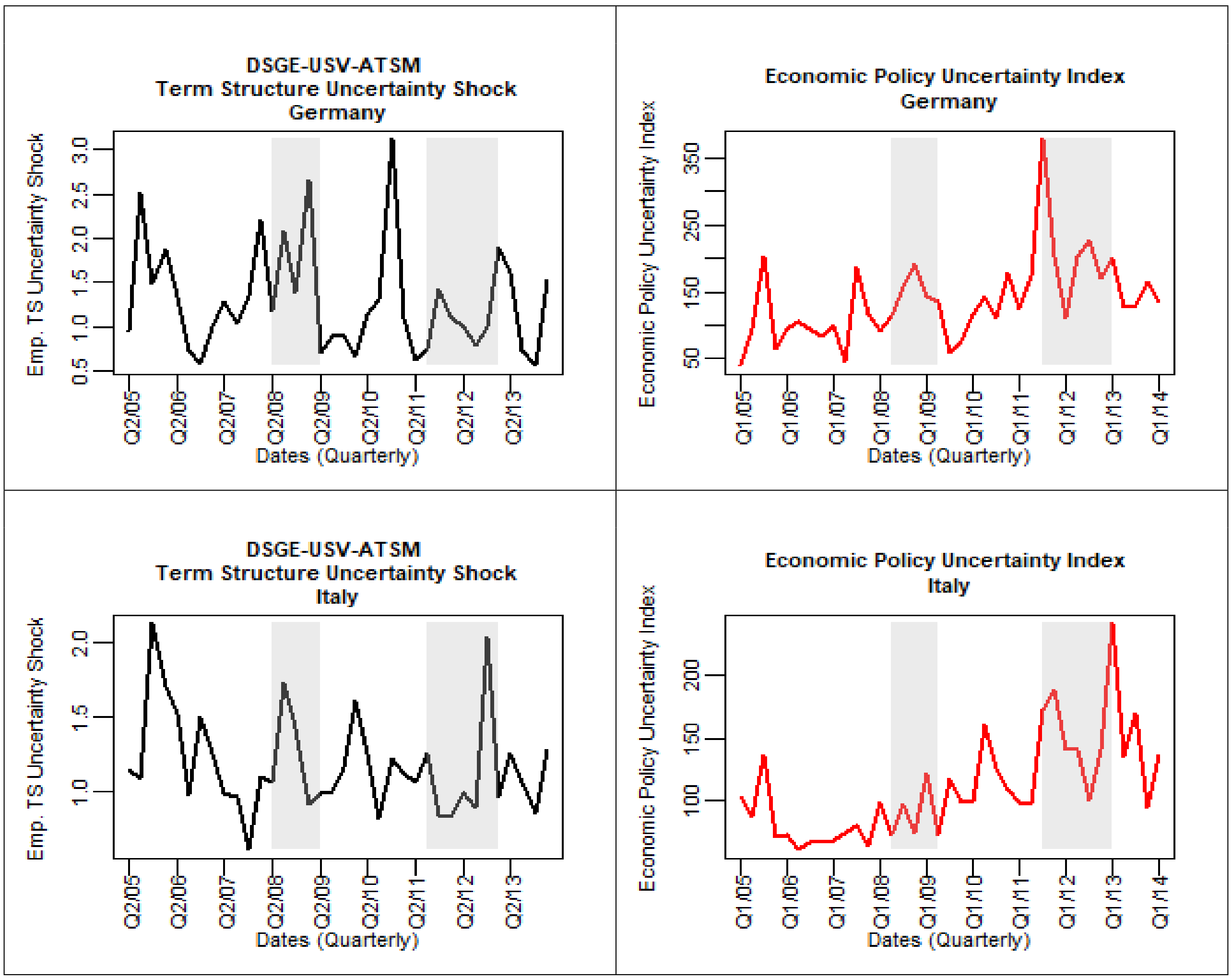


Table 5.10: Empirical term structure of interest rates uncertainty shock $\hat{\varepsilon}_{h,t}$ implied by the DSGE-USV-ATSM evaluated at the posteriors mode and the Economic Policy Uncertainty Index between Q1/2005 and Q1/2014 for Germany and Italy.

points out that there is a second phase of high uncertainty shocks with its peak in Q4/2010, in which the term structure of interest rates starts to increase after a two years lasting period of decreasing interest rates and a few month before the ECB's official decision to increase its short term monetary policy rate for the EMU in April 2011. Italy shows a sharp uncertainty shock in Q4/2012 with the announcement of the resigning of Mario Monti, who headed a non-elected cabinet of economic experts since 2011 that launches the Italian austerity reform program and the dissolution of the Italian parliament that followed the resignation. In Figure 8 we additionally plot the Economic Policy Uncertainty (EPU) index proposed by Baker, Bloom and Davis [2016] measured for Germany and Italy. For Germany the EPU index indicates a phase of higher economic policy uncertainty in the first recession phase between Q2/2008 and Q2/2009 and the three quarters before this phase. The peak of the German EPU index at the beginning of the second recession phase in Q3/2011 reflects the political discussions related to the 50% writedown of the value of Greece's government debt held by private investors announced in October 2011 with strong effects also to the German stock market and its volatilities as shown in 5.5.1. The uncertainty shock pattern revealed for the German term structure of interest rates developments shows an increased uncertainty one to two periods before the German EPU index peaks. For Italy the interest rates uncertainty shock $\hat{\epsilon}_{h,t}$ peaks at the end of 2012 whereas the Italian EPU index peaks at the beginning of 2013 with the Italian election in February 2013.

5.6 Conclusion

In this chapter we have implemented and estimated a large-scale second generation New-Keynesian open economy DSGE model with stochastic volatilities for modeling the economy and the economic uncertainties of the EMU as a whole. Estimation of our stochastic volatility area wide DSGE model has similarities to the estimation procedure applied by Justiano and Primiceri [2008]. We have applied an MCMC Gibbs sampling procedure where our procedure alternates between two large modeling blocks described by two state-space models. Because of the non-linear character of our open economy DSGE we have applied a forward Gibbs particle filter with conditional resampling and backward drawing for extracting the volatility patterns induced by the 21 sources of economic uncertainty. We find that between Q4/2005 and Q3/2008 the uncertainty related to EMU's monetary policy sharply increased. We further find that the ECB's short term interest rates decrease initialized after the bankruptcy of Lehman Brothers in September 2008 reduced the monetary policy related uncertainty in the EMU. Nevertheless we find that monetary policy uncertainty in the EMU still remains historically high. With respect to the Euro and its introduction we find that until 2006 the introduction of the Euro lead at first to historically low uncertainties related to financial risk premiums demanded by investors in the EMU. In 2006 risk premium uncertainties in the EMU began gradually to increase until peaking in Q1/2012, with the agreement of the EMU's finance ministers on the second rescue program for Greece in February 2012 implying the 50% "haircut" of Greece's government debt. Focusing on the EMU's major stock markets we used the theoretical background of the intertemporal capital asset pricing model (iCAPM) proposed by Merton [1973] for analyzing the excess returns' volatilities of stock prices from more than 100 companies listed in the DAX 30, CAC 40, AEX 25, FTSE MIB and in the IBEX 35 where we have applied for estimating the time-varying covariance matrices of the iCAPM the generalized autoregressive score (GAS) model recently proposed by Creal, Koopman and Lucas [2011]. With the GAS implied multivariate Student's t-distribution we regard more extreme financial risks lying in the fat tails of the multivariate t-distribution in our analysis. Here too we find that uncertainty is especially high around the announcement of the 50% "haircut" for Greece by the state representatives of the 17 EMU countries. We further find that Germany shows the largest reaction to this announcement with a sharp increase in its stock markets volatility dominated by an increase of the volatilities of stock price movements of German insurance and banking companies. Taking into account the upcoming of the EMU's sovereign debt crisis since 2010 in this paper we further focused on the term structure of interest rates dynamics and its implied uncertainties in the market of German and Italian government bonds. Here we used the results from the estimation of the medium- to large-scale stochastic volatility New-Keynesian DSGE model combined with an arbitrage-free macro-finance affine term structure of interest rates model with unspanning stochastic volatility factors introduced in the previous chapter. From the estimations for Germany and Italy it follows that the volatilities of the term structure of interest rates sharply increased after the bankruptcy of Lehman Brothers and ECB's initialized interest rates decreasing reaction. The German term structure also reveals higher uncertainty in the phase around Q4/2010 in which the ECB after two years of decreasing interest rates tried to increase again its short term interest rates. For Italy we further find that the resignation of Mario Monti in Q4/2012 who implemented the Italian austerity policy lead to a larger uncertainty shock.

6. Comparison of Yield Curve Forecasting Models

6.1 Introduction

In this chapter we compare the predictive abilities of various term structure of interest rate models. With Germany, France, the Netherlands, Italy, Spain and Portugal, the comparison includes six major countries of the European Monetary Union (EMU). Our data samples with which our analysis are done range from 03/2005 to 02/2014 - a very challenging but nevertheless realized period where the Euro and the EMU institutions become more settled and before ECB initializes its unconventional expanded asset purchase program (EAPP) in the end 2014 and the public sector purchase program (PSPP) in the beginning 2015 - but with critical events such as the upcoming of the international financial crisis with the default of Lehman Brother's in September 2008, the sharp 275 basis points decrease of ECB's controlled short term main refinancing operations rate from 4.25% in September 2008 to 1.50% in March 2009 and the European sovereign debt crisis since 2010 with Mario Draghi's "Whatever it takes" in July 2012.

The term structure models used for our analysis are mainly selected from three classes of term structure models. The first class is defined by models of the dynamic Nelson-Siegel (DNS) type. These models originally based on the parametric approach proposed by Nelson and Siegel [1987]. The parsimonious and static Nelson-Siegel approach aims to provide a good in-sample-fit for bond data observed at a single time t . The dynamic extension of the static approach was methodologically introduced for interest rate predictions by Diebold and Li [2006]. For our out-of-sample experiments done in this chapter we use two specifications of the DNS by Diebold and Li [2006]. Both specifications decompose the yields into a set of latent factors. To combine the latent factors with observed macroeconomic factors we further use the macro-finance DNS developed by Diebold, Rudebusch and Aruoba [2006]. Substituted under the class of DNS models are also the arbitrage-free DNS (AF-DNS) proposed by Christensen, Diebold and Rudebusch [2011]. Christensen, et. al. [2011] reformulate the DNS by introducing a no-arbitrage term in the term structure equations of the DNS. Christensen et. al. [2011] further show that their arbitrage-free formulation of the DNS provides superior forecasts compared to the original proposed DNS.

The second class of term structure models consists of the so called short rate models. Here we specify a three factor version of the Vasicek-short rate model, proposed by Vasicek [1977]. The model assumes a mean-reverting Ornstein-Uhlenbeck transition process for describing the model's factor dynamics. Under the no-arbitrage condition the Vasicek- short rate model implies a closed form analytic expression of the term structure of interest rates in economic equilibrium.

The third class of term structure models implies the arbitrage-free affine term structure models (ATSM) originally proposed by Duffie and Kan [1996]. Here the lines between the three classes become fuzzy because of the arbitrage-free and affine characteristics of term structure models from the other model classes.

In their seminal paper Ang and Piazzesi [2003] formulate two types of arbitrage-free ATSM - the latent factor yields-only ATSM and the macro-finance ATSM with unobservable latent and observable macroeconomic factors. Both types use a Taylor rule like short rate equation for combining the term structure with their respective observable and unobservable factors. For our forecasting purposes we use both model types. With respect to stochastic volatilities we also apply the ATSM with (spanned) stochastic volatilities proposed by Creal and Wu [2015]. Here beside the Gaussian latent factors determining the term structure of interest rates Creal and Wu [2015] introduce a non-Gaussian heteroscedasticity factor which drives the interest rates' volatilities.

We find that all except two of our term structure model implementations show good in-sample fitting qualities over the whole horizon ranging between 03/2005 to 02/2014. With respect to our out-of-sample forecasting experiments we find that in its forecasting performance the non-parametric random walk is hard to beat. Comparatively good are also the forecasting qualities of the two parameter slope-regression - an econometric forecasting model alternative to the more complex term structure of interest rate models. The term structure model with the best forecasting performance is the independent AF-DNS. Our forecasting experiments also reveal that an increase in the complexity of the term structure of interest rates models lead in tendency to a poorer forecasting performance.

This chapter is organized as follows: Section 6.2 introduces in more detail the term structure of interest rates models of the three classes that we use for our term structure forecasting experiments. In section 6.3 we present and discuss in detail our empirical findings. Forecasting experiments are done by applying a recursive two step procedure with the rolling 60-month-sample estimations of the various models in the first step and the forward out-of-sample model iterations for getting the model specific interest rate forecasts and forecasting errors in the second step. Section 6.4 discusses our findings with respect to Ockham's razor. The chapter closes with our conclusions in section 6.5.

6.2 Models

6.2.1 Class of dynamic Nelson-Siegel models

Nelson and Siegel [1987] have shown that a broad range of various forms of the term structure of interest rates can be remarkable well fitted at a particular point t in time by a linear combination of three parsimoniously parametrized smooth functions:

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) + \varepsilon_\tau \tag{6.1}$$

where $y(\tau)$ is the zero-coupon yield with $\tau \geq 0$ month to maturity and β_1, β_2 and β_3 are similar to Litterman and Scheinkman [1991] interpreted as the yield curve's level, slope and curvature factors. T is the bond's expiration date. The parameter λ determines the exponential decay of the β_2 and β_3 factor loadings. The parsimonious character of the Nelson-Siegel model in 6.1 makes the model's application straightforward, so that the model is popular in practice for both financial practitioners and central banks as outlined in Svensson [1995], Bank of International Settlement [2005], Gurkaynak Sack and Wright [2007] and Nyholm and Rebonato [2008].

6.2.1.1 Baseline dynamic Nelson-Siegel model

Diebold and Li [2006] reformulate the static Nelson-Siegel (NS) approach in 6.1 in a dynamic model with time-varying yield factors. The dynamic Nelson-Siegel model (DNS) is defined as:

$$y_t(\tau) = X_{1,t} + X_{2,t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + X_{3,t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) + \varepsilon_{\tau,t} \tag{6.2}$$

Now the NS approach in 6.1 becomes a dynamic factor model with time-varying level, slope and curvature factors $X_{1,t}, X_{2,t}$ and $X_{3,t}$. In state-space form the DNS can be written as:

$$\boldsymbol{y}_t = \boldsymbol{c} + \mathbf{B}\mathbf{X}_t + \boldsymbol{\varepsilon}_t \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}) \quad t = 1, 2, ..., T \tag{6.3}$$

$$\mathbf{X}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{X}_{t-1} + \boldsymbol{\eta}_t \quad \boldsymbol{\eta}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}) \quad t = 1, 2, ..., T \tag{6.4}$$

where $\boldsymbol{y}_t^T = [y_t(\tau_1), y_t(\tau_2), ..., y_t(\tau_N)]$ contains the N zero-coupon yields with maturities $\tau_1, \tau_2, ..., \tau_N$ observed at time $t = 1, 2, ..., T$, $\mathbf{X}_t^T = [X_{1,t}, X_{2,t}, X_{3,t}]$ is the vector of the three yield factors at time t. $\boldsymbol{c}$ and $\boldsymbol{\mu}$ are $N \times 1$ and 3×1 vectors of constants respectively. $\mathbf{B}$ is a $N \times 3$ matrix of factor loadings and $\mathbf{A}$ is the 3×3 transition matrix of the factors $\mathbf{X}_t$. $\boldsymbol{\Sigma}$ is the $N \times N$ diagonal covariance matrix of the N measurement errors $\boldsymbol{\varepsilon}_t$ affecting the zero-coupon yield measurements. $\boldsymbol{\Omega}$ is the 3×3 covariance matrix of the 3×1 factor disturbance vector $\boldsymbol{\eta}_t$, which are independent of the residuals $\boldsymbol{\varepsilon}_t$ $\forall t$.

Analogue to Diebold and Li [2006] we specify the DNS in two ways. In the *independent-factor*

DNS model, the three factors $\mathbf{X}_t^T = [X_{1,t}, X_{2,t}, X_{3,t}]$ follow independent AR[1] processes, such that the transition of the *independent-factor DNS model* is specified as:

$$\mathbf{X}_t = (\mathbf{I} - \mathbf{A})\,\boldsymbol{\mu} + \mathbf{A}\mathbf{X}_{t-1} + \boldsymbol{\eta}_t \quad \boldsymbol{\eta}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}) \tag{6.5}$$

where the transition matrix $\mathbf{A}$ and the covariance matrix $\boldsymbol{\Omega}$ are diagonal matrices.

In the *correlated-factor DNS model* the three yield factors follow 6.5 with a full parametrized transition matrix $\mathbf{A}$ and the covariance matrix $\boldsymbol{\Omega} = \tilde{\boldsymbol{\Omega}}\tilde{\boldsymbol{\Omega}}^T$, where $\tilde{\boldsymbol{\Omega}}$ is lower triangular:

$$\tilde{\boldsymbol{\Omega}} = \begin{bmatrix} \tilde{\omega}_{1,1} & 0 & 0 \\ \tilde{\omega}_{2,1} & \tilde{\omega}_{2,2} & 0 \\ \tilde{\omega}_{3,1} & \tilde{\omega}_{3,2} & \tilde{\omega}_{3,3} \end{bmatrix} \tag{6.6}$$

The measurement equation is identical for both the independent- and the correlated-factor DNS with $\boldsymbol{c} = 0$ and

$$\mathbf{B} = \begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{bmatrix} \tag{6.7}$$

Estimation of the two specifications of the DNS is done by applying maximum likelihood estimation (MLE) based on the Kalman filter.

6.2.1.2 Macro finance dynamic Nelson-Siegel model

Following Diebold, Rudebusch and Aruoba [2006] we extend the DNS outlined in the previous section by regarding the macroeconomic variables: Annual growth rate of industrial production IP_t as the real economy component, annual price inflation π_t determined by the published CPI's as the economy's price component and ECB's main refinancing operations rate r_t for reflecting monetary policy activities. Industrial production and CPI data are both queried from the OECD economic database. Extending 6.5 with respect to IP_t, π_t and r_t leads to the state variables' transition analogue to the baseline DNS in 6.5:

$$\mathbf{X}_t = (\mathbf{I} - \mathbf{A})\,\boldsymbol{\mu} + \mathbf{A}\mathbf{X}_{t-1} + \boldsymbol{\eta}_t \quad \boldsymbol{\eta}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}) \tag{6.8}$$

with factors $\mathbf{X}_t^T = [X_{1,t}, X_{2,t}, X_{3,t}, IP_t, \pi_t, r_t]$, where beside the yield factors $X_{1,t}, X_{2,t}$ and $X_{3,t}$ the factors now also include the macroeconomic factors IP_t, π_t and r_t respectively. $\boldsymbol{\mu}, \mathbf{A}$ and $\boldsymbol{\Omega} = \tilde{\boldsymbol{\Omega}}\tilde{\boldsymbol{\Omega}}^T$ now becoming vectors and matrices of order 6×1 and 6×6 respectively. We specify the macro-finance DNS (MF-DNS) in its correlated version, such that all 36 elements of $\mathbf{A}$ are to be estimated. As for the DNS we specify $\tilde{\boldsymbol{\Omega}}$ as lower triangular. The measurement equation is identical to 6.3. Estimation is done by MLE based on the Kalman filter's likelihood.

6.2.1.3 Arbitrage free dynamic Nelson-Siegel models

Filipovic [1999] outlines that it is impossible to prevent arbitrage in bond-pricing by applying the Nelson-Siegel model. To overcome this theoretical weakness Christensen, Diebold and Rudebusch [2011] have developed the arbitrage-free dynamic Nelson-Siegel (AF-DNS) framework. According to Duffie and Kan [1996] they assume that the yield factors X_t under the risk-neutral measure Q are described in their time-continuous development by the mean-reverting stochastic differential equation (SDE):

$$d\mathbf{X}_t = \mathbf{K}^Q(t)\left(\boldsymbol{\theta}^Q(t) - \mathbf{X}_t\right)dt + \boldsymbol{\Omega}(t)\mathbf{D}(\mathbf{X}_t, t)d\mathbf{W}_t^Q \tag{6.9}$$

where $\mathbf{W}_t^Q$ is a K dimensional standard Brownian motion. K is the number of factors. $\mathbf{K}^Q(t)$ is a $K \times K$ matrix indicating the impact of the idiosyncratic increments of the Brownian Motion $d\mathbf{W}_t^Q$ on $d\mathbf{X}_t$ and $\mathbf{D}(\mathbf{X}_t, t)$ is a $K \times K$ diagonal matrix adjusting the standard deviations $diag\left(\boldsymbol{\Omega}(t)\right)$ of $\boldsymbol{\Omega}(t)$ by:

$$d_{i,i} = \sqrt{\left(\gamma_i(t) + \delta_{i,1}(t)X_{1,t} + \gamma_i(t) + \delta_{i,2}(t)X_{2,t} + ... + \gamma_i(t) + \delta_{i,K}(t)X_{K,t}\right)} \tag{6.10}$$

According to Duffie and Kan [1996] the risk-free short rate is assumed to follow an affine function of the state variables $\mathbf{X}_t$:

$$r_t = \rho_0(t) + \boldsymbol{\rho}_1(t)^T\mathbf{X}_t \tag{6.11}$$

where $\rho_0(t)$ is a scalar and $\boldsymbol{\rho}_1(t)$ is a $K \times 1$ vector. Under these conditions Duffie and Kan [1996] proved that the zero-coupon yields have the closed-form analytic expression:

$$y(t,\tau) = -\frac{1}{\tau}ln\left(\mathbb{E}_t^Q\left[exp\left(-\int_t^T r_s ds\right)\right]\right) = -\frac{1}{\tau}\left(\Gamma(t,T) + \mathbf{B}(t,T)^T\mathbf{X}_t\right) \tag{6.12}$$

T is the expiration date. Time to maturity is $\tau = T - t$. The scalar $\Gamma(t,T)$ and the $K \times 1$ vector of bond loadings $\mathbf{B}(t,T)$ satisfy the system of ordinary differential equations (ODE):

$$\frac{\partial\Gamma(t,T)}{\partial dt} = \rho_0 - \mathbf{B}(t,T)^T\left(\mathbf{K}^Q\right)^T\boldsymbol{\theta}^Q - \frac{1}{2}\sum_{i=1}^{K}\left[\boldsymbol{\Omega}^T\mathbf{B}(t,T)\mathbf{B}(t,T)^T\boldsymbol{\Omega}\right]_{i,i}\gamma_i \tag{6.13}$$

$$\frac{\partial\mathbf{B}(t,T)}{\partial dt} = \boldsymbol{\rho}_1 + \left(\mathbf{K}^Q\right)^T\mathbf{B}(t,T) - \frac{1}{2}\sum_{i=1}^{K}\left[\boldsymbol{\Omega}^T\mathbf{B}(t,T)\mathbf{B}(t,T)^T\boldsymbol{\Omega}\right]_{i,i}\boldsymbol{\delta}_i^T \tag{6.14}$$

with the system's boundary conditions $\Gamma(T,T) = 0$ and $\mathbf{B}(T,T) = \mathbf{0}$. $[]_{i,i}$ denotes the i diagonal matrix of the matrix in brackets. To get the same factor loadings as in the DNS with $K = 3$ Christensen, Diebold and Rudebusch [2011] restrict the vector of bond loadings $\mathbf{B}(t,T)$ in the ODE 6.13 with:

$$B_1(t,T) = -(T - t) \tag{6.15}$$

$$B_2(t,T) = -\frac{1-e^{-\lambda\tau}}{\lambda} \tag{6.16}$$

$$B_3(t,T) = -\frac{1-e^{\lambda\tau}}{\lambda} + \tau e^{-\lambda\tau} \tag{6.17}$$

such that the AF-DNS yield model is:

$$y(t,T) = X_{1,t} + X_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + X_{3,t}\left(\frac{1-e^{\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) - \frac{\Gamma(t,T)}{\tau} \tag{6.18}$$

where the first three terms are similar to the DNS. The fourth term of the AF-DNS is the yield adjustment term to ensure the model is arbitrage-free. Following Singleton [2006] Christensen, Diebold and Rudebusch [2011] propose the following identifying restrictions holding under the risk-neutral measure Q :

$\boldsymbol{\theta}^Q = \mathbf{0}$, $\boldsymbol{\Omega}$ is lower triangular, $\rho_0, \boldsymbol{\rho}_1^T = [1,1,0], \boldsymbol{\delta} = \mathbf{0}, \boldsymbol{\gamma} = \boldsymbol{e}$, where $\boldsymbol{e}$ is the $K \times 1$ unit vector and

$$\mathbf{K}^Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{bmatrix} \tag{6.19}$$

Under these restrictions the yield adjustment term becomes:

$$\frac{\Gamma(t,T)}{\tau} = \frac{1}{2\tau}\sum_{i=1}^{3}\int_t^T \left[\boldsymbol{\Omega}^T\mathbf{B}(t,T)\mathbf{B}(t,T)^T\boldsymbol{\Omega}\right]_{i,i} ds \tag{6.20}$$

Analogue to the DNS model we specify the AF-DNS in two ways. For the *independent-factor AF-DNS* the real-world P measure dynamics are given by:

$$d\mathbf{X}_t = \mathbf{K}^P\left(\boldsymbol{\theta}^P - \mathbf{X}_t\right)dt + \boldsymbol{\Omega}d\mathbf{W}_t^P \tag{6.21}$$

where we specify the state-variable reversion rate $\mathbf{K}^P$ and the covariance $\boldsymbol{\Omega}$ as diagonal matrices. The *correlated-factor AF-DNS* is specified by a full parametrized matrix $\mathbf{K}^P$ and a lower triangular $\boldsymbol{\Omega}$. As for the DNS we use for the AF-DNS ML estimation based on the Kalman filter, so that we have to depart from the continuous-time formulation. Using the AF-DNS P measure conditional mean and covariance:

$$\mathbb{E}^P\left[\mathbf{X}_T|\mathbf{X}_t\right] = \left[\mathbf{I} - exp\left(-\mathbf{K}^P\Delta t\right)\right]\boldsymbol{\theta}^P + exp\left(-\mathbf{K}^P\Delta t\right)\mathbf{X}_t \tag{6.22}$$

$$\mathbf{V}^P\left[\mathbf{X}_T|\mathbf{X}_t\right] = \int_0^{\Delta t} exp\left(-\mathbf{K}^P s\right)\boldsymbol{\Omega}\boldsymbol{\Omega}^T exp\left(-\left(\mathbf{K}^P\right)^T s\right) ds \tag{6.23}$$

we have to compute the conditional covariance matrix $\mathbf{V}^P\left[\mathbf{X}_t|\mathbf{X}_{t-1}\right]$. Calculation of the conditional covariance matrix in using the diagonalization of $\mathbf{K}^P$ is outlined in more detail in Appendix E.1. The generic transition of the AF-DNS for both specifications is:

$$\mathbf{X}_t = \left[\mathbf{I} - exp\left(-\mathbf{K}^P\Delta t\right)\right]\boldsymbol{\theta}^P + exp\left(-\mathbf{K}^P\Delta t\right)\mathbf{X}_{t-1} + \boldsymbol{\eta}_t \qquad \boldsymbol{\eta}_t \sim N(\mathbf{0},\mathbf{Q}) \tag{6.24}$$

with $\mathbf{Q} = \mathbf{V}^P [\mathbf{X}_t | \mathbf{X}_{t-1}]$. We set $\Delta = 1/12$ for our used monthly data. The AF-DNS model's measurement is analogue to the DNS as expressed in 6.3 except for the measurement's $N \times 1$ constant vector $\boldsymbol{c}^T = [-\Gamma(\tau_1)/\tau_1, -\Gamma(\tau_2)/\tau_2, ..., -\Gamma(\tau_N)/\tau_N]$ with $\tau_j = T_j - t$ with $j = 1, 2, ..., N$ contain the yield adjustment terms defined in 6.20. The exact expression for the adjustment terms of our two AF-DNS specifications are outlined in Appendix E.2.

6.2.2 Class of arbitrage-free multi-factor short rate models: Multi-factor Vasicek model

Extending the original formulation of the arbitrage-free mean reverting short rate model proposed by Vasicek [1977] from a one factor to a more general three-factor model leads to the following definition of the term structure of interest rates:

$$y(t, T) = -a_\tau + \boldsymbol{b}_\tau^T \boldsymbol{f}_t \tag{6.25}$$

where $\boldsymbol{f}_t$ is the $K \times 1$ vector of K latent (orthogonal) term structure factors with number of factors $K = 3$. The maturity dependent scalar a_τ and the 3×1 coefficient vector $\boldsymbol{b}_\tau$ are defined as:

$$\mathbf{b}_\tau^T = \frac{1}{\tau}\left[\frac{1}{\kappa_1}\left(1 - exp\left(-\kappa_1 \tau\right)\right), \frac{1}{\kappa_2}\left(1 - exp\left(-\kappa_2 \tau\right)\right), \frac{1}{\kappa_3}\left(1 - exp\left(-\kappa_3 \tau\right)\right)\right] \tag{6.26}$$

$$a_\tau = \frac{1}{\tau}\sum_{i=1}^{3}\left[\kappa_i^2\left(\theta_i - \frac{\sigma_i \lambda_i}{\kappa_i}\right) - \frac{\sigma_i^2}{2}\right]\frac{(b_{\tau,i} - \tau)}{\kappa_i^2} - \frac{\sigma_i^2 b_{\tau,i}^2}{4\kappa_i} \tag{6.27}$$

Integrating the model into a state-space model setting, the measurement and transition equation of the Vasicek three factor model we use is implemented as follows:

$$\boldsymbol{y}_t = \boldsymbol{a} + \mathbf{B}\boldsymbol{f}_t + \boldsymbol{\vartheta}_t \tag{6.28}$$

$\boldsymbol{y}_t$ contains the observed rates with 5 maturities $\tau = 12, 24, 36, 60$ and 120 month, the 5×1 vector of constants is specified as $\boldsymbol{a}^T = [-a_{12}, -a_{24}, ..., -a_{120}]$ and the 5×3 coefficient matrix $\mathbf{B}$ is $\mathbf{B} = [b_{12}, b_{24}, ..., b_{120}]$. $\boldsymbol{\vartheta}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\vartheta)$ is the Gaussian measurement error with diagonal variance-covariance $\boldsymbol{\Sigma}_\vartheta$. The factor dynamics are specified by the transition equation:

$$\boldsymbol{f}_t = \boldsymbol{c} + \mathbf{D}\boldsymbol{f}_{t-1} + \boldsymbol{\epsilon}_t \tag{6.29}$$

where the 3×1 vector $\boldsymbol{c}$ is

$$\boldsymbol{c}^T = \left[\theta_1\left(1 - exp(-\kappa_1)\right), \theta_2\left(1 - exp(-\kappa_2)\right), \theta_3\left(1 - exp(-\kappa_3)\right)\right]$$

the 3×3 matrix $\mathbf{D}$ is diagonal with

$$diag(\mathbf{D}) = [exp(-\kappa_1), exp(-\kappa_2), exp(-\kappa_3)]$$

and the transition's disturbance is Gaussian $\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\epsilon)$ with diagonal covariance $\boldsymbol{\Sigma}_\epsilon$, where the diagonal elements are

$$diag(\boldsymbol{\Sigma}_\epsilon) = \left[\frac{\sigma_1^2}{2\kappa_1}\left(1 - exp(-2\kappa_1)\right), \frac{\sigma_2^2}{2\kappa_2}\left(1 - exp(-2\kappa_2)\right), \frac{\sigma_3^2}{2\kappa_3}\left(1 - exp(-2\kappa_3)\right), \right]$$

The 17 parameters of the three factor Vasicek model

$$\boldsymbol{\theta}^T = [\kappa_1, \kappa_2, \kappa_3, \theta_1, \theta_2, \theta_3, \lambda_1, \lambda_2, \lambda_3, \sigma_1, \sigma_2, \sigma_3, \sigma_{12}, \sigma_{24}, \sigma_{36}, \sigma_{60}, \sigma_{120}]$$

are estimated by maximizing the system's (log) likelihood $\mathscr{L}(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$, which is calculated by the Kalman-filter.

6.2.3 Class of arbitrage-free affine term structure models

6.2.3.1 Latent ATSM

As the first model in the class of arbitrage-free affine term-structure models (ATSM) we use the arbitrage-free vector autoregressive ATSM, proposed by Ang and Piazessi [2003] and Hamilton and Wu [2012, 2014]. In the latent ATSM specification we use $N_f = 3$ latent factors $\boldsymbol{f}_t^T = \boldsymbol{f}_t^{l\,T} = \left[f_t^{l,1}, f_t^{l,2}, f_t^{l,3}\right]$ which follow the Gaussian VAR[1] process:

$$\boldsymbol{f}_t = \boldsymbol{\mu} + \boldsymbol{\Psi}\boldsymbol{f}_t + \boldsymbol{\Sigma}\boldsymbol{\eta}_t \tag{6.30}$$

The model's risk-free short rate used by the investors for discounting the bond prices is described by:

$$r_t = \delta_0 + \boldsymbol{\delta}_1^T \boldsymbol{f}_t \tag{6.31}$$

The term structure of the ATSM is defined as:

$$y(t,T) = a_\tau + \boldsymbol{b}_\tau^T \boldsymbol{f}_t \tag{6.32}$$

where a_τ and $\boldsymbol{b}_\tau$ are determined by the recursive pricing scheme applied by a risk averse investor outlined by Ang and Piazessi [2003] or in Appendix A.5.3. For estimating the parameters of both the latent and the macro-finance ATSM outlined in the following section we use the MLE procedure proposed by Chen and Scott [1993]. In this procedure the estimation is based on a state space model where the measurement equation:

$$\begin{bmatrix} \mathbf{Y}_t^1 \\ \mathbf{Y}_t^2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}^1 \\ \mathbf{B}^2 \end{bmatrix} \boldsymbol{f}_t + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Sigma}_e \end{bmatrix} \boldsymbol{\varepsilon}_t \tag{6.33}$$

divides into two parts: The $N_l = 3$ observed yields $\mathbf{Y}_t^1$ with maturities $\tau = 6, 24, 108$ month, the $N_l \times 1$ vector $\mathbf{A}^1$ and the $N_l \times N_l$ matrix $\mathbf{B}^1$ define the first part, whereas the $N_e = 3$ observed yields $\mathbf{Y}_t^2$ with maturities $\tau = 12, 60, 120$ month, the $N_e \times 1$ vector $\mathbf{A}^2$ and the $N_e \times N_l$ matrix $\mathbf{B}^2$ define the second part. For the first part Chen and Scott [1993] assume that $\mathbf{Y}_t^1$ are measured without error, while $\mathbf{Y}_t^2$ imply a Gaussian measurement error $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{I}_{N_e \times N_e})$. $\mathbf{A}^1$ and $\mathbf{A}^2$ contain the constants a_τ for the yields of maturities $\mathbf{Y}_t^1$ and $\mathbf{Y}_t^2$ and the matrices $\mathbf{B}^1$ and $\mathbf{B}^2$ contain the transposed vectors $\boldsymbol{b}_\tau^T$. $\boldsymbol{\Sigma}_e$ is diagonal so that it is assumed that the measurement errors are uncorrelated to each other. The ATSM parameter vector

$$\boldsymbol{\theta}^T = [\boldsymbol{\mu}, vec(\boldsymbol{\Psi}), vech(\boldsymbol{\Sigma}), \boldsymbol{\lambda}_0, vec(\lambda_1), diag(\boldsymbol{\Sigma}_e), \delta_0, \boldsymbol{\delta}_1]$$

is estimated by conventional MLE:

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \mathscr{L}(\boldsymbol{\theta}) = \sum_{t=1}^{n} ln\left(f\left(\mathbf{Y}_t \boldsymbol{f}_t | \mathbf{Y}_{t-1}, \boldsymbol{f}_{t-1}, ..., \boldsymbol{\theta}\right)\right) \tag{6.34}$$

with $\mathbf{Y}_t^T = [\mathbf{Y}_t^1, \mathbf{Y}_t^2]$. As in Dai and Singleton [2000], Singleton [2006] and in Hamilton and Wu [2012] for reasons of identification, the parameters of the latent ATSM are restricted with $\mathbf{I}_{N_l \times N_l}, \boldsymbol{\delta}_1 \geq 0$ and $\boldsymbol{\mu} = 0$. Further the coefficient matrix $\boldsymbol{\Psi}$ is lower triangular, so that the parameter vector $\boldsymbol{\theta}$ becomes

$$\boldsymbol{\theta}^T [vech(\boldsymbol{\Psi}), \boldsymbol{\lambda}_0, vec(\boldsymbol{\lambda}_1), diag(\boldsymbol{\Sigma}_e), \delta_0, \boldsymbol{\delta}_1]$$

With the number of observations $n > 0$ the (log) likelihood of the latent ATSM for the MLE estimation is:

$$\begin{aligned}
\mathscr{L}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_e) &= \sum_{t=1}^{n} -ln\left(|\mathbf{J}|\right) + ln\left(f\left(\boldsymbol{f}_t | \boldsymbol{f}_{t-1}, \boldsymbol{\theta}\right)\right) + ln\left(f(\boldsymbol{\varepsilon}_t, \boldsymbol{\Sigma}_e)\right) \\
&= \sum_{t=1}^{n} -ln\left(|\mathbf{J}|\right) + ln\left(\frac{1}{\sqrt{(2\pi)^{N_l}}} exp\left(-\frac{1}{2}\boldsymbol{u}_t^T \boldsymbol{u}_t\right)\right) \\
&\quad + ln\left(\frac{1}{\sqrt{(2\pi)^{N_e}} |\boldsymbol{\Sigma}_e \boldsymbol{\Sigma}_e^T|} exp\left(-\frac{1}{2}\boldsymbol{\varepsilon}_t^T \left(\boldsymbol{\Sigma}_e \boldsymbol{\Sigma}_e^T\right)^{-1} \boldsymbol{\varepsilon}_t\right)\right)
\end{aligned} \tag{6.35}$$

the residuals $\boldsymbol{u}_t = \boldsymbol{f}_t - \boldsymbol{\mu} - \boldsymbol{\Psi} \boldsymbol{f}_{t-1}$ and the $(N_l + N_e) \times (N_l + N_e)$ Jacobian:

$$\mathbf{J} = \begin{bmatrix} \mathbf{B}^1 & \mathbf{0} \\ \mathbf{B}^2 & \boldsymbol{\Sigma}_e \end{bmatrix}$$

$|\mathbf{C}|$ denotes the determinant of some matrix $\mathbf{C}$. From the inversion of the correctly measured yields the latent factors are derived by:

$$\boldsymbol{f}_t = \left(\mathbf{B}^1\right)^{-1} \left[\mathbf{Y}_t^1 - \mathbf{A}^1\right] \tag{6.36}$$

and the measurement errors are:

$$\begin{aligned}
\boldsymbol{\varepsilon}_t &= \boldsymbol{\Sigma}_e^{-1} \left[\mathbf{Y}_t^2 - \mathbf{A}^2 - \mathbf{B}^2 \left(\mathbf{B}^1\right)^{-1} \left(\mathbf{Y}_t^1 - \mathbf{A}^1\right)\right] \\
&= \boldsymbol{\Sigma}_e^{-1} \left[\mathbf{Y}_t^2 - \mathbf{A}^2 - \mathbf{B}^2 \mathbf{f}_t\right]
\end{aligned} \tag{6.37}$$

6.2.3.2 Macro-finance ATSM

Additional to the three factors of the latent ATSM, in the macro-finance ATSM proposed by Ang and Piazzesi [2003] two macroeconomic variables are integrated into the modeling framework. Beside the latent factors $\boldsymbol{f}_t^{l\,T} = \left[f_t^{l,1}, f_t^{l,2}, f_t^{l,3}\right]$, $\boldsymbol{f}_t = \left[\boldsymbol{f}_t^l, \boldsymbol{f}_t^m\right]$ contains in this extended specification the additional observable macro factors $\boldsymbol{f}_t^{m\,T} = [f_t^\pi, f_t^\gamma]$ where f_t^π and f_t^γ are the economy's price and real component. The VAR[1]-process for the factor dynamics in the macro-finance ATSM is defined as:

$$\begin{bmatrix} \boldsymbol{f}_t^l \\ \boldsymbol{f}_t^m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_l \\ \boldsymbol{\mu}_m \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Psi}_{lm} & \boldsymbol{\Psi}_{ll} \\ \boldsymbol{\Psi}_{mm} & \boldsymbol{\Psi}_{ml} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_{t-1}^l \\ \boldsymbol{f}_{t-1}^m \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma}_{lm} & \boldsymbol{\Sigma}_{ll} \\ \boldsymbol{\Sigma}_{mm} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_t^l \\ \boldsymbol{\varepsilon}_t^m \end{bmatrix} \tag{6.38}$$

where we apply the identification restrictions $\boldsymbol{\Sigma}_{lm} = \mathbf{0}, \boldsymbol{\Sigma}_{ll} = \mathbf{I}_{N_l \times N_l}$, $\boldsymbol{\mu}_l - [\boldsymbol{\Psi}_{lm}, \boldsymbol{\Psi}_{ll}]\lambda_0 = \mathbf{0}$ and $\boldsymbol{\Sigma}_{mm}$ as lower triangular purposed by Pericoli and Taboga [2008]. The short rate dynamic is modified as:

$$r_t = \delta_0 + \left[\boldsymbol{\delta}_{1,l}^T, \boldsymbol{\delta}_{1,m}^T\right] \begin{bmatrix} \boldsymbol{f}_t^l \\ \boldsymbol{f}_t^m \end{bmatrix} \tag{6.39}$$

with the additional restriction $\boldsymbol{\delta}_{1,l} \geq 0$. The model's measurement equation becomes in its macro-finance extended form:

$$\begin{bmatrix} \mathbf{Y}_t^1 \\ \mathbf{Y}_t^2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_l^1 & \mathbf{B}_m^1 \\ \mathbf{B}_m^2 & \mathbf{B}_m^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_t^l \\ \boldsymbol{f}_t^m \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Sigma}_e \end{bmatrix} \boldsymbol{\varepsilon}_t \tag{6.40}$$

For the MLE of the parameter vector

$$\boldsymbol{\theta}^T = [\boldsymbol{\mu}_l, \boldsymbol{\mu}_m, vec(\boldsymbol{\Psi}_{lm}), vec(\boldsymbol{\Psi}_{ll}), vec(\boldsymbol{\Psi}_{mm}), vec(\boldsymbol{\Psi}_{ml}), vech(\boldsymbol{\Sigma}_{mm}), \boldsymbol{\lambda}_0, vec(\boldsymbol{\lambda}_1), diag(\boldsymbol{\Sigma}_e), \delta_0, \boldsymbol{\delta}_{1,l}, \boldsymbol{\delta}_{1,m}]$$

the (log) likelihood of the macro finance ATSM is:

$$\begin{aligned} \mathscr{L}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_e) &= \sum_{t=1}^{n} -ln\left(|\mathbf{J}|\right) + ln\left(f\left(\boldsymbol{f}_t | \boldsymbol{f}_{t-1}, \boldsymbol{\theta}\right)\right) + ln\left(f(\boldsymbol{\varepsilon}_t, \boldsymbol{\Sigma}_e)\right) \\ &= \sum_{t=1}^{n} -ln\left(|\mathbf{J}|\right) + ln\left(\frac{1}{\sqrt{(2\pi)^{N_m}|\boldsymbol{\Sigma}_{mm}\boldsymbol{\Sigma}_{mm}^T|}} exp\left(-\frac{1}{2}\boldsymbol{u}_t^{m\,T}(\boldsymbol{\Sigma}_{mm}\boldsymbol{\Sigma}_{mm}^T)^{-1}\boldsymbol{u}_t^m\right)\right) \\ &\quad + ln\left(\frac{1}{\sqrt{(2\pi)^{N_l}}} exp\left(-\frac{1}{2}\boldsymbol{u}_t^{l\,T}\boldsymbol{u}_t^l\right)\right) + ln\left(\frac{1}{\sqrt{(2\pi)^{N_e}|\boldsymbol{\Sigma}_e\boldsymbol{\Sigma}_e^T|}} exp\left(-\frac{1}{2}\boldsymbol{\varepsilon}_t^T(\boldsymbol{\Sigma}_e\boldsymbol{\Sigma}_e^T)^{-1}\boldsymbol{\varepsilon}_t\right)\right) \end{aligned} \tag{6.41}$$

with $\boldsymbol{u}_t^m = \boldsymbol{f}_t - \boldsymbol{\mu}_m - [\boldsymbol{\Psi}_{ml}, \boldsymbol{\Psi}_{mm}]\,\mathbf{f}_{t-1}$, $\boldsymbol{u}_t^l = \boldsymbol{f}_t - \boldsymbol{\mu}_l - [\boldsymbol{\Psi}_{ll}, \boldsymbol{\Psi}_{lm}]\,\mathbf{f}_{t-1}$ and the Jacobian:

$$\mathbf{J} = \begin{bmatrix} \mathbf{B}_l^1 & \mathbf{0} \\ \mathbf{B}_l^2 & \boldsymbol{\Sigma}_e \end{bmatrix}$$

6.2.3.3 Latent ATSM with spanned stochastic volatility

As an additional extension of the arbitrage-free latent ATSM outlined in section 6.2.3.1 we have implemented the latent ATSM with spanned stochastic volatility purposed by Creal and Wu [2014]. Here the latent factors $\boldsymbol{f}_t^T = [\boldsymbol{g}_t, \boldsymbol{h}_t]$ are composed by the $G \times 1$ yield factors $\boldsymbol{g}_t$ and the $H \times 1$ heteroscedasticity factors $\boldsymbol{h}_t$, where the dynamics of the factors are described by the following processes:

$$\boldsymbol{g}_{t+1} = \boldsymbol{\mu}_g + \boldsymbol{\Psi}_g \boldsymbol{g}_t + \boldsymbol{\Psi}_{gh} \boldsymbol{h}_t + \boldsymbol{\Sigma}_{gh} \boldsymbol{\varepsilon}_{h,t+1} + \boldsymbol{\varepsilon}_{g,t+1} \tag{6.42}$$

$$\boldsymbol{\Sigma}_{g,t} \boldsymbol{\Sigma}_{g,t}^T = \boldsymbol{\Sigma}_{g,0} \boldsymbol{\Sigma}_{g,0}^T + \sum_{i=1}^{H} h_{i,t} \boldsymbol{\Sigma}_{i,g} \boldsymbol{\Sigma}_{i,g}^T \tag{6.43}$$

$$\boldsymbol{\varepsilon}_{g,t+1} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{g,t} \boldsymbol{\Sigma}_{g,t}^T) \tag{6.44}$$

$$\boldsymbol{\varepsilon}_{h,t+1} = \boldsymbol{h}_{t+1} - (\mathbf{I}_{H \times H} - \boldsymbol{\Psi}_h)\, \boldsymbol{\mu}_h - \boldsymbol{\Sigma}_h \mathbf{v}_h + \boldsymbol{\Psi}_h \boldsymbol{h}_t \tag{6.45}$$

and

$$\boldsymbol{h}_{t+1} = \boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h \boldsymbol{\omega}_{t+1} \tag{6.46}$$

with

$$\omega_{i,t+1} \sim Gamma(v_{h,i} + z_{i,t+1}, 1) \quad i = 1, ..., H$$
$$z_{i,t+1} \sim Poisson(\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1} \boldsymbol{\Psi}_h \boldsymbol{\Sigma}_h \boldsymbol{\omega}_t) \quad i = 1, ..., H$$

The process of $\boldsymbol{g}_{t+1}$ has non-Gaussian terms $\boldsymbol{\Psi}_{gh} \boldsymbol{h}_t$ and Gaussian terms $\boldsymbol{\Sigma}_{gh} \boldsymbol{\varepsilon}_{h,t+1}$. $\boldsymbol{h}_{t+1}$ is driven by the Gamma distributed error term $\boldsymbol{\omega}_{t+1}$, where the shape parameters $v_{h,i} + z_{i,t+1}$ of the i-th elements $\omega_{i,t+1}$ are itself driven by the i-th Poisson distributed $z_{i,t+1}$ with $i = 1, ..., H$. $\boldsymbol{\delta}_i$ is a $H \times 1$ vector with 1 at the i-th position and zero else. To ensure non-negativity of the heteroscedasticity factor $\boldsymbol{h}_{t+1}$ all elements of $\boldsymbol{\mu}_h, \boldsymbol{\Sigma}_h$ and $\boldsymbol{\Sigma}_h^{-1} \boldsymbol{\Psi}_h \boldsymbol{\Sigma}_h$ have to be non-negative. Following Creal and Wu we set $v_{i,h} \geq 1$. Further parameter restrictions regarding the arbitrage-free recursive pricing scheme are outlined in Appendix E.3. Similar to the latent ATSM with constant volatility the short-rate is given by:

$$r_t = \delta_0 + \boldsymbol{\delta}_1^T \boldsymbol{f}_t \tag{6.47}$$

where $\boldsymbol{\delta}_1^T [\boldsymbol{\delta}_{1,g}, \boldsymbol{\delta}_{1,h}]$. The term structure is given by the affine linear equation:

$$y(t, T) = a_\tau + \boldsymbol{b}_\tau^T \boldsymbol{f}_t \tag{6.48}$$

where the constant a_τ and the loadings $\boldsymbol{b}_\tau^T = [\boldsymbol{b}_{g,\tau}, \boldsymbol{b}_{h,\tau}]$ for maturity τ are determined by the arbitrage-free recursive pricing scheme outlined in more detail in Appendix E.3. Estimation of the latent ATSM with spanned stochastic volatility

$$\begin{aligned}\boldsymbol{\theta}^T = [&diag(\boldsymbol{\Sigma}_e), \delta_0, vec(\boldsymbol{\Psi}_g), vec(\boldsymbol{\Psi}_{gh}), vech(\boldsymbol{\Psi}_{0g}), vech(\boldsymbol{\Psi}_{1g}), ..., vech(\boldsymbol{\Psi}_{Hg}), vec(\boldsymbol{\Psi}_h), diag(\boldsymbol{\Sigma}_h),\\ &\boldsymbol{v}_h, \boldsymbol{\lambda}_g, \boldsymbol{\lambda}_h, vec(\boldsymbol{\Lambda}_g), vec(\boldsymbol{\Lambda}_h), vec(\boldsymbol{\Lambda}_{gh})]\end{aligned}$$

is also similar to the MLE procedure of the latent ATSM with constant volatility. Here we now use the parameters a_τ and $\boldsymbol{b}_\tau^T$ of 6.48 with respect to the pricing scheme outlined in Appendix E.3 in $\boldsymbol{A}^1, \boldsymbol{A}^2, \boldsymbol{B}^1$ and $\boldsymbol{B}^2$ of the measurement equation. Given the models parameters $\boldsymbol{\theta}$ the latent yield and heteroscedasticity factors $\boldsymbol{f}_t^T = [\boldsymbol{g}_t, \boldsymbol{h}_t]$ are determined by inversion:

$$\boldsymbol{f}_t = \left(\mathbf{B}^1\right)^{-1}\left(\mathbf{Y}_t^1 - \mathbf{A}^1\right) \tag{6.49}$$

The (log) likelihood function of the latent ATSM with spanned stochastic volatility factors is given by:

$$\begin{aligned}
\mathscr{L}(\boldsymbol{\theta}) = {} & CONST - (n-1)\, ln\left(|\mathbf{B}^1|\right) - \frac{1}{2}\sum_{t=2}^{n} tr\left(\boldsymbol{\Sigma}_e^{-1}\boldsymbol{\eta}_t\boldsymbol{\eta}_t^T\right) - \frac{1}{2}\sum_{t=2}^{n} ln\left(\boldsymbol{\Sigma}_{g,t-1}\boldsymbol{\Sigma}_{g,t-1}^T\right) \\
& - \frac{1}{2}\sum_{t=2}^{n} tr\left(\left(\boldsymbol{\Sigma}_{g,t-1}\boldsymbol{\Sigma}_{g,t-1}^T\right)^{-1}\boldsymbol{\varepsilon}_{g,t}\boldsymbol{\varepsilon}_{g,t}^T\right) - (n-1)ln(\boldsymbol{\Sigma}_h) - \sum_{t=2}^{n}\sum_{i=1}^{H}\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\left(\boldsymbol{h}_t - \boldsymbol{\mu}_h\right) \\
& - \sum_{t=2}^{n}\sum_{i=1}^{H}\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\boldsymbol{\Psi}_h\left(\boldsymbol{h}_{t-1} - \boldsymbol{\mu}_h\right) + \sum_{t=2}^{n}\sum_{i=1}^{H}\frac{(v_{h,i}-1)}{2} ln\left(\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\left(\boldsymbol{h}_t - \boldsymbol{\mu}_h\right)\right) \\
& - \sum_{t=2}^{n}\sum_{i=1}^{H}\frac{(v_{h,i}-1)}{2} ln\left(\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\boldsymbol{\Psi}_h\left(\boldsymbol{h}_{t-1} - \boldsymbol{\mu}_h\right)\right) \\
& + \sum_{t=2}^{n}\sum_{i=1}^{H} ln\left(B_{v_{h,i}-1}\left(2\sqrt{\left(\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\left(\boldsymbol{h}_t - \boldsymbol{\mu}_h\right)\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\left(\boldsymbol{h}_{t-1} - \boldsymbol{\mu}_h\right)\right)}\right)\right)
\end{aligned} \tag{6.50}$$

where:

$$B_\lambda(z) = \left(\frac{1}{2}z\right)^\lambda \sum_{k=0}^{\infty}\frac{\left(\frac{z^2}{4}\right)^k}{K!\Gamma(\lambda + k + 1)}$$

is the modified Bessel function of the first kind with $\lambda = v_{h,i} - 1$ and

$$z = 2\sqrt{\left(\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\left(\boldsymbol{h}_t - \boldsymbol{\mu}_h\right)\boldsymbol{\delta}_i^T\boldsymbol{\Sigma}_h^{-1}\left(\boldsymbol{h}_{t-1} - \boldsymbol{\mu}_h\right)\right)}$$

$\Gamma(*)$ denotes the Gamma function.

6.2.4 Class of alternative econometric approaches

Beside the term structure of interest rate models we implement alternative econometric forecasting models for a broader evaluation of the models' forecasting performances.

6.2.4.1 Driftless random walk

According to the driftless random walk, the $h > 0$ periods-ahead forecast of the zerocoupon spot rate with maturity $\tau = T - t$:

$$\hat{y}(t+h, T+h) = y(t, T) \tag{6.51}$$

at time t for the future period $t + h$ is the spot rate $y(t, T)$ observed at time t.

6.2.4.2 Slope regression

Forecasting the zero-coupon rates with maturity $\tau > 6$ by slope regression, is defined as:

$$\hat{y}(t+h, T+h) - y(t, T) = \hat{\alpha} + \hat{\beta}\left(y(t, t+\tau) - y(t, t+6)\right) \tag{6.52}$$

where the slope of the term structure is approximated by the spread between the spot rate $y(t, t+\tau)$ with maturity $\tau > 6$ month and the 6 -month spot rate $y(t, t+6)$. Estimates of the regression parameters α and β are simply the OLS estimates

$$\left[\hat{\alpha}, \hat{\beta}\right]^T = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\boldsymbol{y}$$

with the $t \times 2$ matrix $\mathbf{X}$, where the first column consists of ones and the second contains the 6 month spreads

$$\mathbf{X}^T_{[*,2]} = [y(1, 1+\tau) - y(1, 1+6), ..., y(t, t+\tau) - y(t, t+6)]$$

with respect to the selected maturity $\tau > 6$ observed until t. The $t \times 1$ vector $\boldsymbol{y}$ collects the h periods spreads

$$\boldsymbol{y} = [y(h+1, h+1+\tau) - y(1, 1+\tau), ..., y(t, t+\tau) - y(t-h, t-h+\tau)]$$

with the selected maturities $\tau > 6$ month observed until time t.

6.2.4.3 Vector autoregression in levels

By using a conventional VAR[1]-process the time t h period-ahead forecast of the $M \times 1$ vector $\hat{\boldsymbol{y}}_{t,h}$ of spot rates with $M = 5$ maturities $\tau = 12, 24, 36, 60, 120$ month is defined as:

$$\hat{\boldsymbol{y}}_{t,h} = \hat{\boldsymbol{c}} + \hat{\boldsymbol{\Gamma}}\boldsymbol{y}_t \tag{6.53}$$

where $\hat{\boldsymbol{c}}$ and $\hat{\boldsymbol{\Gamma}}$ are the estimated VAR[1]-parameters based on the zero-coupon rates observed until time t. For parameter estimation the spot rates $y(t, T)$ for all $\tau = 12, 24, 36, 60, 120$ month are regressed on their lagged values $y(t-h, T-h)$.

6.2.4.4 Principal components autoregression

Principal components forecasting is done in two steps. In step one we determine the covariance $\mathbf{\Sigma}$ of zero coupon rates with the $M = 10$ maturities $\tau = 12, 24, ..., 120$ month observed until time t. In the second step we decompose $\mathbf{\Sigma}$ with $\mathbf{\Sigma} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T$, where $\mathbf{\Lambda}$ is a $M \times M$ diagonal matrix with main diagonal containing the M eigenvalues of $\mathbf{\Sigma}$. The $M \times M$ matrix $\mathbf{Q}$ contains the associated M eigenvectors of $\mathbf{\Sigma}$. The eigenvalues $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_M$ are ordered decreasingly. The ordering is mapped to the eigenvectors $\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2, ..., \boldsymbol{\vartheta}_M$, such that e.g. $\boldsymbol{\vartheta}_1$ is the eigenvector associated to the largest eigenvector λ_1. Analogue to Litterman and Scheinkman [1991] we select the first 3 eigenvalues and their eigenvectors to define the first 3 principal components $\boldsymbol{p}_t = [p_{1,t}, p_{2,t}, p_{3,t}]$ with:

$$p_{k,t} = \boldsymbol{\vartheta}_k^T \boldsymbol{y}_t \quad k = 1, 2, 3 \tag{6.54}$$

with $\boldsymbol{y}_t^T = [y(t, 12), ..., y(t, 120)]$. For modeling the dynamic behaviour of the principal components, three separate AR[1] processes are used:

$$\hat{p}_{k,t+h} = \hat{\alpha}_k + \hat{\beta}_{k,h} p_{k,t} \tag{6.55}$$

where the regression parameters $\hat{\alpha}_k$ and $\hat{\beta}_{k,h}$ of the principal components $p_{k,t}$ on the lagged components $p_{k,t-h}$ with horizon h are estimated for $k = 1, 2, 3$ by OLS with date observed until time t. For the spot rate with maturity τ the forecast with the first 3 components is:

$$\hat{y}(t + h, T + h) = \sum_{k=1}^{3} \vartheta_k(\tau) \hat{p}_{k,t+h} \tag{6.56}$$

where $\vartheta_k(\tau)$ denotes the time to maturity τ element of the k-th eigenvector $\boldsymbol{\vartheta}_k$. Beside only using 3 principal components, we also use 6 components for our forecasting purposes.

6.3 In-sample-fitting and out-of-sample forecasting

6.3.1 Bond data and term structure of interest rates

For our forecasting experiments we use government bond prices for the six EMU countries Germany, France, Netherlands, Italy, Spain and Portugal between 03/2005 to 02/2014. From the bond prices we extract for every country the monthly term structure of interest rates. A month is approximated by its last banking day. The term structure of interest rates we use in this paper is denoted in terms of zero-coupon rates. The zero-coupon rates are extracted from the observed bond prices by applying the Nelson-Siegel-Svensson (NSS) approach proposed by Nelson and Siegel [1987] and Svensson [1994, 1995]. In Appendix A.2 we give an overview about the countries' bond prices we use. We further outline in more detail in Appendix A.3 the NSS approach for extracting the zero-coupon rates from the observed bond prices.

6.3.2 Model comparison of in-sample estimations

We start our analysis of the forecasting performance of the different term structure of interest rate models at fist by comparing the in-sample fitting quality of our implemented term-structure models. In-sample-fitting is important for the forecasting performance because the in-sample-estimation calibrates the respective models' free parameters for its out-of-sample forecasting purposes. In this section we have a look at the models' overall data in-sample-fit. As mentioned above overall data ranges between 03/2005 and 02/2014. For measuring the models' fitting quality for every time to maturity τ we use the root mean squared yield error (RMSYE) and the mean absolute deviation (MAD):

$$RMSYE_\tau = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y(t,t+\tau) - \hat{y}(t,t+\tau)\right)^2} \tag{6.57}$$

and

$$MAD_\tau = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left|y(t,t+\tau) - \hat{y}(t,t+\tau)\right|} \tag{6.58}$$

Table 6.2 lists the RMSYEs and MADs for ten term structure models introduced theoretically in the previous section for Germany, France, Italy and Spain with respect to the five maturities $\tau = 12, 24, 36, 60$ and 120 month. For the Netherlands and Portugal the insample RMSYEs and MADs are listed in the table of Appendix E.4. Additionally Figure 6.1 shows the observed and the estimated model implied yields for the five maturities $\tau = 12, 24, 60$ and 120 month for Germany and Italy. For France, the Netherlands, Spain and Portugal the graphical comparison of the observed and estimated yields are shown in the figure of Appendix E.4. Obviously the comparison of the in-sample estimates of the models in all six EMU countries measured by the RMSYE as well as the MAD suggests that the latent ATSM and the MF-DNS have the best in-sample fit to the observed yield data. For these two models the RMSYEs and MADs are the lowest and lie on average under ten basis points. Different to these models, the stochastic volatility ATSM with two latent yield and a single heteroscedasticity factor and the three factor Vasicek model show the highest deviations from the observed yields. For the stochastic volatility ATSM the fit to the observed data becomes poorer for higher maturities. The Vasicek model shows a poorer in-sample fit at the short end. Especially the three factor Vasicek implied 12 month yield shows a poor fit to the data.

With focus on the class of dynamic Nelson-Siegel models an interesting aspect in comparing these models is the comparison between the AF-DNS and the DNS in general and the comparison between the correlated and the independent specifications of AFDNS and DNS respectively. The first comparison reveals that except for the Netherlands the DNS models in their correlated and independent specification show a better fit to the observed yields than the correlated and independent AF-DNS models. The second comparison reveals that for Germany there is on average no difference in the in-sample performance between the correlated and the independent specifications of the AF-DNS and the DNS models. For Italy and Spain the correlated specifications show a better insample fit than the independent specifications. For France the independent specifications show a better fit to the data. For the Netherlands and Portugal the independent AF-DNS reveals a better fit than the correlated AF-DNS whereas the correlated DNS fits the data better than the independent DNS. Different to Christensen et.al. [2011] who find that the correlated specifications are more accurate than the independent specification our findings from the country comparison are more heterogeneous and not so clear in its conclusion.

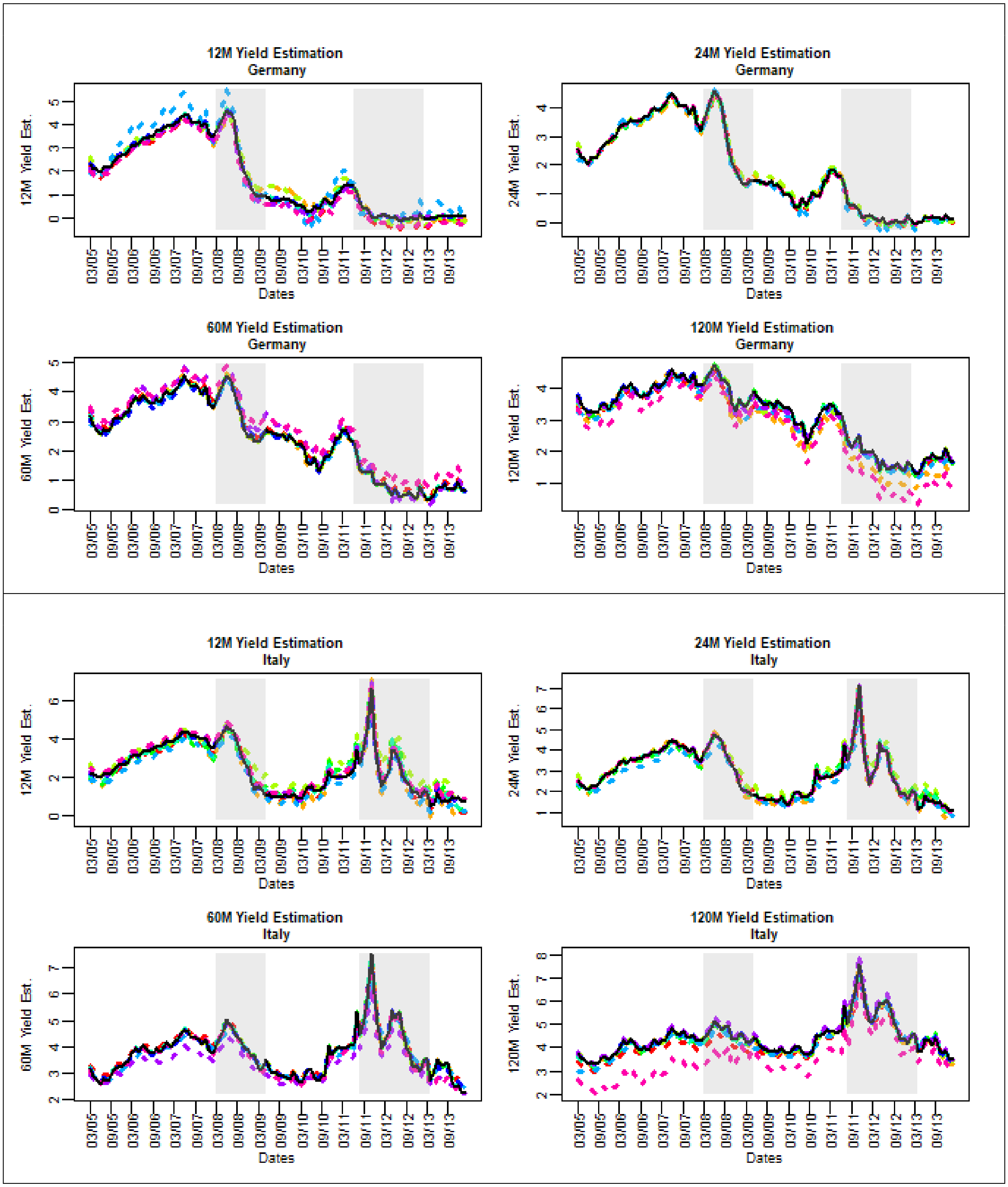


Table 6.1: Observed and estimated yields with $\tau = 12, 24, 60$ and 120 month maturities for Germany and Italy between 03/2005 and 02/2014.

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
In-Sample-Fitting RMSYE Germany 03/2005 - 02/2014									
12	0.239	0.206	0.238	0.060	0.073	0.453	0.041	0.134	0.270
24	0.089	0.118	0.122	0.054	0.016	0.135	0.000	0.000	0.000
36	0.095	0.055	0.057	0.051	0.015	0.138	0.045	0.173	0.208
60	0.133	0.046	0.000	0.019	0.017	0.073	0.072	0.272	0.402
120	0.160	0.381	0.112	0.074	0.038	0.212	0.022	0.082	0.567
In-Sample-Fitting RMSYE France 03/2005 - 02/2014									
12	0.251	0.319	0.149	0.303	0.209	0.434	0.010	0.054	0.236
24	0.107	0.375	0.042	0.132	0.061	0.204	0.000	0.001	0.000
36	0.075	0.259	0.000	0.060	0.014	0.098	0.022	0.102	0.163
60	0.055	0.097	0.015	0.000	0.015	0.071	0.037	0.214	0.283
120	0.148	0.248	0.043	0.122	0.022	0.147	0.015	0.072	0.246
In-Sample-Fitting RMSYE Italy 03/2005 - 02/2014									
12	0.245	0.381	0.559	0.350	0.254	0.353	0.063	0.119	0.247
24	0.116	0.180	0.289	0.178	0.104	0.339	0.000	0.001	0.000
36	0.130	0.062	0.122	0.060	0.015	0.299	0.067	0.286	0.190
60	0.146	0.022	0.022	0.021	0.038	0.235	0.090	0.455	0.164
120	0.322	0.075	0.062	0.105	0.086	0.267	0.021	0.144	0.994
In-Sample-Fitting RMSYE Spain 03/2005 - 02/2014									
12	0.193	0.319	0.643	0.331	0.293	0.476	0.041	0.153	0.252
24	0.059	0.127	0.331	0.125	0.093	0.472	0.000	0.001	0.000
36	0.034	0.038	0.146	0.036	0.010	0.385	0.042	0.066	0.194
60	0.040	0.023	0.002	0.008	0.027	0.241	0.065	0.103	0.144
120	0.139	0.093	0.416	0.097	0.070	0.234	0.023	0.040	1.094

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
In-Sample-Fitting MAD Germany 03/2005 - 02/2014									
12	0.209	0.161	0.184	0.040	0.054	0.374	0.029	0.115	0.264
24	0.074	0.101	0.089	0.047	0.013	0.105	0.000	0.000	0.000
36	0.071	0.048	0.040	0.044	0.012	0.125	0.035	0.150	0.201
60	0.122	0.037	0.000	0.013	0.013	0.057	0.057	0.234	0.395
120	0.134	0.323	0.080	0.049	0.028	0.183	0.017	0.070	0.530
In-Sample-Fitting MAD France 03/2005 - 02/2014									
12	0.203	0.279	0.109	0.219	0.145	0.348	0.007	0.044	0.221
24	0.089	0.312	0.031	0.103	0.046	0.164	0.000	0.001	0.000
36	0.054	0.209	0.000	0.045	0.011	0.078	0.018	0.084	0.146
60	0.045	0.087	0.012	0.000	0.012	0.056	0.028	0.183	0.266
120	0.120	0.224	0.034	0.096	0.018	0.119	0.011	0.063	0.238
In-Sample-Fitting MAD Italy 03/2005 - 02/2014									
12	0.201	0.301	0.421	0.284	0.198	0.345	0.044	0.089	0.236
24	0.078	0.147	0.205	0.139	0.078	0.284	0.000	0.001	0.000
36	0.111	0.050	0.084	0.047	0.011	0.234	0.045	0.259	0.173
60	0.125	0.017	0.017	0.017	0.029	0.170	0.063	0.407	0.108
120	0.284	0.057	0.051	0.088	0.073	0.217	0.016	0.119	0.931
In-Sample-Fitting MAD Spain 03/2005 - 02/2014									
12	0.145	0.244	0.512	0.237	0.206	0.476	0.025	0.139	0.248
24	0.042	0.101	0.276	0.095	0.069	0.425	0.000	0.001	0.000
36	0.025	0.030	0.126	0.028	0.008	0.321	0.030	0.052	0.190
60	0.030	0.018	0.002	0.006	0.021	0.192	0.048	0.080	0.125
120	0.112	0.076	0.327	0.072	0.051	0.187	0.017	0.030	1.028

Table 6.2: In-sample RMSYEs and MADs for Germany, France, Italy and Spain - Estimation period 03/2005 - 02/2014.

6.3.3 Model comparison of out-of-sample forecasting

6.3.3.1 Rolling sample procedure

The forecasting experiments are done by using a rolling sample procedure. We choose the length l of our rolling samples with five years. The procedure starts with the data sample ranging from 03/2005 to 03/2010 and iterates forward in monthly steps finishing with the sample ranging between 01/2009 and 01/2014. For every of these rolling five years data samples we separately estimate the term-structure of interest rate models outlined in section 6.2. Depending on the required data input for estimating the specific term structure of interest rates model the rolling data samples include observed yield data as well as observed macroeconomic data.

6.3.3.2 Point yield forecasts

Based on the rolling sample procedure we generate for all of the outlined term structure of interest rate models point yield forecasts for various forecasting horizons h ranging from 1 to 12 month. For the state space models including the terms structure models of the class of dynamic Nelson-Siegel models from section 6.2.1, the class of multi-factor short-rate models from section 6.2.2 and the models from the class of arbitrage-free ATSM from section 6.2.3 the forecasting is done by iterating h times forward the state variables transition equation based on the rolling model parameter estimates regarding data of the previous five years back from the date of doing the forecast at time t with $t = l+1, l+2, ..., n-h$ according to our rolling sample scheme. n is the number of observations. For the Gaussian state-space models with $M \times 1$ and $N \times 1$ measurement $\mathbf{Y}_t$ and state variable $\mathbf{X}_t$ respectively:

$$\boldsymbol{Y}_t = \boldsymbol{d} + \boldsymbol{D}\boldsymbol{X}_t + \boldsymbol{\eta}_t \quad \boldsymbol{\eta}_t \sim N(\mathbf{0}, \mathbf{S}) \tag{6.59}$$

$$\mathbf{X}_t = \boldsymbol{c} + \mathbf{C}\mathbf{X}_{t-1} + \boldsymbol{\epsilon}_t \quad \boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}) \tag{6.60}$$

h times forward iteration of the first order transition dynamics at time t conditional to the information between $t-l$ and t leads to:

$$\mathbb{E}_{t-l:t}\left[\mathbf{X}_{t+h}\right] = \left(\mathbf{I} + \sum_{i=1}^{h-1} \mathbf{C}^i\right) \boldsymbol{c} + \mathbf{C}^h \mathbf{X}_t \tag{6.61}$$

such that the h month ahead point forecast at time t is:

$$\mathbb{E}_{t-l:t}\left[\mathbf{X}_{t+h}\right] = \boldsymbol{d} + \mathbf{D}\left[\left(\mathbf{I} + \sum_{i=1}^{h-1} \mathbf{C}^i\right) \boldsymbol{c} + \mathbf{C}^h \mathbf{X}_t\right] \tag{6.62}$$

For every of the six EMU countries we specify the measurement $\boldsymbol{Y}_t$ as the 5×1 column vector of observed yields with maturities $\tau = 12, 24, 36, 60$ and 120 month.

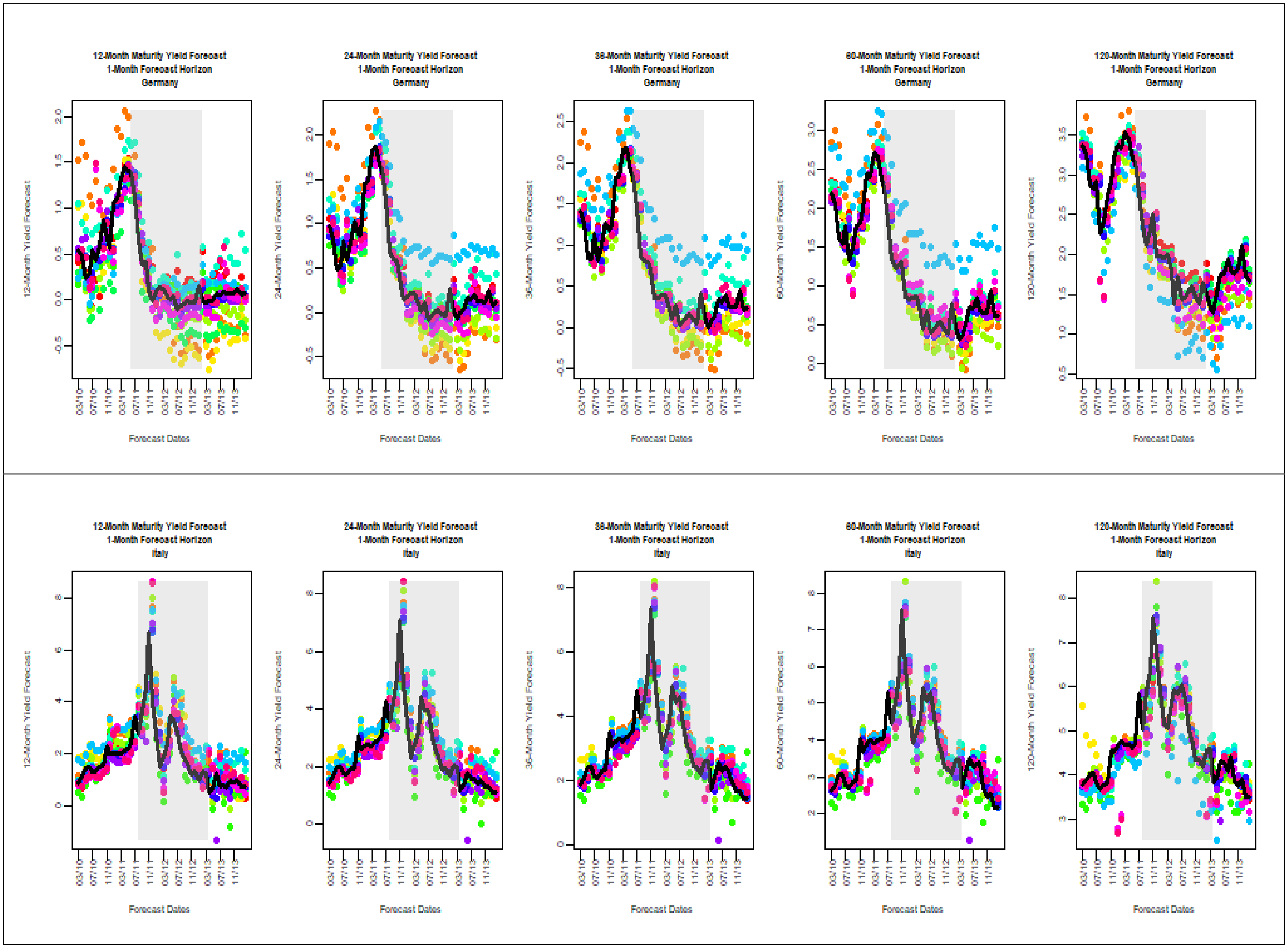


Table 6.3: Observed and 1-month ahead predicted yields with $\tau = 12, 24, 36, 60$ and 120 month maturities for Germany and Italy between 03/2010 and 02/2014.

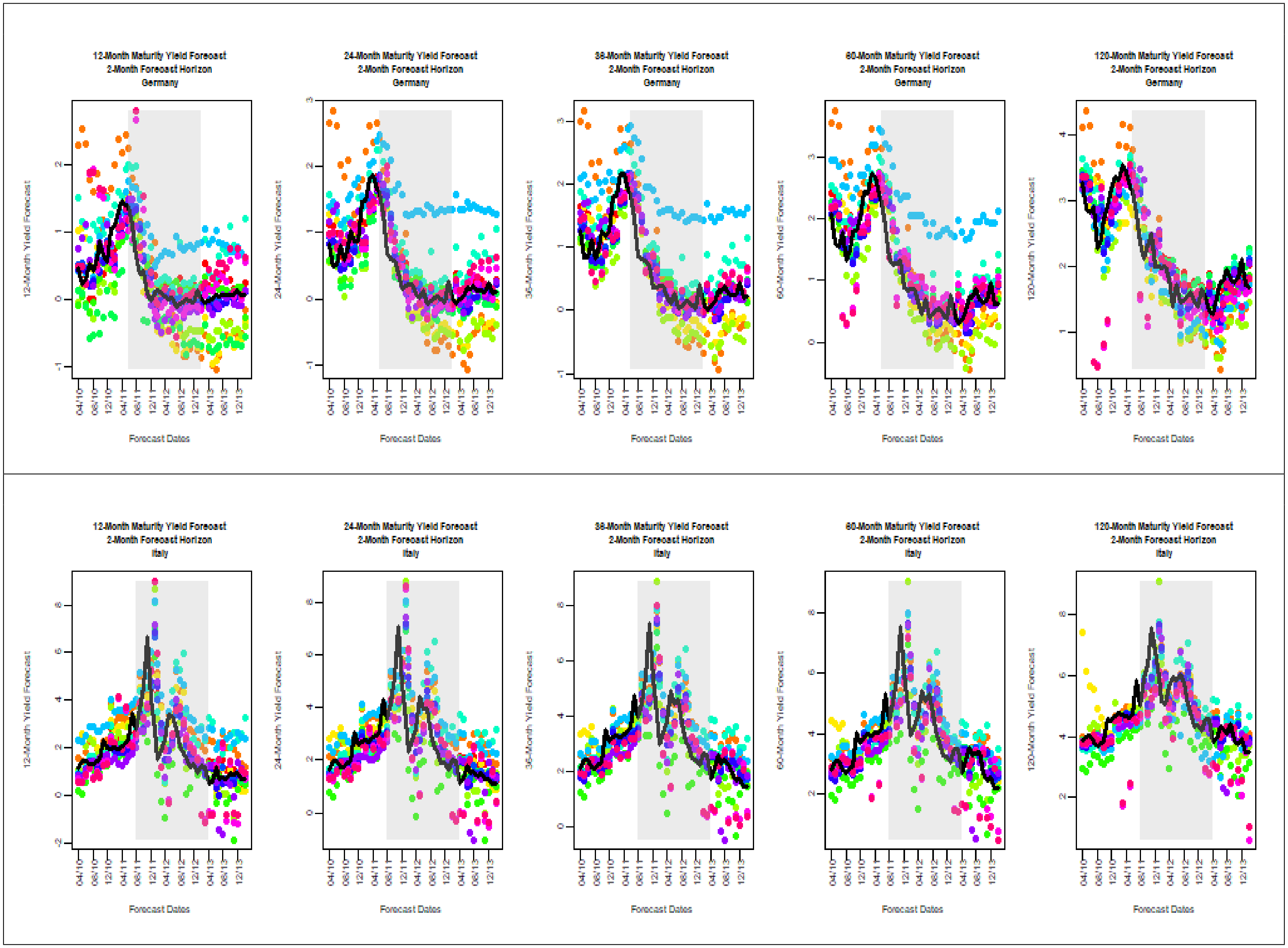


Table 6.4: Observed and 2-month ahead predicted yields with $\tau = 12, 24, 36, 60$ and 120 month maturities for Germany and Italy between 08/2010 and 02/2014.

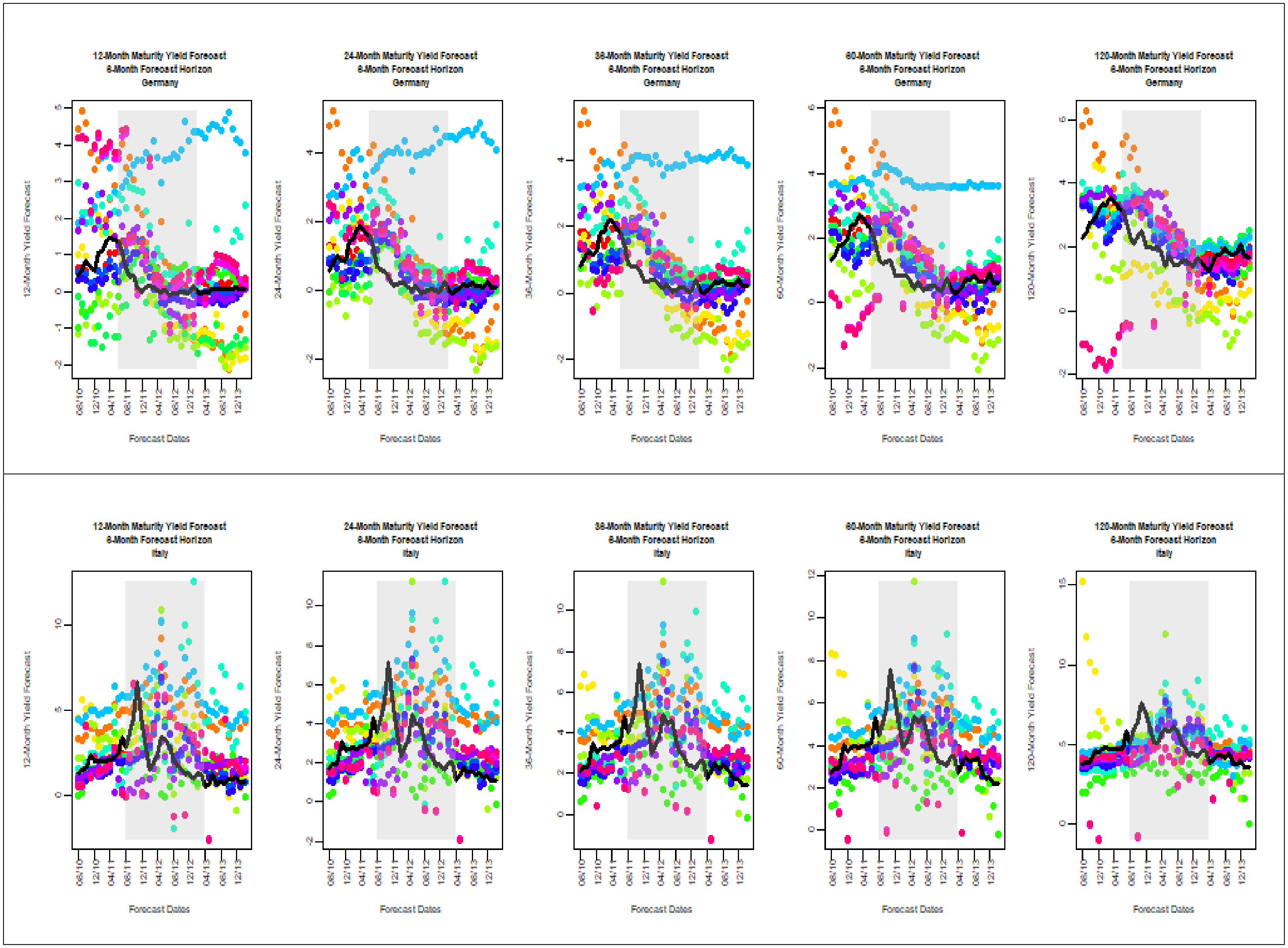


Table 6.5: Observed and 6-month ahead predicted yields with $\tau = 12, 24, 36, 60$ and 120 month maturities for Germany and Italy between 08/2010 and 02/2014

As the metric for evaluating the term structure of interest rates models' forecasting performance we apply the root mean squared yield forecasting error (RMSYFE). The RMSYFE with respect to the time to maturity τ and the forecasting horizon h is defined as:

$$RMSYFE_{\tau}^{h} = \sqrt{\frac{1}{n-l-h}\sum_{t=l+1}^{n-h}\left(y(t+h,t+h+\tau) - \mathbb{E}_{t-l:t}\left[y(t+h,t+h+\tau)\right]\right)^2} \quad (6.63)$$

In Figures 6.3, 6.4 and 6.5 for Germany and Italy we show the observed and predicted zero-coupon rates with maturities $\tau = 12, 24, 36, 60$ and 120 month implied by 13 term structure and alternative econometric forecasting models over the forecasting horizons $h = 1, 2$ and 6 month. In the following we often speak about good and poor forecasting qualities. These terms are relative expressions - meaning that we take a model comparing perspective in using these terms. The analogue Figures of the predicted zero-coupon rates over the horizons $h = 1, 2$ and 6 month for France, Netherlands, Spain and Portugal are shown in Appendix E.5. From the scattering of the forecasts around the observed interest rates from the Figures it becomes clear, that the forecasts becomes poorer with an increasing h. Obviously for Germany, France and Netherlands especially the stochastic volatility ATSM has larger problems in predicting the out-of-sample zero-coupon rates. For these three countries the model strongly deviates in its predicted dynamics from the observed interest rate dynamics as well as from the dynamics predicted by the other models. In Table 6.6 and 6.7 as well as in the tables of Appendix E.5 we list the RMSYFEs for the six EMU countries over the horizons $h = 1, 2, 6$ and 12 month. Not surprisingly for all countries the stochastic volatility ATSM shows the highest RMSYFEs. From the Figures of the observed and predicted interest rate dynamics as well as from the Tables with the models RMSYFEs it becomes clear that for all countries the non-parametric (driftless) random walk shows the lowest RMSYFEs followed by the two parameter slope regression. The term structure model with the best forecasting performance is the independent AF-DNS. The performance of the independent AF-DNS is robust over all six countries. Latent ATSM and MF-DNS follow the independent AF-DNS in showing good forecasting performances. But these two models are not robust in their performance with respect to the different interest rate dynamics observed for our six EMU countries. The latent ATSM performs well for Italy and Netherlands but shows a poorer forecasting performance for Spain and Portugal. The MF-DNS shows a good performance in predicting the interest rate dynamics of Germany, France and Spain though the forecasting performance of the MF-DNS becomes poorer compared to other models with longer horizons h . From Table 6.6 and 6.7 and tables of Appendix E.5 it becomes also clear that beside the stochastic ATSM the models with a larger number of free parameters in tendency show high RMSYEs. Here especially the correlated- DNS and AF-DNS as well as the macro-finance ATSM show high RMSYEs over the various forecasting horizons h .

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	RW	Slope Regression	VAR[1]	PC-3-AR	PC-6-AR	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
Panel A: 1-month ahead RMSFYEs for Germany														
12	0.260	0.532	0.440	0.258	0.213	0.781	0.158	0.162	0.181	0.259	0.248	0.393	0.350	0.224
24	0.233	0.525	0.388	0.282	0.208	0.580	0.176	0.179	0.196	0.247	0.231	0.323	0.332	0.413
36	0.230	0.513	0.355	0.296	0.210	0.496	0.189	0.192	0.208	0.240	0.229	0.293	0.330	0.526
60	0.233	0.495	0.334	0.303	0.214	0.450	0.200	0.202	0.219	0.252	0.234	0.283	0.320	0.613
120	0.239	0.477	0.328	0.310	0.220	0.421	0.208	0.209	0.226	0.269	0.244	0.276	0.308	0.604
Panel B: 2-month ahead RMSFYEs for Germany														
12	0.315	0.879	0.550	0.481	0.352	0.721	0.245	0.252	0.298	0.529	0.571	0.690	0.618	0.666
24	0.299	0.876	0.506	0.508	0.335	0.557	0.267	0.274	0.315	0.434	0.466	0.555	0.579	0.921
36	0.304	0.866	0.480	0.524	0.330	0.505	0.283	0.290	0.328	0.399	0.423	0.492	0.564	1.026
60	0.315	0.848	0.463	0.534	0.330	0.481	0.296	0.303	0.340	0.418	0.433	0.465	0.543	1.072
120	0.321	0.815	0.467	0.544	0.334	0.465	0.306	0.310	0.349	0.491	0.493	0.449	0.516	0.981
Panel C: 6-month ahead RMSFYEs for Germany														
12	0.536	1.965	1.096	1.353	0.754	0.716	0.502	0.515	0.818	1.772	1.848	1.432	1.385	3.470
24	0.542	1.971	1.054	1.383	0.700	0.699	0.544	0.568	0.863	1.359	1.423	1.139	1.294	3.571
36	0.560	1.967	1.027	1.405	0.678	0.735	0.572	0.603	0.899	1.212	1.260	1.009	1.252	3.493
60	0.583	1.949	1.012	1.427	0.675	0.756	0.595	0.628	0.926	1.298	1.334	0.957	1.206	3.314
120	0.590	1.889	1.074	1.465	0.676	0.755	0.601	0.631	0.923	1.590	1.608	0.926	1.137	2.982
Panel C: 12-month ahead RMSFYEs for Germany														
12	0.836	2.922	2.021	2.222	1.221	0.761	0.685	1.030	1.339	2.436	2.506	1.429	2.276	7.767
24	0.887	2.930	2.054	2.246	1.565	0.929	0.741	1.021	1.315	2.051	2.105	1.163	2.135	7.474
36	0.934	2.931	2.094	2.268	1.833	1.046	0.779	1.002	1.298	1.872	1.912	1.119	2.076	7.068
60	0.978	2.922	2.163	2.304	2.011	1.101	0.805	0.978	1.282	1.815	1.850	1.162	2.022	6.557
120	0.977	2.835	2.528	2.399	1.961	1.105	0.803	0.935	1.243	1.903	1.922	1.189	1.916	5.911

Table 6.6: Out-of-sample RMSFYEs for Germany - Forecasting period 03/2010 - 02/2014.

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	RW	Slope Regression	VAR[1]	PC-3-AR	PC-6-AR	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
Panel A: 1-month ahead RMSFYEs for Italy														
12	0.755	1.009	1.001	0.883	0.868	0.938	0.696	0.729	0.862	0.968	0.952	0.722	0.949	1.157
24	0.732	0.936	0.897	0.834	0.861	0.939	0.689	0.715	0.844	0.908	0.896	0.698	0.914	1.019
36	0.712	0.879	0.835	0.802	0.857	0.905	0.679	0.702	0.826	0.864	0.852	0.681	0.878	0.927
60	0.684	0.825	0.787	0.765	0.840	0.850	0.658	0.678	0.798	0.828	0.815	0.658	0.830	0.861
120	0.645	0.767	0.750	0.718	0.804	0.791	0.623	0.640	0.751	0.796	0.784	0.622	0.778	0.854
Panel B: 2-month ahead RMSFYEs for Italy														
12	1.063	1.577	1.266	1.329	1.329	1.353	1.015	1.089	1.300	1.663	1.667	1.054	1.669	1.855
24	1.040	1.465	1.175	1.267	1.335	1.314	1.003	1.066	1.286	1.571	1.579	1.012	1.599	1.653
36	1.015	1.374	1.120	1.223	1.335	1.253	0.986	1.041	1.268	1.497	1.504	0.979	1.516	1.506
60	0.977	1.281	1.077	1.172	1.320	1.173	0.953	1.004	1.230	1.424	1.433	0.941	1.421	1.393
120	0.921	1.186	1.057	1.106	1.283	1.091	0.901	0.946	1.163	1.369	1.367	0.888	1.325	1.304
Panel C: 6-month ahead RMSFYEs for Italy														
12	1.358	3.259	1.883	2.064	1.448	2.098	1.325	1.519	1.745	2.180	2.171	1.357	4.215	4.058
24	1.339	2.968	1.813	1.975	1.534	1.992	1.319	1.500	1.731	2.052	2.046	1.290	3.907	3.632
36	1.316	2.725	1.784	1.910	1.601	1.882	1.303	1.471	1.694	1.977	1.975	1.249	3.611	3.315
60	1.280	2.489	1.802	1.851	1.679	1.757	1.272	1.427	1.634	1.942	1.943	1.208	3.331	3.053
120	1.221	2.274	2.015	1.790	1.768	1.634	1.218	1.362	1.551	1.945	1.940	1.155	3.098	2.763
Panel C: 12-month ahead RMSFYEs for Italy														
12	1.944	5.280	3.459	3.524	2.948	3.062	1.874	2.989	1.982	3.855	3.840	1.794	7.718	7.004
24	1.943	4.760	3.417	3.421	2.641	2.922	1.907	2.820	2.029	3.683	3.669	1.718	6.820	6.354
36	1.916	4.323	3.419	3.345	2.403	2.774	1.894	2.688	2.036	3.554	3.542	1.666	6.165	5.856
60	1.864	3.904	3.541	3.273	2.218	2.602	1.851	2.563	2.008	3.427	3.421	1.614	5.645	5.412
120	1.783	3.549	4.230	3.199	2.179	2.430	1.785	2.434	1.945	3.287	3.275	1.556	5.256	4.894

Table 6.7: Out-of-sample RMSFYEs for Italy - Forecasting period 03/2010 - 02/2014.

6.4 Ockham's Razor

In section 6.3 we have seen that term structure models with a larger number of free parameters in tendency show high RMSYFEs. In Figure 6.1 for Germany and Italy we plot the number of the forecasting models' free parameters against their respective RMSYFEs. We focus here on the horizon $h = 1$ where the models show their best forecasting performances. The suggestions and conclusions derived here are also true for the horizons $h = 2, 6$ and 12 . Figure 5 is motivated by Ockham's Razor which in general terms commands model approaches with reduced complexity in describing well specified phenomena. From Figure

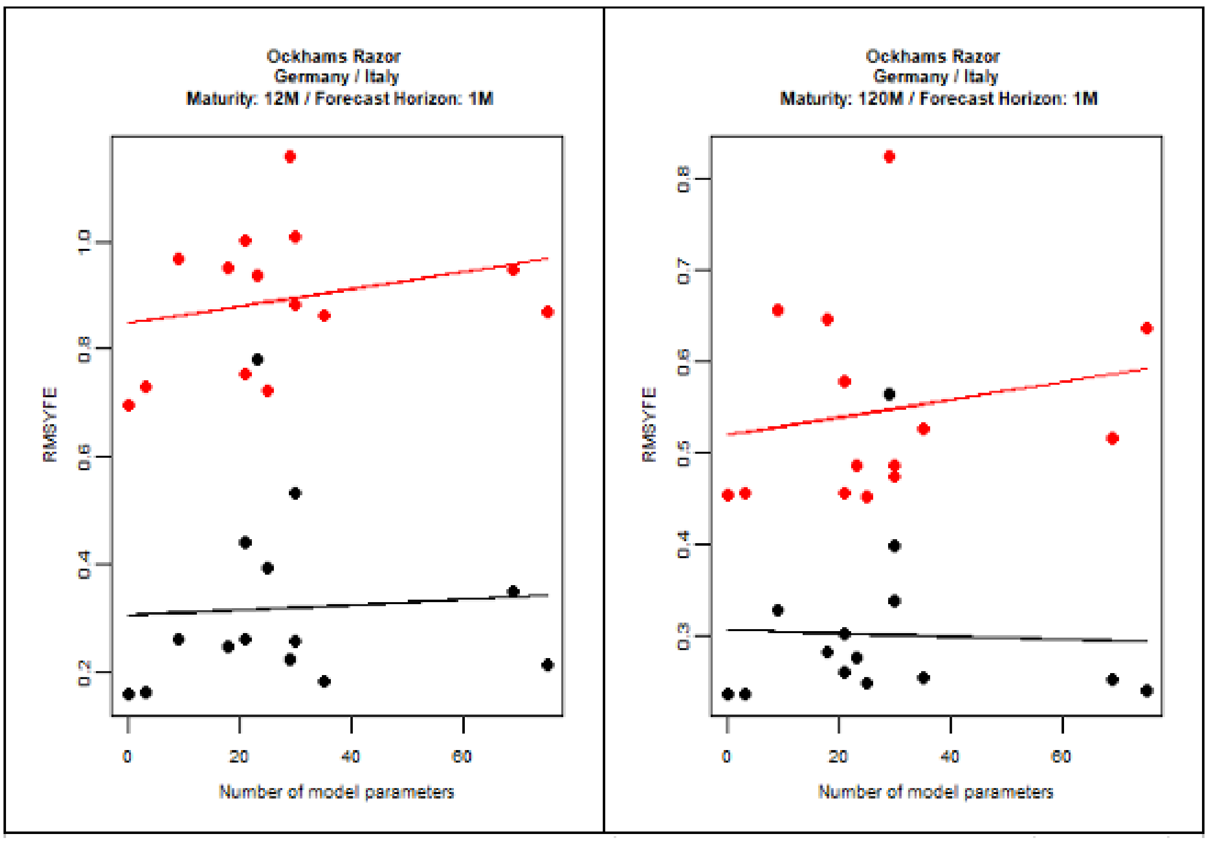


Figure 6.1: Ockham's Razor expressed as the number of model parameters and the RM-SYFEs over the $h = 1$ month forecasting horizon for the maturities 12 and 120 month for Germany (black) and Italy (red)

6.1 it becomes clear that there is no gain with respect to the forecasting performance of our interest rate prediction models in increasing the models' complexity measured in their free parameters. For both countries the regression lines (with insignificant slope parameters) suggest a constant or a poorer forecasting performance due to an increase in the model's complexity

6.5 Conclusion

For this chapter we have implemented a large number of term structure of interest rate models as well as alternative econometric forecasting models. With focus on the overall in-sample fitting quality where the observed data ranges between 03/2005 and 02/2014 we find that most of our implemented term structure models show good in-sample fits with RMSYEs lower than ten basis points. Exception here are the three factor Vasicek short rate model and the stochastic volatility ATSM model. These two models show poor overall in-sample-fitting qualities with high RMSYEs compared to the alternative terms structure models. Focusing on our rolling horizon out-of-sample forecasting experiments we find that the non-parametric driftless random walk is in its forecasting performance hard to beat. But also the two parameter slope-regression reveals good forecasting results. The term structure of interest rates model which shows the best forecasting performance is the independent AF-DNS. This model reveals a high robustness in its forecasting performance for all six EMU countries and for all forecasting horizons. The latent ATSM and the MF-DNS follow the independent AF-DNS in also showing good forecasting performances. But these two models are not robust in their performances with respect to the different interest rate dynamics observed for the six EMU countries. In our forecasting experiments we also find that in tendency more complex term structure of interest rates models with a larger number of free model parameters perform poorer in forecasting interest rates than models with lower complexity measured by a lower number of free model parameters.

Appendices

A. Appendix Chapter 2

A.1 Macroeconomic data

The quarterly data cover 2005/Q1 to 2014/Q1, where the countries regarded here are Germany, France and Italy. Source of the macroeconomic data for the four EMU countries is the FED St. Louis. The preparation of the macroeconomic time series we used for our estimations are as follows:

1. Real Output Growth

$$\Delta GDP_t = 100 \times \left[ln\left(GDP_t\right) - ln\left(GDP_{t-1}\right)\right]$$

where GDP_t is the quarterly level of the real gross domestic product at quarter t.

2. Real Consumption Growth

$$\Delta CONS_t = 100 \times \left[ln\left(\frac{CONS_t}{GDPP_t}\right) - ln\left(\frac{CONS_{t-1}}{GDPP_{t-1}}\right)\right]$$

where $CONS_t$ is the quarterly level of private final consumption expenditures of the gross domestic product and $GDPP_t$ the quarterly level of the gross domestic product price deflator.

3. Real Investment Growth

$$\Delta INV_t = 100 \times \left[ln\left(\frac{INV_t}{GDPP_t}\right) - ln\left(\frac{INV_{t-1}}{GDPP_{t-1}}\right)\right]$$

with INV_t standing for the level of the quarterly seasonally adjusted gross fixed capital formation expenditures of the gross domestic product.

4. Real Wage Growth

$$\Delta WAGE_t = 100 \times \left[ln\left(\frac{WAGE_t}{GDPP_t}\right) - ln\left(\frac{WAGE_{t-1}}{GDPP_{t-1}}\right)\right]$$

where $WAGE_t$ are the seasonally adjusted private sector hourly earnings.

5. Labour

$$LABOUR_t = 100 \times ln\left(HOURS_t\right)$$

with $HOURS_t$ as the average annual hours worked by persons engaged for respective country.

6. Inflation

$$\Delta INF_t = 100 \times ln\left(\frac{GDPP_t}{GDPP_{t-1}}\right)$$

7. Short rate

For the short rate we take the one month EONIA swap rate published by the German Bundesbank where we calculate the quarterly value by averaging over the three end-ofmonth quotes of the quarter.

A.2 Composition of EMU bond price data

Germany

Year	Total	$\tau \leq 1Y$	$1Y < \tau \leq 2Y$	$2Y < \tau \leq 5Y$	$5Y < \tau \leq 10Y$	$10Y < \tau \leq 20Y$	$\tau > 20$
2005	54	10	8	15	10	4	7
2006	52	8	9	14	11	3	7
2007	52	9	9	12	12	2	8
2008	51	9	8	12	12	4	6
2009	50	8	8	12	12	4	6
2010	53	8	8	13	12	6	6
2011	55	8	8	15	13	6	5
2012	57	8	8	17	12	6	6
2013	60	8	9	17	14	6	6
2014	63	8	9	18	16	6	6

France

Year	Total	$\tau \leq 1Y$	$1Y < \tau \leq 2Y$	$2Y < \tau \leq 5Y$	$5Y < \tau \leq 10Y$	$10Y < \tau \leq 20Y$	$\tau > 20$
2005	34	6	5	12	9	1	1
2006	32	5	5	12	8	1	1
2007	29	5	5	12	6	0	1
2008	27	5	5	13	3	0	1
2009	35	5	4	12	6	5	3
2010	42	4	4	12	11	7	4
2011	45	5	6	11	11	7	5
2012	41	4	4	11	11	6	5
2013	38	2	4	10	11	6	5
2014	42	5	3	11	13	5	5

Netherlands

Year	Total	$\tau \leq 1Y$	$1Y < \tau \leq 2Y$	$2Y < \tau \leq 5Y$	$5Y < \tau \leq 10Y$	$10Y < \tau \leq 20Y$	$\tau > 20$
2005	21	4	5	7	5	0	0
2006	19	4	3	7	4	1	0
2007	15	3	3	5	3	1	0
2008	13	3	3	4	2	1	0
2009	16	3	2	4	4	3	0
2010	20	2	2	5	5	4	2
2011	19	2	2	5	5	3	2
2012	18	2	2	5	4	3	2
2013	15	0	1	5	5	2	2
2014	18	2	1	6	5	2	2

Italy

Year	Total	$\tau \leq 1Y$	$1Y < \tau \leq 2Y$	$2Y < \tau \leq 5Y$	$5Y < \tau \leq 10Y$	$10Y < \tau \leq 20Y$	$\tau > 20$
2005	37	8	7	12	7	2	1
2006	33	6	6	10	7	3	1
2007	32	6	5	11	6	4	0
2008	30	4	5	11	6	4	0
2009	38	4	4	11	8	6	5
2010	42	4	5	11	10	7	5
2011	45	5	6	12	10	8	4
2012	38	4	4	10	8	8	4
2013	49	5	7	12	13	8	4
2014	46	5	6	10	14	7	4

Spain

Year	Total	$\tau \leq 1Y$	$1Y < \tau \leq 2Y$	$2Y < \tau \leq 5Y$	$5Y < \tau \leq 10Y$	$10Y < \tau \leq 20Y$	$\tau > 20$
2005	19	3	3	7	6	0	0
2006	16	2	3	6	5	0	0
2007	15	2	3	6	4	0	0
2008	14	3	2	8	1	0	0
2009	22	2	2	9	5	1	3
2010	28	2	4	9	7	2	4
2011	33	4	4	9	8	4	4
2012	29	3	3	9	6	4	4
2013	29	1	4	8	7	6	3
2014	32	2	4	8	7	7	4

Portugal

Year	Total	$\tau \leq 1Y$	$1Y < \tau \leq 2Y$	$2Y < \tau \leq 5Y$	$5Y < \tau \leq 10Y$	$10Y < \tau \leq 20Y$	$\tau > 20$
2005	12	2	2	4	4	0	0
2006	11	2	2	4	3	0	0
2007	9	2	1	4	2	0	0
2008	7	1	1	4	1	0	0
2009	10	1	2	3	3	0	1
2010	11	2	1	3	4	0	1
2011	12	1	1	4	5	0	1
2012	11	0	1	4	4	1	1
2013	12	1	1	4	3	2	1
2014	13	2	1	4	4	1	1

Table A.1: Annual distribution of (average) number of government bonds issued by the EMU countries Germany, France, Italy, the Netherlands, Spain and Portugal along the maturity spectrum between 2005 and 2014.

A.3 NSS term structure estimation

Based on the time t price $P(t,T)$ of a coupon-bond with time to maturity $\tau = T - t$ and a nominal (scaled) to 1:

$$P\left(t,T\right)=\frac{c}{\left(1+y(t,t_1)\right)^{(t_1-t)}}+\frac{c}{\left(1+y(t,t_2)\right)^{(t_2-t)}}+...\frac{c}{\left(1+y(t,t_{n-1})\right)^{(t_{n-1}-t)}}+\frac{(1+c)}{\left(1+y(t,T)\right)^{(T-t)}} \tag{A.1}$$

where T and c are the bond's redemption date and its coupon rate with coupon payments at the n dates $t_1, t_2, ..., t_{n-1}, T$ with $t < t_1 < t_2 < ... < t_{n-1} < T$. For discounting the bond's cash flow on the RHS of A.1 there are n discount factors $d(t,t_i) = 1/\left(1+y(t,t_i)\right)^{t_i-t}$ with $i = 1,2,...,n$ and $t_n = T$. $y(t,t_i)$ is the zero-coupon rate with time to maturity $\tau_i = t_i - t$. These rates are not directly observable. For extracting the zero-coupon rates from the county's bond prices we use the Nelson-Siegel-Svensson (NSS) parametric approach by Svensson [1994, 1995] which extends the approach originally formulated by Nelson and Siegel [1987] and is widely used among central banks and international institutions BIS [2005]. According to the NSS approach a zero-coupon rate $y(t,t_i,\boldsymbol{\theta})$ with time to maturity $\tau = t_i - t$ depends on the 6×1 vector of parameters $\boldsymbol{\theta}^T = [\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]$ and is defined as:

$$\begin{aligned} y(t,T,\boldsymbol{\theta}) = \beta_0 &+ \beta_1 \frac{\left(1-exp\left(-\frac{(T-t)}{\tau_1}\right)\right)}{\frac{(T-t)}{\tau:1}} + \beta_2\left[\frac{\left(1-exp\left(-\frac{(T-t)}{\tau_1}\right)\right)}{\frac{(T-t)}{\tau_1}} - exp\left(-\frac{(T-t)}{\tau_1}\right)\right] \\ &+ \beta_3\left[\frac{\left(1-exp\left(-\frac{(T-t)}{\tau_2}\right)\right)}{\frac{(T-t)}{\tau_2}} - exp\left(-\frac{(T-t)}{\tau_2}\right)\right] \end{aligned} \tag{A.2}$$

From A.2 it becomes clear that with $\boldsymbol{\theta}$ the zero-coupon rates of every maturity $\tau = T - t$ can be calculated, receiving the term structure of interest rates over a whole spectrum of various maturities. For estimating the parameter vector $\boldsymbol{\theta}$ at time t we use the bond prices of the N government bonds issued by the considered country at t. Depending on the coupon dates of the N government bonds t_{i_j}, where i_j is the index of the $i_j = 1,2,...,n_j$ coupon payments for the $j = 1,2,...,N$ government bonds, we define for the M different dates t_{i_j} for all $j = 1,2,...,N$ bonds - increasingly sorted and re-indexed $t_1 \le t_2 \le ... \le t_M$ - the $N \times M$ matrices $\mathbf{D}_c(\boldsymbol{\theta})$ and $\mathbf{D}_R(\boldsymbol{\theta})$. Every element $d_c\left(\boldsymbol{\theta}\right)_{j,m}$ of $\mathbf{D}_c(\boldsymbol{\theta})$ is defined as:

$$d_c\left(\boldsymbol{\theta}\right)_{j,m} = \begin{cases} 1/\left(1+y(t,t_m,\boldsymbol{\theta})\right)^{(t_m-t)} & if \;\; \exists t_{i_j} = t_m \;\; \forall i_j \neq n_j \\ 0 & else \end{cases} \tag{A.3}$$

whereas the elements $d_R\left(\boldsymbol{\theta}\right)_{j,m}$ of $\mathbf{D}_R(\boldsymbol{\theta})$ are defined as:

$$d_R\left(\boldsymbol{\theta}\right)_{j,m} = \begin{cases} 1/\left(1+y(t,t_m,\boldsymbol{\theta})\right)^{(t_m-t)} & if \;\; t_{n_j} = t_m \\ 0 & else \end{cases} \tag{A.4}$$

such that $\mathbf{D}_c(\boldsymbol{\theta})$ and $\mathbf{D}_R(\boldsymbol{\theta})$ are the matrices of discount factors at the bonds coupon and redemption dates respectively. With $\mathbf{D}_c(\boldsymbol{\theta})$, $\mathbf{D}_R(\boldsymbol{\theta})$ and the bond price equation A.1 we get the NSS bond prices:

$$\mathbf{P}_t\left(\boldsymbol{\theta}\right) = \tilde{\mathbf{D}}_c(\boldsymbol{\theta})\tilde{\boldsymbol{c}} + \tilde{\mathbf{D}}_R(\boldsymbol{\theta})\boldsymbol{e}_{NM} \tag{A.5}$$

where $\mathbf{P}_t(\boldsymbol{\theta})$ is the $N \times 1$ vector of bond prices implied by the NSS approach depending on $\boldsymbol{\theta}$. $\tilde{\mathbf{D}}_c(\boldsymbol{\theta}) = \sum_{j=1}^{N} \boldsymbol{\delta}_t^T \otimes \boldsymbol{\delta}_t \boldsymbol{\delta}_t^T \mathbf{D}_c(\boldsymbol{\theta})$ and $\tilde{\mathbf{D}}_R(\boldsymbol{\theta}) = \sum_{j=1}^{N} \boldsymbol{\delta}_t^T \otimes \boldsymbol{\delta}_t \boldsymbol{\delta}_t^T \mathbf{D}_R(\boldsymbol{\theta})$ are $N \times NM$ matrices. $\tilde{\boldsymbol{c}} = \sum_{j=1}^{N} \boldsymbol{\delta}_j \otimes c_j \boldsymbol{e}_M$ is an $NM \times 1$ vector and $\boldsymbol{\delta}_j$ is an $N \times 1$ vector with 1 at position j and 0 else. $\boldsymbol{e}_M$ and $\boldsymbol{e}_{NM}$ are $M \times 1$ and $NM \times 1$ unit vectors respectively. With the system of NSS prices in A.5 we get the estimate $\hat{\boldsymbol{\theta}}$ by:

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \sqrt{||\mathbf{P}_t(\boldsymbol{\theta}) - \mathbf{P}_t||} \tag{A.6}$$

where $||\ ||$ denotes the Euclidian norm and $\mathbf{P}_t$ is the $N \times 1$ vector containing the government bond prices observed at time t. We minimize the price differences in A.6 by using the heuristic differential-evolution algorithm developed by Storn and Price [1997] for solving larger non-linear optimization problems.

A.4 Equilibrium conditions of the decision problems

A.4.1 Final goods sector decisions

The final good Y_t is composed of a continuum of intermediate goods $Y_t(i)$ produced in the sector of intermediate goods i, which is outlined in its organization in the next section. The final goods producers sell their products to consumers, investors and the government and act as price takers in a perfectly competitive market, where they face the following profit maximization problem with respect to the decision about the amount Y_t to sell on the market for final goods and the amount $Y_t(i)$ to buy from the intermediate producers i:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \tag{A.7}$$

subject to the final goods production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{(1+\varepsilon_t^p)}} di \right)^{(1+\varepsilon_t^p)} \tag{A.8}$$

P_t and $P_t(i)$ are the prices in the final and intermediate goods sectors respectively. ε_t^P is a price mark-up shock which influences the production process of the final goods producers and is specified as:

$$ln\left(\varepsilon_t^p\right) = (1-\rho) ln\left(\epsilon_p\right) \rho_P ln\left(\varepsilon_{t-1}^p\right) + \sigma_p \epsilon_t^p \quad \epsilon_t^p \sim N\left(0, 1\right) \tag{A.9}$$

The final goods producers Lagrange function is given by:

$$\Lambda\left(Y_t, Y_t(i), \lambda_t^{FG}(i)\right) = P_t Y_t - \int_0^1 P_t(i) Y_t(i) di + \lambda_t^{FG}\left[\left(\int_0^1 Y_t(i)^{\frac{1}{(1+\varepsilon_t^p)}} di\right)^{\left(1+\varepsilon_t^p\right)} - Y_t\right] \quad \text{(A.10)}$$

where the optimization problem's FOCs are:

$$\partial\Lambda/\partial Y_t : \qquad \lambda_t^{FG} = P_t \quad \text{(A.11)}$$

$$\partial\Lambda/\partial Y_t(i) : \qquad P_t(i) = \lambda_t^{FG}\left(\int_0^1 Y_t(i)^{\frac{1}{(1+\varepsilon_t^p)}} di\right)^{\varepsilon_t^p} Y_t(i)^{-\frac{\varepsilon_t^p}{(1+\varepsilon_t^p)}} \quad \text{(A.12)}$$

where λ_t^{FG} is the problem's Lagrangian multiplier. Combining both FOCs yields the final goods producers demand for intermediate goods $Y_t(i)$:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1-\varepsilon_t^p)}{\varepsilon_t^p}} Y_t \quad \text{(A.13)}$$

The aggregate price level under the zero profit condition $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di = 0$ with respect to the optimal demand $Y_t(i)$ then becomes:

$$P_t = \left(\int_0^1 Y_t(i)^{\frac{1}{\varepsilon_t^p}} di\right)^{\varepsilon_t^p} \quad \text{(A.14)}$$

A.4.2 Intermediate goods sector decisions

At every time t the intermediate goods producers i have to solve the following profit maximization problem:

$$\max_{Y_t(i), L_t(i), K_t^s(i)} P_t(i) Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i) \quad \text{(A.15)}$$

subject to i th intermediate producers used production technology:

$$Y_t(i) = \varepsilon_t^a K_t^s(i)^{\alpha}\left(\gamma^t L_t(i)\right)^{(1-\alpha)} - \gamma^t \phi \quad \text{(A.16)}$$

where the production factors are the capital service used in the economy's production process $K_t^s(i)$ and labour $L_t(i)$. W_t and R_t^k are the aggregated nominal wage and the rental rate on capital.γ^t is the labour augmented deterministic growth rate of the economy and ϕ is a general fixed cost factor which negatively effects the production process. The production process in the intermediate sector is disturbed by an exogenous log-normal process:

$$ln\left(\varepsilon_t^a\right) = \rho_a ln\left(\varepsilon_{t-1}^a\right) + \sigma_a \epsilon_t^a \quad \text{(A.17)}$$

where $\epsilon_t^a \sim N(0,1)$ is standard normal. The decision problem's Lagrange function is:

$$\begin{aligned}\Lambda\left(Y_t(i), L_t(i), K_t^s(i), \lambda_t^{IG}(i)\right) = {} & P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i) \\ & + \lambda_t^{IG}(i)\left(\varepsilon_t^a K_t^s(i)^\alpha \left(\gamma^t L_t(i)\right)^{(1-\alpha)} - \gamma^t \phi - Y_t(i)\right)\end{aligned} \tag{A.18}$$

where the FOCs are:

$$\partial\Lambda/\partial Y_t(i): \quad P_t(i) = \lambda_t^{IG}(i) \tag{A.19}$$

$$\partial\Lambda/\partial L_t(i): \quad \lambda_t^{IG}(i)\gamma^{(1-\alpha)t}(1-\alpha)\varepsilon_t^a K_t^s(i)^\alpha L_t(i)^{-\alpha} = W_t \tag{A.20}$$

$$\partial\Lambda/\partial K_t^s(i): \quad \lambda_t^{IG}(i)\gamma^{(1-\alpha)t}\alpha\varepsilon_t^a K_t^s(i)^{(\alpha-1)} L_t(i)^{1-\alpha} = R_t^k \tag{A.21}$$

In a competitive market where the firms set their prices equal to their marginal costs from the FOC in A.19 it is obvious that the Lagrangian multiplier λ_t^{IG} equals the marginal costs of production. Reformulating the FOC in A.21 with respect to λ_t^{IG} leads to:

$$\lambda_t^{IG}(i) = R_t^k \gamma^{-(1-\alpha)t}\alpha^{-1}\left(\varepsilon_t^a\right)^{-1} K_t^s(i)^{-(\alpha-1)} L_t(i)^{(1-\alpha)} \tag{A.22}$$

Setting $\lambda_t^{IG}(i)$ into FOC A.20 we get the capital-to-labor ratio:

$$K_t^s = \frac{\alpha}{(1-\alpha)}\frac{W_t}{R_t^k}L_t \tag{A.23}$$

which is equal to all firms, such that the producer's index i drops out. With respect to the capital-to-labor constant for all firms, the marginal costs of production in the intermediate sector λ_t^{IG} become:

$$\lambda_t^{IG} = \gamma^{-(1-\alpha)t}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{(1-\alpha)}R_t^{k\,\alpha}\left(\varepsilon_t^a\right)^{-1} \tag{A.24}$$

Price setting in the intermediate sector faces nominal rigidities. We consider price-setting as proposed by Calvo [1983] where only a fraction $(1-\xi_P)$ with $0 \le \xi_P \le 1$ of contracts expire each period and are renegotiated by the participants. The renegotiating firms set their prices according to their optimal nominal price. All other firms set their prices according to:

$$P_{t+s}(i) = \tilde{P}_t(i)X_{t,s} \tag{A.25}$$

where

$$X_{t,s} = \begin{cases} 1 \quad for \quad s = 0 \\ \prod_{m=1}^{s}\gamma\pi_{t+m-1}^{\iota_p}\bar{\pi}^{(1-\iota_p)} \quad \forall s \in \{1,2,...,\infty\} \end{cases} \tag{A.26}$$

so that the fraction ξ_p of firms in the intermediate sector, which are not part of the renegotiations passively adjust their prices according to a weighted average of the steady-state inflation rate $\bar{\pi}$, last period's inflation rate π_{t-1} and the general growth rate γ of the economy. The prices setting of the producers in the intermediate sector is described by the following optimization problem:

$$\max_{\tilde{P}_t(i)} \Lambda\left(\tilde{P}_t(i)\right) = \max_{\tilde{P}_t(i)} \mathbb{E}_t\left[\frac{\xi_p^s \beta^s \lambda_{t+s}^{HI} P_t}{\lambda_t^{HI} P_{t+s}}\left(\tilde{P}_t(i) X_{t,s} - \lambda_{t+s}^{IG}\right) Y_{t+s}(i)\right] \tag{A.27}$$

subject to the final goods producers optimal demand for intermediate goods:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1-\varepsilon_t^p)}{\varepsilon_t^p}} Y_t \tag{A.28}$$

in A.13.

From the intermediate producer's objective function $\Lambda\left(\tilde{P}_t(i)\right)$ the FOC is:

$$\begin{aligned}\partial\Lambda/\partial\tilde{P}_t(i): \quad & \mathbb{E}_t\left[\frac{\xi_p^s \beta^s \lambda_{t+s}^{HI} P_t}{\lambda_t^{HI} P_{t+s}} X_{t,s} Y_{t+s}(i) Y_{t+s}\right] \\ & - \mathbb{E}_t\left[\frac{(1+\varepsilon_t^p)}{\varepsilon_t^p}\left(\tilde{P}_t(i) X_{t,s} - \lambda_t^{IG}\right)\left(\frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}}\right)^{\frac{(1+\varepsilon_t^p)}{\varepsilon_t^p}} \frac{X_{t,s}}{P_{t+s}} Y_{t+s}\right] = 0\end{aligned} \tag{A.29}$$

With the outlined price setting the aggregate price level becomes:

$$P_t = \left[(1-\xi_p)\,\tilde{P}_t^{\frac{1}{\varepsilon_t^p}} + \xi_p\left(\gamma \pi_{t-1}^{\iota_p} \bar{\pi}^{(1-\iota_p)} P_{t-1}\right)^{\frac{1}{\varepsilon_t^p}}\right]^{\varepsilon_t^p} \tag{A.30}$$

A.4.3 Household decisions

At every time t household j faces the following utility maximization problem:

$$\max_{C_t(j), L_t(j), B_t(j), I_t(j), Z_t(J)} U\left(C_t(j), C_{t-1}(j), L_t(j)\right) \tag{A.31}$$

where the household's time t maximization problem is embedded in the intertemporal optimization problem of maximizing the expected utility:

$$\mathbb{E}_t\left[\sum_{h=0}^{\infty} \beta^h U\left(C_{t+h}(j), C_{t+h-1}(j), L_{t+h}(j)\right)\right] \tag{A.32}$$

with the household's utility function specified as:

$$U\left(C_{t+h}(j), C_{t+h-1}(j), L_{t+h}(j)\right) = \frac{\left(C_{t+h}(j) - \lambda C_{t+h-1}(j)\right)^{(1-\sigma_c)}}{(1-\sigma_c)} exp\left(\frac{(\sigma_c - 1)}{(1+\sigma_l)} L_{t+h}(j)^{(1+\sigma_l)}\right) \tag{A.33}$$

For this maximization problem the following two constraints hold:

$$\begin{aligned} C_{t+h}(j) + I_{t+h}(j) + \frac{B_{t+h}(j)}{\varepsilon_t^b R_{t+h} P_{t+h}} - T_{t+h} \leq & \frac{B_{t+h-1}(j)}{P_{t+h}} + \frac{W_{t+h}(j) L_{t+h}(j)}{P_{t+h}} \\ & + \frac{R_{t+h}^k(j) Z_{t+h}(j) K_{t+h-1}(j)}{P_{t+h}} \\ & - a\left(Z_{t+h}(j)\right) K_{t+h-1}(j) + \frac{D_{t+h}}{P_{t+h}} \end{aligned} \tag{A.34}$$

$$K_{t+h}(j) = (1-\delta) K_{t+h-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_{t+h}(j)}{I_{t+h-1}(j)}\right)\right] I_{t+h}(j) \tag{A.35}$$

where the first constraint is the household's budget restriction with respect to the household's consumption C_{t+h}, investment I_{t+h} and saving behavior (netted by regarding the lump sum tax T_{t+h}) on the one side and the income cash flows from saving, labor, direct capital investments and dividends D_{t+h} on the other side. Saving is done by buying one period bonds B_{t+h} with yield R_{t+h} . Labor income is determined by the working hours L_{t+h} and wage W_{t+h}. Capital income is determined by the effective capital service $K_{t+h}^s = Z_{t+h} K_{t+h-1}$ directly used in the production process and the cost of capital utilization $a\left(Z_{t+h}\right) K_{t+h-1}$. Z_{t+h} indicates the degree of the economy's capital utilization. The second constraint is the capital accumulation equation. δ is the depreciation ratio of capital and S (...) is the adjustment cost function, indicating the fraction of investment $S(...)I_{t+h}$ necessary to increase the economy's capital stock by the investments I_{t+h} done at time $t+h$ for some $h \geq 0$.The time t Lagrange function of the optimization problem is:

$$\begin{aligned} & \Lambda\left(C_t(j), L_t(j), B_t(j), I_t(j), Z_t(j), \lambda_t^{H1}, \lambda_t^{H2}\right) = \\ & = \mathbb{E}_t \left[\sum_{h=0}^{\infty} \beta^h \frac{\left(C_{t+h}(j) - \lambda C_{t+h-1}(j)\right)^{(1-\sigma_c)}}{(1-\sigma_c)} exp\left(\frac{(\sigma_c - 1)}{(1+\sigma_l)} L_{t+h}(j)^{(1+\sigma_l)}\right)\right] \\ & + \mathbb{E}_t \left[\sum_{h=0}^{\infty} \beta^h \lambda_t^{H1} \left(\frac{B_{t+h-1}(j)}{P_{t+h}} + \frac{W_{t+h}(j) L_{t+h}(j)}{P_{t+h}} + \frac{R_{t+h}^k(j) Z_{t+h}(j) K_{t+h-1}(j)}{P_{t+h}}\right)\right] \\ & + \mathbb{E}_t \left[\sum_{h=0}^{\infty} \beta^h \lambda_t^{H1} \left(-a\left(Z_{t+h}(j)\right) K_{t+h-1}(j) + \frac{D_{t+h}}{P_{t+h}} - C_{t+h}(j) - I_{t+h}(j) - \frac{B_{t+h}(j)}{\varepsilon_t^b R_{t+h} P_{t+h}} + T_{t+h}\right)\right] \\ & + \mathbb{E}_t \left[\sum_{h=0}^{\infty} \beta^h \lambda_t^{H2} \left((1-\delta) K_{t+h-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_{t+h}(j)}{I_{t+h-1}(j)}\right)\right] I_{t+h}(j) - K_{t+h}(j)\right)\right] \end{aligned} \tag{A.36}$$

with the Lagrangian multipliers λ_t^{H1} and λ_t^{H2} for the budget restriction and the capital accumulation constraint and $Q_t = \lambda_t^{H2}/\lambda_t^{H1}$ representing Tobin's Q. The FOCs are (note that in equilibrium all households j make the same choices, so that the index j can be dropped):

$$\partial\Lambda/\partial C_t : \lambda_t^{H1} = \left(C_t - \lambda C_{t-1}\right)^{-\sigma_c} exp\left(\frac{(\sigma_c - 1)}{(1+\sigma_l)} L_t^{(1+\sigma_l)}\right) \tag{A.37}$$

$$\partial\Lambda/\partial L_t : -\lambda_t^{H1}\frac{W_t}{P_t} = \frac{\left(C_t - \lambda C_{t-1}\right)^{(1-\sigma_c)}}{(1-\sigma_c)} exp\left(\frac{(\sigma_c - 1)}{(1+\sigma_l)} L_t^{(1+\sigma_l)}\right)(\sigma_c - 1) L_t^{\sigma_l} \tag{A.38}$$

$$\partial\Lambda/\partial B_t : \lambda_t^{H1} = \beta\varepsilon_t^b R_t \, \mathbb{E}_t\left[\frac{\lambda_{t+1}^{H1} P_t}{P_{t+1}}\right] \tag{A.39}$$

$$\partial\Lambda/\partial I_t : \lambda_t^{H1} = \lambda_t^{H2}\varepsilon_t^i\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \beta\,\mathbb{E}_t\left[\lambda_{t+1}^{H2}\varepsilon_{t+1}^i S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{A.40}$$

$$\partial\Lambda/\partial K_t : \lambda_t^{H2} = \beta\,\mathbb{E}_t\left[\lambda_{t+1}^{H1}\left(\frac{R_{t+1}^k}{P_{t+1}} Z_{t+1} - a\left(Z_{t+1}\right)\right) + \lambda_{t+1}^{H2}\left(1-\delta\right)\right] \tag{A.41}$$

$$\partial\Lambda/\partial Z_t : a'\left(Z_t\right) = \frac{R_t^k}{P_t} \tag{A.42}$$

where the processes of the exogenous shocks ε_t^b and ε_t^i are specified as:

$$ln\left(\varepsilon_t^b\right) = \rho_b ln\left(\varepsilon_{t-1}^b\right) + \sigma_b\epsilon_t^b \qquad \epsilon_t^b \sim N(0,1) \tag{A.43}$$

$$ln\left(\varepsilon_t^i\right) = \rho_i ln\left(\varepsilon_{t-1}^i\right) + \sigma_i\epsilon_t^i \qquad \epsilon_t^i \sim N(0,1) \tag{A.44}$$

A.4.4 Labor market decisions

Demand and supply side of the labour market are organized as follows. The supply side consists of differentiated labour services $L_t(l)$ offered by the households. On the demand side there are the intermediate goods producer which are confronted with the various labour services. For reducing the complexity resulting from the labor fragmentation, there are labor packers as intermediaries between the households and the goods producers. The labor packers bundled the differentiated labor services to labor service packages L_t and offer them to the producers in the intermediate goods sector. For negotiation purposes every labor service l is represented by a union which negotiates their wages with the labor packers. Labor packers act profit orientated and therefore face the following profit maximization problem:

$$\max_{L_t, L_t(j)} W_t L_t - \int_0^1 L_t(l) W_t(l) dl \tag{A.45}$$

subject to:

$$L_t = \left(\int_0^1 L_t(l)^{\frac{1}{(1+\varepsilon_t^w)}} dl \right)^{(1+\varepsilon_t^w)} \tag{A.46}$$

where the exogenous shock process of ε_t^w is specified as:

$$ln\left(\varepsilon_t^w\right) = (1-\rho_w)ln\left(\varepsilon_w\right) + \rho_w ln\left(\varepsilon_{t-1}^w\right)\sigma_w \epsilon_t^w \qquad \epsilon_t^w \sim N(0,1) \tag{A.47}$$

so that the labor packer's Lagrange function is:

$$\Lambda\left(L_t, L_t(l), \lambda_t^{LP}\right) = W_t L_t - \int_0^1 L_t(l) W_t(l) dl + \lambda_t^{LP}\left[\left(\int_0^1 L_t(l)^{\frac{1}{(1+\varepsilon_t^w)}} dl \right)^{(1+\varepsilon_t^w)} - L_t\right] \tag{A.48}$$

with the FOCs:

$$\partial\Lambda/\partial L_t : \lambda_t^{LP} = W_t \tag{A.49}$$

$$\partial\Lambda/\partial L_t(l) : W_t(l) = \lambda_t^{LP} L_t(l)^{-\frac{\varepsilon_t^w}{(1+\varepsilon_t^w)}} \left(\int_0^1 L_t(l)^{\frac{1}{(1+\varepsilon_t^w)}} dl \right)^{\varepsilon_t^w} \tag{A.50}$$

Taking into account $L_t^{\frac{1}{(1+\varepsilon_t^w)}} dl = \int_0^1 L_t(l)^{\frac{1}{(1+\varepsilon_t^w)}} dl$ differentiated labor $L_t(l)$ then becomes:

$$L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\frac{(1+\lambda_{w,t})}{\varepsilon_t^w}} L_t \tag{A.51}$$

Assuming perfect competition between the labor packers so that the packer's profit equals zero:

$$W_t L_t - \int_0^1 L_t(l) W_t(l) dl = 0 \tag{A.52}$$

the wage costs for the intermediate goods producers are:

$$W_t = \left(\int_0^1 W_t(l)^{\frac{1}{\varepsilon_t^w}} \right)^{\varepsilon_t^w} \tag{A.53}$$

In their wage negotiations labor unions face nominal wage rigidities. Union's wage negotiations are described by using a Calvo scheme with partial indexation, where $(1-\xi_w)$ with $0 \le \xi_w \le 1$ labor unions can actively readjust their wages and set them to $\tilde{W}_t(l)$ each period. On the contrary this implies that ξ_w unions do not readjust their wages. They passively set their prices $W_t(l)$ according to the growth rate γ and a weighted average of the steady-state inflation rate $\bar{\pi}$ and last period's inflation rate π_{t-1}, so that the wage setting decision between labor unions and labor packers is determined by:

$$\max_{\tilde{W}_t(l)} \Lambda\left(\tilde{W}_t(l)\right) = \max_{\tilde{W}_t(l)} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \frac{\xi_w^s \beta^s \lambda_{t+s}^{HI} P_t}{\lambda_t^{HI} P_{t+s}} \left(\tilde{W}_{t+s}(i) - W_{t+s} \right) L_{t+s} \right] \tag{A.54}$$

with the labor packers optimal demand for differentiated labor services:

$$L_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\frac{(1+\lambda_{w,t})}{\varepsilon_t^w}} L_t \tag{A.55}$$

and the mentioned passive price-setting rule with respect to growth and inflation:

$$W_{t+s}(l) = X_{t,s}\tilde{W}_t(l) \tag{A.56}$$

with:

$$X_{t,s} = \begin{cases} 1 \quad for \quad s = 0 \\ \prod_{m=1}^{s} \gamma\pi_{t+m-1}^{\iota_w}\bar{\pi}^{(1-\iota_w)} \quad \forall s \in \{1, 2, ..., \infty\} \end{cases} \tag{A.57}$$

the labour unions FOC derived from the problem's objective function $\Lambda\left(\tilde{W}_t(l)\right)$ is:

$$\partial\Lambda/\partial\tilde{W}_t(l):$$

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\xi_w\frac{\beta^s\lambda_{t+1}^{H1}P_t}{\lambda_t^{H1}P_{t+1}}\left[X_{t,s}L_{t+s}(l) - \frac{(1+\varepsilon_t^w)}{\varepsilon_t^w}\frac{X_{t,s}}{W_{t+s}(l)}L_{t+s}(l)\left(W_{t+s}(l) - W_{t+s}\right)\left(\frac{X_{t,s}\tilde{W}_t(l)}{W_{t+s}(l)}\right)^{\left(\frac{1-\varepsilon_{t+s}^w}{\varepsilon_{t+s}^w}-1\right)}\right]\right] = 0 \tag{A.58}$$

The economy's aggregated wage index with respect to actively and passively set wages under the Calvo scheme and the zero profit assumption for the labor packers is given by:

$$W_t = \left[(1-\xi_w)\tilde{W}_t^{\frac{1}{\varepsilon_t^w}} + \xi_w\left(\gamma\pi_{t-1}^{\iota_w}\bar{\pi}^{(1-\iota_w)}W_{t-1}\right)^{\frac{1}{\varepsilon_t^w}}\right]^{\varepsilon_t^w} \tag{A.59}$$

A.4.5 Monetary policy and fiscal policy decisions

Similar to current work by Andreasen, Fernandez-Villaverde and Rubio-Ramirez [2017] and deviating from approaches used by DeGreave, Emiris and Wouters [2007], Rudebusch and Swanson [2008, 2012], Beakert, Cho and Moreno [2010], van Binsenberg, Fernandez-Villaverde, Koijen and Rubio-Ramirez [2012] or Kliem and Meyer-Gohde [2017] for integrating the term structure of interest rates into the macroeconomic modeling framework of the DSGE we specify a Taylor rule type monetary policy decision rule in which the central bank sets the short term rate dependent to the inflation and output gap, the change in the output gap and to a term structure component. Andreasen, Fernandez-Villaverde and Rubio-Ramirez [2017] use as a term structure component a measure for the term-premia priced in the term structure. In our paper in using only first order approximation for solving our model, we try to keep the solution and computation effort as low as possible. Disadvantage of using only a first order approximation is that we cannot focus on time-varying risk-premia because of the lack of non-linearities necessary for the modeling of time-varying term-premia. So instead of focusing on term-premia we deviate from Andreasen, Fernandez-Villaverde and Rubio-Ramirez [2017] in using three termstructure factors, which are the driving forces of our yield curve as the term structure related component in our central bank's monetary policy decision rule. The central bank's decision rule is as follows:

$$\begin{aligned} r_t = \rho r_{t-1} - \rho \tilde{r} + (1-\rho)\left(r_\pi ln\left(\frac{\pi_t}{\tilde{\pi}}\right) + r_y\left(\frac{y_t}{\tilde{y}_t}\right)\right) + r_{\Delta y} ln\left(\frac{(y_t/\tilde{y}_t)}{(y_{t-1}/\tilde{y}_{t-1})}\right) \\ + \omega_l f_{l,t} + \omega_s f_{s,t} + \omega_c f_{c,t} + ln\left(\varepsilon_t^r\right) \end{aligned} \tag{A.60}$$

where $\tilde{r}$ and $\tilde{\pi}$ are short term interest rate and inflation rate in the steady-state. $\tilde{y}$ is the potential output under full price and wage flexibility. π_t and y_t are inflation rate and output respectively and $f_{l,t}, f_{s,t}, f_{c,t}$ are the three latent term structure factors the central bank focuses on in its decision finding.

$$ln\left(\varepsilon_t^r\right) = \rho_r ln\left(\varepsilon_{t-1}^r\right) + \sigma_r \epsilon_t^r \qquad \epsilon_t^r \sim N(0,1) \tag{A.61}$$

is the log-normally distributed monetary policy shock. The government faces in its fiscal policy decisions the following budget constraint:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \tag{A.62}$$

where the LHS indicates the government's expenditures for general public sector activities and debt redemption, whereas the RHS indicates the government's revenues from taxes and credit. The government expenditures G_t are described by the random process:

$$G_t = \varepsilon_t^g \tag{A.63}$$

with:

$$ln\left(\varepsilon_t^g\right) = \rho_g ln\left(\varepsilon_{t-1}^g\right) + \sigma_g \epsilon_t^g \qquad \epsilon_t^g \sim N(0,1) \tag{A.64}$$

A.5 SW-DSGE-ATSM implementation

A.5.1 SW-DSGE-ATSM under price and wage flexibility

Under full price and wage flexibility determined by $\xi_p = \xi_w = 0$ and in the absence of the price and wage mark-up disturbances ε_t^p and ε_t^w the equations of the SW-2007 DSGE model become:

$$\tilde{y}_t = c_y \tilde{c}_t + i_y \tilde{i}_t + z_y \tilde{z}_t + \varepsilon_t^g \tag{A.65}$$

$$\tilde{i}_t = \frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\tilde{i}_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\mathbb{E}_t\left[\tilde{i}_{t+1}\right] + \frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi}\tilde{q}_t + \varepsilon_t^i \tag{A.66}$$

$$\tilde{q}_t = \beta(1-\delta)\gamma^{-\sigma_c}\mathbb{E}_t[\tilde{q}_{t+1}] - r_t + \mathbb{E}_t[\tilde{\pi}_{t+1}] + \left(1-\beta(1-\delta)^{-\sigma_c}\right)\mathbb{E}_t\left[\tilde{r}^k_{t+1}\right] - \varepsilon^b_t \tag{A.67}$$

$$\tilde{y}_t = \Phi\left(\alpha\tilde{k}^s_t + (1-\alpha)\tilde{l}_t + \varepsilon^a_t\right) \tag{A.68}$$

$$\tilde{k}^s_t = \tilde{k}_{t-1} + \tilde{z}_t \tag{A.69}$$

$$\tilde{z}_t = z_t\tilde{r}^k_t \tag{A.70}$$

$$\tilde{k}_t = \frac{(1-\delta)}{\gamma}\tilde{k}_{t-1} + \frac{(\gamma-1+\delta)}{\gamma}\left(\tilde{i}_t + \left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi\varepsilon^i_t\right) \tag{A.71}$$

$$\tilde{w}_t = a\left(\tilde{k}^s_t - \tilde{l}_t\right) + \varepsilon^a_t \tag{A.72}$$

$$\tilde{r}^k_t = \tilde{l}_t + \tilde{w}_t + \tilde{k}_t \tag{A.73}$$

$$\tilde{w}_t = \sigma_c\tilde{l}_t + \frac{1}{1-\lambda/\gamma}(\tilde{c}_t - \lambda/\gamma\tilde{c}_{t-1}) \tag{A.74}$$

A.5.2 Canonical rational expectations form of the SW-DSGE-ATSM

The matrices $\mathbf{\Gamma}_0, \mathbf{\Gamma}_1, \mathbf{\Psi}$ and $\mathbf{\Pi}$ of the canonical linear rational expectations form in 2.26 are row-wise specified as follows (where all other elements in the rows are set to zero):

$$\mathbf{\Gamma}_0[1,1] = 1, \mathbf{\Gamma}_0[1,2] = -c_y, \mathbf{\Gamma}_0[1,3] = -i_y, \mathbf{\Gamma}_0[1,6] = -z_y, \mathbf{\Gamma}_0[1,17] = -1$$

$$\mathbf{\Gamma}_0[2,2] = 1, \mathbf{\Gamma}_0[2,14] = -\frac{wl_c(\sigma_c-1)}{\sigma_c(1+\lambda/\gamma)}, \mathbf{\Gamma}_0[2,22] = -\frac{1}{1+\lambda/\gamma}, \mathbf{\Gamma}_0[2,24] = \frac{wl_c(\sigma_c-1)}{\sigma_c(1+\lambda/\gamma)}$$

$$\mathbf{\Gamma}_0[2,13] = \frac{(1-\lambda/\gamma)}{\sigma_c(1+\lambda/\gamma)}, \mathbf{\Gamma}_0[2,25] = \frac{(1-\lambda/\gamma)}{\sigma_c(1+\lambda/\gamma)}, \Gamma_0[2,16] = \frac{(1-\lambda/\gamma)}{\sigma_c(1+\lambda/\gamma)}$$

$$\mathbf{\Gamma}_1[2,2] = \frac{\lambda/\gamma}{1+\lambda/\gamma}$$

$$\mathbf{\Gamma}_0[3,3] = 1, \mathbf{\Gamma}_0[3,23] = -\frac{\beta\gamma^{(1-\sigma_c)}}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}, \mathbf{\Gamma}_0[3,4] = -\frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi}, \mathbf{\Gamma}_0[3,18] = -1$$

$$\mathbf{\Gamma}_1[3,3] = \frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}$$

$$\mathbf{\Gamma}_0[4,4] = 1, \mathbf{\Gamma}_0[4,26] = -\beta\left(1-\delta\right)\gamma^{-\sigma_c}, \mathbf{\Gamma}_0[4,27] = -\left(1-\beta\left(1-\delta\right)\gamma^{-\sigma_c}\right), \mathbf{\Gamma}_0[4,13] = 1,$$

$$\mathbf{\Gamma}_0[4,25] = -1, \mathbf{\Gamma}_0[4,16] = 1$$

$$\mathbf{\Gamma}_0[5,1] = 1, \mathbf{\Gamma}_0[5,5] = -\Phi\alpha, \mathbf{\Gamma}_0[5,14] = -\Phi(1-\alpha), \mathbf{\Gamma}_0[5,15] = -\Phi$$

$$\mathbf{\Gamma}_0[6,5] = 1, \mathbf{\Gamma}_0[6,6] = -1$$

$$\mathbf{\Gamma}_1[6,7] = 1$$

$$\mathbf{\Gamma}_0[7,6] = 1, \mathbf{\Gamma}_0[7,10] = -\frac{(1-\psi)}{\psi}$$

$$\mathbf{\Gamma}_0[8,7] = 1, \mathbf{\Gamma}_0[8,3] = -\frac{(\gamma-1+\delta)}{\gamma}, \mathbf{\Gamma}_0[8,18] = -\left(\gamma-1+\delta\right)\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma\psi,$$

$$\mathbf{\Gamma}_1[8,7] = \frac{(1-\delta)}{\gamma}$$

$$\mathbf{\Gamma}_0[9,8] = 1, \mathbf{\Gamma}_0[9,5] = -\alpha, \mathbf{\Gamma}_0[9,14] = \alpha, \mathbf{\Gamma}_0[9,15] = -1, \mathbf{\Gamma}_0[9,12] = 1$$

$$\mathbf{\Gamma}_0[10,9] = 1, \mathbf{\Gamma}_0[10,25] = -\frac{\beta\gamma^{(1-\sigma_c)}}{\left(1+\beta\gamma^{(1-\sigma_c)}\iota_p\right)}, \mathbf{\Gamma}_0[10,8] = \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_p\right)\left(1-\xi_p\right)}{\left(1+\beta\gamma^{(1-\sigma_c)}\iota_p\right)\xi_p\left(\left(\Phi-1\right)\varepsilon_p+1\right)}$$

$$\mathbf{\Gamma}_0[10,20] = -1$$

$$\mathbf{\Gamma}_1[10,9] = \frac{\iota_p}{(1+\beta\gamma^{(1-\sigma_c)}\iota_p)}$$

$$\mathbf{\Gamma}_0[11,10] = 1, \mathbf{\Gamma}_0[11,7] = 1, \mathbf{\Gamma}_0[11,14] = -1, \mathbf{\Gamma}_0[11,12] = -1$$

$$\mathbf{\Gamma}_0[12,11] = 1, \mathbf{\Gamma}_0[12,12] = -1, \mathbf{\Gamma}_0[12,14] = \sigma_l, \mathbf{\Gamma}_0[12,2] = \frac{1}{(1-\lambda)}$$

$$\mathbf{\Gamma}_1[12,2] = \frac{\lambda}{(1-\lambda)}$$

$$\mathbf{\Gamma}_0[13,12] = 1, \mathbf{\Gamma}_0[13,28] = \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})} - 1, \mathbf{\Gamma}_0[13,25] = 1 - \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})},$$

$$\mathbf{\Gamma}_0[13,9] = \frac{\left(1+\beta\gamma^{(1-\sigma_c)}\iota_w\right)}{(1+\beta\gamma^{(1-\sigma_c)})}, \mathbf{\Gamma}_0[13,11] = \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_w\right)(1-\xi_w)}{(1+\beta\gamma^{(1-\sigma_c)})\,\xi_w\left((\lambda_w i - 1)\,\varepsilon_w + 1\right)}, \mathbf{\Gamma}_0[13,21] = -1$$

$$\mathbf{\Gamma}_1[13,12] = \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})}, \mathbf{\Gamma}_1[13,9] = \frac{\iota_w}{(1+\beta\gamma^{(1-\sigma_c)})}$$

$$\mathbf{\Gamma}_0[14,13] = 1, \mathbf{\Gamma}_0[14,9] = (1-\rho)r_\pi, \mathbf{\Gamma}_0[14,1] = -\left((1-\rho)r_y + r_{\Delta y}\right)$$

$$\mathbf{\Gamma}_0[14,19] = -1, \mathbf{\Gamma}_0[14,29] = \left((1-\rho)r_y + r_{\Delta y}\right), \mathbf{\Gamma}_0[14,50] = \omega_l, \mathbf{\Gamma}_0[14,51] = \omega_s, \mathbf{\Gamma}_0[14,52] = \omega_c$$

$$\mathbf{\Gamma}_1[14,13] = \rho, \mathbf{\Gamma}_1[14,45] = -r_{\Delta y}, \mathbf{\Gamma}_1[14,49] = r_{\Delta y}$$

$$\mathbf{\Gamma}_0[15,15] = 1, \mathbf{\Gamma}_1[15,15] = \rho_a, \mathbf{\Psi}[15,1] = \sigma_a$$

$$\mathbf{\Gamma}_0[16,16] = 1, \mathbf{\Gamma}_1[16,16] = \rho_b, \mathbf{\Psi}[16,2] = \sigma_b$$

$$\mathbf{\Gamma}_0[17,17] = 1, \mathbf{\Gamma}_1[17,17] = \rho_g, \mathbf{\Psi}[17,3] = \sigma_g$$

$$\mathbf{\Gamma}_0[18,18] = 1, \mathbf{\Gamma}_1[18,18] = \rho_i, \mathbf{\Psi}[18,4] = \sigma_i$$

$$\mathbf{\Gamma}_0[19,19] = 1, \mathbf{\Gamma}_1[19,19] = \rho_r, \mathbf{\Psi}[19,5] = \sigma_r$$

$\mathbf{\Gamma}_0[20,20] = 1, \mathbf{\Gamma}_1[20,20] = \rho_p, \mathbf{\Psi}[20,6] = \sigma_p$

$\mathbf{\Gamma}_0[21,21] = 1, \mathbf{\Gamma}_1[21,21] = \rho_w, \mathbf{\Psi}[21,7] = \sigma_w$

$\mathbf{\Gamma}_0[22,2] = 1, \mathbf{\Gamma}_1[22,22] = 1, \mathbf{\Pi}[22,2] = 1$

$\mathbf{\Gamma}_0[23,3] = 1, \mathbf{\Gamma}_1[23,23] = 1, \mathbf{\Pi}[23,6] = 1$

$\mathbf{\Gamma}_0[24,14] = 1, \mathbf{\Gamma}_1[24,24] = 1, \mathbf{\Pi}[24,3] = 1$

$\mathbf{\Gamma}_0[25,9] = 1, \mathbf{\Gamma}_1[25,25] = 1, \mathbf{\Pi}[25,1] = 1$

$\mathbf{\Gamma}_0[26,4] = 1, \mathbf{\Gamma}_1[26,26] = 1, \mathbf{\Pi}[26,4] = 1$

$\mathbf{\Gamma}_0[27,10] = 1, \mathbf{\Gamma}_1[27,27] = 1, \mathbf{\Pi}[27,5] = 1$

$\mathbf{\Gamma}_0[28,12] = 1, \mathbf{\Gamma}_1[28,28] = 1, \mathbf{\Pi}[28,7] = 1$

$\mathbf{\Gamma}_0[29,45] = 1, \mathbf{\Gamma}_1[29,1] = 1$

$\mathbf{\Gamma}_0[30,46] = 1, \mathbf{\Gamma}_1[30,2] = 1$

$\mathbf{\Gamma}_0[31,47] = 1, \mathbf{\Gamma}_1[31,3] = 1$

$\mathbf{\Gamma}_0[32,48] = 1, \mathbf{\Gamma}_1[32,12] = 1$

$\mathbf{\Gamma}_0[33,49] = 1, \mathbf{\Gamma}_1[33,29] = 1$

$\mathbf{\Gamma}_0[34,29] = 1, \mathbf{\Gamma}_0[34,30] = -c_y, \mathbf{\Gamma}_0[34,31] = -i_y, \mathbf{\Gamma}_0[34,34] = -z_y, \mathbf{\Gamma}_0[34,17] = -1$

$\mathbf{\Gamma}_0[35,30] = 1, \mathbf{\Gamma}_0[35,40] = -\dfrac{1}{(1+\lambda/\gamma)}, \mathbf{\Gamma}_0[35,39] = -\dfrac{wl_c(\sigma_c - 1)}{\sigma_c\,(1+\lambda/\gamma)}, \Gamma_0[35,42] = \dfrac{wl_c(\sigma_c - 1)}{\sigma_c\,(1+\lambda/\gamma)}$

$$\mathbf{\Gamma}_0[35,38] = \frac{(1-\lambda/\gamma)}{\sigma_c\,(1+\lambda/\gamma)}, \mathbf{\Gamma}_0[35,16] = \frac{(1-\lambda/\gamma)}{\sigma_c\,(1+\lambda/\gamma)}$$

$$\mathbf{\Gamma}_1[35,30] = \frac{\lambda/\gamma}{(1+\lambda/\gamma)}$$

$$\mathbf{\Gamma}_0[36,31] = 1, \mathbf{\Gamma}_0[36,41] = -\frac{\beta\gamma^{(1-\sigma_c)}}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}, \mathbf{\Gamma}_0[36,32] = -\frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi}, \mathbf{\Gamma}_0[36,18] = -1$$

$$\mathbf{\Gamma}_1[36,31] = \frac{1}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)}$$

$$\mathbf{\Gamma}_0[37,32] = 1, \mathbf{\Gamma}_0[37,43] = -\beta\,(1-\delta)\,\gamma^{-\sigma_c}, \mathbf{\Gamma}_0[37,44] = -\left(1-\beta\,(1-\delta)\,\gamma^{-\sigma_c}\right), \mathbf{\Gamma}_0[37,38] = 1$$

$$\mathbf{\Gamma}_0[37,16] = 1$$

$$\mathbf{\Gamma}_0[38,29] = 1, \mathbf{\Gamma}_0[38,33] = -\Phi\alpha, \mathbf{\Gamma}_0[38,39] = -\Phi\,(1-\alpha)\,, \mathbf{\Gamma}_0[38,15] = -\Phi$$

$$\mathbf{\Gamma}_0[39,33] = 1, \mathbf{\Gamma}_0[39,34] = -1$$

$$\mathbf{\Gamma}_1[39,35] = 1$$

$$\mathbf{\Gamma}_0[40,34] = 1, \mathbf{\Gamma}_0[40,36] = -\frac{(1-\psi)}{\psi}$$

$$\mathbf{\Gamma}_0[41,35] = 1, \mathbf{\Gamma}_0[41,31] = -\frac{(\gamma-1+\delta)}{\gamma}, \mathbf{\Gamma}_0[41,18] = -\,(\gamma-1+\delta)\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma\varphi$$

$$\mathbf{\Gamma}_1[41,35] = \frac{(1-\delta)}{\gamma}$$

$$\mathbf{\Gamma}_0[42,37] = 1, \mathbf{\Gamma}_0[42,33] = -\alpha, \mathbf{\Gamma}_0[42,39] = \alpha, \mathbf{\Gamma}_0[42,15] = -1$$

$$\mathbf{\Gamma}_0[43,36] = 1, \mathbf{\Gamma}_0[43,39] = -1, \mathbf{\Gamma}_0[43,35] = 1, \mathbf{\Gamma}_0[43,37] = -1$$

$$\mathbf{\Gamma}_0[44,37]=1,\mathbf{\Gamma}_0[44,39]=-\sigma_l,\mathbf{\Gamma}_0[44,30]=-\frac{1}{(1-\lambda)}$$

$$\mathbf{\Gamma}_1[44,30]=-\frac{1}{(1-\lambda)}$$

$$\mathbf{\Gamma}_0[45,30]=1,\mathbf{\Gamma}_1[45,40]=1,\mathbf{\Pi}[45,8]=1$$

$$\mathbf{\Gamma}_0[46,31]=1,\mathbf{\Gamma}_1[46,41]=1,\mathbf{\Pi}[46,12]=1$$

$$\mathbf{\Gamma}_0[47,39]=1,\mathbf{\Gamma}_1[47,42]=1,\mathbf{\Pi}[47,9]=1$$

$$\mathbf{\Gamma}_0[48,32]=1,\mathbf{\Gamma}_1[48,43]=1,\mathbf{\Pi}[48,10]=1$$

$$\mathbf{\Gamma}_0[49,38]=1,\mathbf{\Gamma}_1[49,44]=1,\mathbf{\Pi}[49,11]=1$$

$$\mathbf{\Gamma}_0[50,50]=1,\mathbf{\Gamma}_1[50,1]=\psi_{l,m,1,1},\mathbf{\Gamma}_1[50,9]=\psi_{l,m,1,2},\mathbf{\Gamma}_1[50,50]=\psi_{l,1,1}\mathbf{\Psi}[50,8]=1$$

$$\mathbf{\Gamma}_0[51,51]=1,\mathbf{\Gamma}_1[51,1]=\psi_{l,m,2,1},\mathbf{\Gamma}_1[51,9]=\psi_{l,m,2,2},\mathbf{\Gamma}_1[51,50]=\psi_{l,2,1},\mathbf{\Gamma}_1[51,51]=\psi_{l,2,2},$$

$$\mathbf{\Psi}[51,9]=1$$

$$\mathbf{\Gamma}_0[52,52]=1,\mathbf{\Gamma}_1[52,1]=\psi_{l,m,3,1},\mathbf{\Gamma}_1[52,9]=\psi_{l,m,3,2},\mathbf{\Gamma}_1[52,50]=\psi_{l,3,1},\mathbf{\Gamma}_1[52,51]=\psi_{l,3,2}$$

$$\mathbf{\Gamma}_1[52,52]=\psi_{l,3,3},\mathbf{\Psi}[52,10]=1$$

$$\mathbf{\Gamma}_0[53,53]=1,\mathbf{\Gamma}_1[53,1]=\psi_{l,m,1,1},\mathbf{\Gamma}_1[53,29]=\psi_{l,m,1,2},\mathbf{\Gamma}_1[53,53]=\psi_{l,1,1},\mathbf{\Psi}[53,8]=1$$

$$\mathbf{\Gamma}_0[54,54]=1,\mathbf{\Gamma}_1[54,1]=\psi_{l,m,2,1},\mathbf{\Gamma}_1[54,29]=\psi_{l,m,2,2},\mathbf{\Gamma}_1[54,53]=\psi_{l,2,1},\mathbf{\Gamma}_1[54,54]=\psi_{l,2,2},$$

$$\mathbf{\Psi}[54,9]=1$$

$$\mathbf{\Gamma}_0[55,55]=1,\mathbf{\Gamma}_1[55,1]=\psi_{l,m,3,1},\mathbf{\Gamma}_1[55,29]=\psi_{l,m,3,2},\mathbf{\Gamma}_1[55,53]=\psi_{l,3,1},\mathbf{\Gamma}_1[55,54]=\psi_{l,3,2}$$

$$\mathbf{\Gamma}_1[55,55]=\psi_{l,3,3},\mathbf{\Psi}[55,10]=1$$

A.5.3 Recursive bond pricing scheme in the SW-DSGE-ATSM

According to Ang and Piazzesi [2003] the maturity dependent scalar A_τ^{RA} and the $M \times 1$ vector $\mathbf{B}_\tau^{RA}$ are derived as follows:
Using 2.48 and 2.52:

$$\begin{aligned}
P(t,T,\boldsymbol{s}_t) &= \mathbb{E}_t\left[M_{t+1}P\left(t+1,T-1,\boldsymbol{s}_{t+1}\right)\right)] \\
&= \mathbb{E}_t\left[exp\left(-r_t-\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t-\boldsymbol{\lambda}_t^T\boldsymbol{\epsilon}_{t+1}\right)exp\left(A_\tau^{RA}+\left(\mathbf{B}_\tau^{RA}\right)^T\boldsymbol{s}_{t+1}\right)\right] \\
&= \mathbb{E}_t\left[exp\left(-r_t-\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t-\boldsymbol{\lambda}_t^T\boldsymbol{\epsilon}_{t+1}\right)exp\left(A_\tau+\left(\mathbf{B}_\tau^{RA}\right)^T\left(\boldsymbol{\theta}_c+\boldsymbol{\Theta}_0\boldsymbol{s}_t+\boldsymbol{\varepsilon}_{t+1}\right)\right)\right]
\end{aligned} \tag{A.75}$$

With $\mathbb{E}_t\left[\boldsymbol{\varepsilon}_{t+1}\right]=\mathbf{0}, Var_t\left[\boldsymbol{\varepsilon}_{t+1}\right]=\boldsymbol{\Theta}_1$ and the dynamics of the state variables for the solution of the canonical rational expectation system from 2.43 it can be written:

$$\begin{aligned}
P\left(t,T,\boldsymbol{s}_t\right) &= exp\left(-r_t-\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t+A_{\tau-1}^{RA}+\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\left(\boldsymbol{\theta}_c+\boldsymbol{\Theta}_0\boldsymbol{s}_t\right)\right) \\
&\times \mathbb{E}_t\left[exp\left(\left(-\boldsymbol{\lambda}_t^T+\left(B_{\tau-1}^{RA}\right)^T\right)\boldsymbol{\varepsilon}_{t+1}\right)\right] \\
&= exp\left(-r_t-\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t+A_{\tau-1}^{RA}+\mathbf{B}_{\tau-1}^{RA}\left(\boldsymbol{\theta}_c+\boldsymbol{\Theta}_0\boldsymbol{s}_t\right)\right) \\
&\times \mathbb{E}_t\left[exp\left(-\boldsymbol{\lambda}_t^T+\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\right)\boldsymbol{\varepsilon}_{t+1}\right] \\
&= exp\left(-r_t-\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t+A_{\tau-1}^{RA}+\mathbf{B}_{\tau-1}^{RA}\left(\boldsymbol{\theta}_c+\boldsymbol{\Theta}_0\boldsymbol{s}_t\right)\right) \\
&\times exp\left(\mathbb{E}_t\left[\left(-\boldsymbol{\lambda}_t^T+\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\right)\boldsymbol{\varepsilon}_{t+1}\right]+\frac{1}{2}Var_t\left[\left(-\boldsymbol{\lambda}_t^T+\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\right)\boldsymbol{\varepsilon}_{t+1}\right]\right) \\
&= exp\left(-r_t-\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t+A_{\tau-1}^{RA}+\mathbf{B}_{\tau-1}^{RA}\left(\boldsymbol{\theta}_c+\boldsymbol{\Theta}_0\boldsymbol{s}_t\right)\right) \\
&\times exp\left(\frac{1}{2}\left(-\boldsymbol{\lambda}_t^T+\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\right)\mathbb{E}_t\left[\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}_{t+1}^T\right]\left(-\boldsymbol{\lambda}_t+\left(\mathbf{B}_{\tau-1}^{RA}\right)\right)\right) \\
&= exp\left(-r_t-\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t+A_{\tau-1}^{RA}+\mathbf{B}_{\tau-1}^{RA}\left(\boldsymbol{\theta}_c+\boldsymbol{\Theta}_0\boldsymbol{s}_t\right)\right) \\
&\times exp\left(\frac{1}{2}\boldsymbol{\lambda}_t^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t-\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\boldsymbol{\Theta}_1\boldsymbol{\lambda}_t+\frac{1}{2}\left(\mathbf{B}_{\tau-1}^{RA}\right)^T\boldsymbol{\Theta}_1\mathbf{B}_{\tau-1}^{RA}\right)
\end{aligned} \tag{A.76}$$

Using the dynamics of the market prices of risk $\boldsymbol{\lambda}_t$ from 2.49 and the short rate expressed as $r_t = \boldsymbol{\delta}_r^T \boldsymbol{s}_t$ the bond price is:

$$P(t, T, \boldsymbol{s}_t) =$$
$$exp\left(A_{\tau-1}^{RA} + \left(\mathbf{B}_{\tau-1}^{RA}\right)^T (\boldsymbol{\theta}_c + \boldsymbol{\Theta}_0 \boldsymbol{\lambda}_0) + \frac{1}{2}\left(\mathbf{B}_{\tau-1}^{RA}\right)^T \boldsymbol{\Theta}_1 \mathbf{B}_{\tau-1}^{RA} + \left(\left(\mathbf{B}_{\tau-1}^{RA}\right)^T (\boldsymbol{\theta}_c + \boldsymbol{\Theta}_0 \boldsymbol{\lambda}_0) - \boldsymbol{\delta}_1^T\right) \boldsymbol{s}_t\right) \tag{A.77}$$

According to the affine definition of the bond price $P(t, T, \boldsymbol{s}_t)$ in 2.52 the following recursive equations for A_τ^{RA} and $\mathbf{B}_\tau^{RA}$ can be derived:

$$A_\tau^{RA} = A_{\tau-1}^{RA} + \left(\mathbf{B}_{\tau-1}^{RA}\right)^T (\boldsymbol{\theta}_c - \boldsymbol{\Theta}_1 \boldsymbol{\lambda}_0) + \frac{1}{2}\left(\mathbf{B}_{\tau-1}^{RA}\right)^T \boldsymbol{\Theta}_1 \mathbf{B}_{\tau-1}^{RA} \tag{A.78}$$

$$\left(\mathbf{B}_\tau^{RA}\right)^T = \left(\mathbf{B}_{\tau-1}^{RA}\right)^T (\boldsymbol{\Theta}_0 - \boldsymbol{\Theta}_1 \boldsymbol{\lambda}_1) - \boldsymbol{\delta}_r^T \tag{A.79}$$

A.5.4 SW-DSGE-ATSM's measurement coefficient matrix

With the bond loadings $\boldsymbol{b}_\tau$ for the maturities $\tau = 12, 24, ..., 120$ month outlined above the measurement's coefficient matrix $\mathbf{M}$ of the state space system in 2.60 is specified as:

$$
\mathbf{M}^T = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,1} & b_{24,1} & b_{36,1} & b_{48,1} & b_{60,1} & b_{72,1} & b_{84,1} & b_{96,1} & b_{108,1} & b_{120,1} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & b_{12,2} & b_{24,2} & b_{36,2} & b_{48,2} & b_{60,2} & b_{72,2} & b_{84,2} & b_{96,2} & b_{108,2} & b_{120,2} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & b_{12,3} & b_{24,3} & b_{36,3} & b_{48,3} & b_{60,3} & b_{72,3} & b_{84,3} & b_{96,3} & b_{108,3} & b_{120,3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,4} & b_{24,4} & b_{36,4} & b_{48,4} & b_{60,4} & b_{72,4} & b_{84,4} & b_{96,4} & b_{108,4} & b_{120,4} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,5} & b_{24,5} & b_{36,5} & b_{48,5} & b_{60,5} & b_{72,5} & b_{84,5} & b_{96,5} & b_{108,5} & b_{120,5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,6} & b_{24,6} & b_{36,6} & b_{48,6} & b_{60,6} & b_{72,6} & b_{84,6} & b_{96,6} & b_{108,6} & b_{120,6} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,7} & b_{24,7} & b_{36,7} & b_{48,7} & b_{60,7} & b_{72,7} & b_{84,7} & b_{96,7} & b_{108,7} & b_{120,7} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,8} & b_{24,8} & b_{36,8} & b_{48,8} & b_{60,8} & b_{72,8} & b_{84,8} & b_{96,8} & b_{108,8} & b_{120,8} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & b_{12,9} & b_{24,9} & b_{36,9} & b_{48,9} & b_{60,9} & b_{72,9} & b_{84,9} & b_{96,9} & b_{108,9} & b_{120,9} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,10} & b_{24,10} & b_{36,10} & b_{48,10} & b_{60,10} & b_{72,10} & b_{84,10} & b_{96,10} & b_{108,10} & b_{120,10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,11} & b_{24,11} & b_{36,11} & b_{48,11} & b_{60,11} & b_{72,11} & b_{84,11} & b_{96,11} & b_{108,11} & b_{120,11} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & b_{12,12} & b_{24,12} & b_{36,12} & b_{48,12} & b_{60,12} & b_{72,12} & b_{84,12} & b_{96,12} & b_{108,12} & b_{120,12} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & b_{12,13} & b_{24,13} & b_{36,13} & b_{48,13} & b_{60,13} & b_{72,13} & b_{84,13} & b_{96,13} & b_{108,13} & b_{120,13} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & b_{12,14} & b_{24,14} & b_{36,14} & b_{48,14} & b_{60,14} & b_{72,14} & b_{84,14} & b_{96,14} & b_{108,14} & b_{120,14} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,15} & b_{24,15} & b_{36,15} & b_{48,15} & b_{60,15} & b_{72,15} & b_{84,15} & b_{96,15} & b_{108,15} & b_{120,15} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,16} & b_{24,16} & b_{36,16} & b_{48,16} & b_{60,16} & b_{72,16} & b_{84,16} & b_{96,16} & b_{108,16} & b_{120,16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,17} & b_{24,17} & b_{36,17} & b_{48,17} & b_{60,17} & b_{72,17} & b_{84,17} & b_{96,17} & b_{108,17} & b_{120,17} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,18} & b_{24,18} & b_{36,18} & b_{48,18} & b_{60,18} & b_{72,18} & b_{84,18} & b_{96,18} & b_{108,18} & b_{120,18} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,19} & b_{24,19} & b_{36,19} & b_{48,19} & b_{60,19} & b_{72,19} & b_{84,19} & b_{96,19} & b_{108,19} & b_{120,19} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,20} & b_{24,20} & b_{36,20} & b_{48,20} & b_{60,20} & b_{72,20} & b_{84,20} & b_{96,20} & b_{108,20} & b_{120,20} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,21} & b_{24,21} & b_{36,21} & b_{48,21} & b_{60,21} & b_{72,21} & b_{84,21} & b_{96,21} & b_{108,21} & b_{120,21} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,22} & b_{24,22} & b_{36,22} & b_{48,22} & b_{60,22} & b_{72,22} & b_{84,22} & b_{96,22} & b_{108,22} & b_{120,22} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,23} & b_{24,23} & b_{36,23} & b_{48,23} & b_{60,23} & b_{72,23} & b_{84,23} & b_{96,23} & b_{108,23} & b_{120,23} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,24} & b_{24,24} & b_{36,24} & b_{48,24} & b_{60,24} & b_{72,24} & b_{84,24} & b_{96,24} & b_{108,24} & b_{120,24} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,25} & b_{24,25} & b_{36,25} & b_{48,25} & b_{60,25} & b_{72,25} & b_{84,25} & b_{96,25} & b_{108,25} & b_{120,25} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,26} & b_{24,26} & b_{36,26} & b_{48,26} & b_{60,26} & b_{72,26} & b_{84,26} & b_{96,26} & b_{108,26} & b_{120,26} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,27} & b_{24,27} & b_{36,27} & b_{48,27} & b_{60,27} & b_{72,27} & b_{84,27} & b_{96,27} & b_{108,27} & b_{120,27} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,28} & b_{24,28} & b_{36,28} & b_{48,28} & b_{60,28} & b_{72,28} & b_{84,28} & b_{96,28} & b_{108,28} & b_{120,28} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,29} & b_{24,29} & b_{36,29} & b_{48,29} & b_{60,29} & b_{72,29} & b_{84,29} & b_{96,29} & b_{108,29} & b_{120,29} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,30} & b_{24,30} & b_{36,30} & b_{48,30} & b_{60,30} & b_{72,30} & b_{84,30} & b_{96,30} & b_{108,30} & b_{120,30} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,31} & b_{24,31} & b_{36,31} & b_{48,31} & b_{60,31} & b_{72,31} & b_{84,31} & b_{96,31} & b_{108,31} & b_{120,31} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,32} & b_{24,32} & b_{36,32} & b_{48,32} & b_{60,32} & b_{72,32} & b_{84,32} & b_{96,32} & b_{108,32} & b_{120,32} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,33} & b_{24,33} & b_{36,33} & b_{48,33} & b_{60,33} & b_{72,33} & b_{84,33} & b_{96,33} & b_{108,33} & b_{120,33} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,34} & b_{24,34} & b_{36,34} & b_{48,34} & b_{60,34} & b_{72,34} & b_{84,34} & b_{96,34} & b_{108,34} & b_{120,34} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,35} & b_{24,35} & b_{36,35} & b_{48,35} & b_{60,35} & b_{72,35} & b_{84,35} & b_{96,35} & b_{108,35} & b_{120,35} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,36} & b_{24,36} & b_{36,36} & b_{48,36} & b_{60,36} & b_{72,36} & b_{84,36} & b_{96,36} & b_{108,36} & b_{120,36} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,37} & b_{24,37} & b_{36,37} & b_{48,37} & b_{60,37} & b_{72,37} & b_{84,37} & b_{96,37} & b_{108,37} & b_{120,37} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,38} & b_{24,38} & b_{36,38} & b_{48,38} & b_{60,38} & b_{72,38} & b_{84,38} & b_{96,38} & b_{108,38} & b_{120,38} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,39} & b_{24,39} & b_{36,39} & b_{48,39} & b_{60,39} & b_{72,39} & b_{84,39} & b_{96,39} & b_{108,39} & b_{120,39} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,40} & b_{24,40} & b_{36,40} & b_{48,40} & b_{60,40} & b_{72,40} & b_{84,40} & b_{96,40} & b_{108,40} & b_{120,40} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,41} & b_{24,41} & b_{36,41} & b_{48,41} & b_{60,41} & b_{72,41} & b_{84,41} & b_{96,41} & b_{108,41} & b_{120,41} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,42} & b_{24,42} & b_{36,42} & b_{48,42} & b_{60,42} & b_{72,42} & b_{84,42} & b_{96,42} & b_{108,42} & b_{120,42} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,43} & b_{24,43} & b_{36,43} & b_{48,43} & b_{60,43} & b_{72,43} & b_{84,43} & b_{96,43} & b_{108,43} & b_{120,43} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,44} & b_{24,44} & b_{36,44} & b_{48,44} & b_{60,44} & b_{72,44} & b_{84,44} & b_{96,44} & b_{108,44} & b_{120,44} \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,45} & b_{24,45} & b_{36,45} & b_{48,45} & b_{60,45} & b_{72,45} & b_{84,45} & b_{96,45} & b_{108,45} & b_{120,45} \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & b_{12,46} & b_{24,46} & b_{36,46} & b_{48,46} & b_{60,46} & b_{72,46} & b_{84,46} & b_{96,46} & b_{108,46} & b_{120,46} \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & b_{12,47} & b_{24,47} & b_{36,47} & b_{48,47} & b_{60,47} & b_{72,47} & b_{84,47} & b_{96,47} & b_{108,47} & b_{120,47} \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & b_{12,48} & b_{24,48} & b_{36,48} & b_{48,48} & b_{60,48} & b_{72,48} & b_{84,48} & b_{96,48} & b_{108,48} & b_{120,48} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,49} & b_{24,49} & b_{36,49} & b_{48,49} & b_{60,49} & b_{72,49} & b_{84,49} & b_{96,49} & b_{108,49} & b_{120,49} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,50} & b_{24,50} & b_{36,50} & b_{48,50} & b_{60,50} & b_{72,50} & b_{84,50} & b_{96,50} & b_{108,50} & b_{120,50} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,51} & b_{24,51} & b_{36,51} & b_{48,51} & b_{60,51} & b_{72,51} & b_{84,51} & b_{96,51} & b_{108,51} & b_{120,51} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,52} & b_{24,52} & b_{36,52} & b_{48,52} & b_{60,52} & b_{72,52} & b_{84,52} & b_{96,52} & b_{108,52} & b_{120,52} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,53} & b_{24,53} & b_{36,53} & b_{48,53} & b_{60,53} & b_{72,53} & b_{84,53} & b_{96,53} & b_{108,53} & b_{120,53} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,54} & b_{24,54} & b_{36,54} & b_{48,54} & b_{60,54} & b_{72,54} & b_{84,54} & b_{96,54} & b_{108,54} & b_{120,54} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{12,55} & b_{24,55} & b_{36,55} & b_{48,55} & b_{60,55} & b_{72,55} & b_{84,55} & b_{96,55} & b_{108,55} & b_{120,55}
\end{bmatrix}
$$

A.5.5 Kalman filter likelihood function

Defining the linear state-space model:

$$\boldsymbol{y}_t = \boldsymbol{\Psi} \boldsymbol{f}_t + \boldsymbol{\eta}_t \tag{A.80}$$

$$\boldsymbol{f}_t = \boldsymbol{\Phi} \boldsymbol{f}_{t-1} + \boldsymbol{\varepsilon}_t \tag{A.81}$$

where $\boldsymbol{y}_t$ and $\boldsymbol{f}_t$ are the $n_o \times 1$ and $n_f \times 1$ vectors of observed measurement and latent state variables respectively. The measurement and transition errors are Gaussian with $\boldsymbol{\eta}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\eta)$ and $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$, where $\boldsymbol{\Sigma}_\eta$ is diagonal. The Kalman filter implied likelihood function is:

$$\begin{aligned}
\mathscr{L}(\boldsymbol{\theta}) = 1/ & \left((2\pi)^{n_o} \left|\boldsymbol{\Psi} \mathbf{P}_{t|t-1} \boldsymbol{\Psi}^T + \boldsymbol{\Sigma}_\eta\right|\right)^{T/2} \\
& \times \prod_{t=1}^{T} exp\left(-\frac{1}{2}\left(\boldsymbol{y}_t - \boldsymbol{\Psi}\hat{\boldsymbol{f}}_{t|t-1}\right)^T \left(P_{t|t-1}\Psi\boldsymbol{\Psi}^T + \boldsymbol{\Sigma}_\eta\right)^{-1}\left(\boldsymbol{y}_t - \boldsymbol{\Psi}\hat{\boldsymbol{f}}_{t|t-1}\right)\right) \\
& = 1/\left((2\pi)^{n_o}\left|\mathbf{S}_t\right|\right)^{T/2} \prod_{t=1}^{T} exp\left(-\frac{1}{2}\hat{\boldsymbol{v}}_t^T \mathbf{S}_t^{-1} \hat{\boldsymbol{v}}_t\right)
\end{aligned} \tag{A.82}$$

with T observations, where the Kalman filter's (a priori) state estimate $\hat{\boldsymbol{f}}_{t|t-1}$, the (a priori) estimate of the states covariance matrix $\boldsymbol{\Psi}\mathbf{P}_{t|t-1}$, the residuals $\boldsymbol{v}_t$ and the covariance $\mathbf{S}_t$ of the residuals $\boldsymbol{v}_t$ in the Kalman filter likelihood in A.82 are given by:

$$\hat{\boldsymbol{f}}_{t|t-1} = \boldsymbol{\Phi}\hat{\boldsymbol{f}}_{t-1|t-1} \tag{A.83}$$

$$\mathbf{P}_{t|t-1} = \boldsymbol{\Phi}\mathbf{P}_{t-1|t-1}\boldsymbol{\Phi}^T + \boldsymbol{\Sigma}_\varepsilon \tag{A.84}$$

which defines the prediction step of the Kalman filter and:

$$\hat{\boldsymbol{v}}_t = \boldsymbol{y}_t - \boldsymbol{\Psi}\hat{\boldsymbol{f}}_{t|t-1} \tag{A.85}$$

$$\mathbf{S}_t = \boldsymbol{\Psi}\mathbf{P}_{t|t-1}\boldsymbol{\Psi}^T + \boldsymbol{\Sigma}_\eta \tag{A.86}$$

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}\boldsymbol{\Psi}^T + \mathbf{S}_t^{-1} \tag{A.87}$$

$$\hat{\boldsymbol{f}}_{t|t} = \hat{\boldsymbol{f}}_{t|t-1} + \mathbf{K}_t\hat{\boldsymbol{v}}_t \tag{A.88}$$

$$\mathbf{P}_{t|t} = \left(\mathbf{I} - \mathbf{K}_t\boldsymbol{\Psi}\right)\mathbf{P}_{t|t-1} \tag{A.89}$$

from the Kalman filter's update step, where $\mathbf{K}_t$ is the Kalman gain and $\hat{\boldsymbol{f}}_{t|t}$ and $\mathbf{P}_{t|t}$ are the (a posteriori) estimates of the state variables and its updated covariance matrices.

A.6 Algorithms of the mixed MH-MCMC

A.6.1 Random-Walk-Metropolis-Hastings Algorithm

The random block (RB)-RW-MH algorithm is defined by the proposal distribution:

$$q\left(*\middle|\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)=f\left(*\middle|\boldsymbol{\mu}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right),\boldsymbol{\Sigma}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)\right) \tag{A.90}$$

where f is the density function of the multivariate normal distribution with the moments $\boldsymbol{\mu}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)$ and $\boldsymbol{\Sigma}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)$ defined as:

$$\boldsymbol{\mu}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)=\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right] \tag{A.91}$$

and

$$\boldsymbol{\Sigma}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)=c^2\hat{\boldsymbol{\Sigma}} \tag{A.92}$$

where according to Herbst and Schorfheide [2016] we set the hyper-parameter $c = 0.5$. The parameter's covariance matrix $\hat{\boldsymbol{\Sigma}}$ is derived from a pre-estimation step where estimation is done by maximizing the (log) posterior p determined by the Kalman filter as outlined in Appendix A.5.5. Maximization is done by using the hybridized genetic Nelder-Mead global optimization algorithm proposed by Chelouah and Siarry [2003] outlined in more detail in Appenidx A.7.1. $\hat{\boldsymbol{\Sigma}}$ is constant over all MCMC iterations $i = 1, 2, ..., N$ with:

$$\hat{\boldsymbol{\Sigma}}=-\mathbf{H}(\hat{\boldsymbol{\theta}})^{-1} \tag{A.93}$$

where $\mathbf{H}$ is the (log) posterior's Hessian:

$$\mathbf{H}(\hat{\boldsymbol{\theta}})=\frac{\partial}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^T}ln\left(p\left(\boldsymbol{\theta}|\mathbf{Y}\right)\right)|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \tag{A.94}$$

evaluated at $\hat{\boldsymbol{\theta}}$ found in the pre-estimation step.

A.6.2 Newton-Metropolis-Hastings Algorithm

As for the RB-RW-MH algorithm the proposal distribution of the RB-Newton-MH algorithm follows the definition in A.90. According to Qi and Minka [2002] the moments $\boldsymbol{\mu}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)$ and $\boldsymbol{\Sigma}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)$ of the Gaussian proposal are specified for the RB-Newton-MH algorithm as follows:

$$\begin{aligned}\boldsymbol{\mu}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)=\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]^T-s\mathbf{H}\left(\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)^{-1}\\ \times\frac{\partial}{\partial\boldsymbol{\theta}}ln\left(p\left(\boldsymbol{\theta}|\mathbf{Y}\right)\right)|_{\boldsymbol{\theta}=\left[\boldsymbol{\theta}_{<b}^{(i)},\boldsymbol{\theta}_{b}^{(i-1)},\boldsymbol{\theta}_{>b}^{(i-1)}\right]}\end{aligned} \tag{A.95}$$

$$\mathbf{\Sigma}\left(\left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\theta}_{b}^{(i-1)}, \boldsymbol{\theta}_{>b}^{(i-1)}\right]\right) = c^2 \mathbf{H}\left(\left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\theta}_{b}^{(i-1)}, \boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)^{-1} \tag{A.96}$$

where $\mathbf{H}\left(\left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\theta}_{b}^{(i-1)}, \boldsymbol{\theta}_{>b}^{(i-1)}\right]\right)$ is the (log) posterior's Hessian defined as in A.94 evaluated at
$\boldsymbol{\theta} = \left[\boldsymbol{\theta}_{<b}^{(i)}, \boldsymbol{\theta}_{b}^{(i-1)}, \boldsymbol{\theta}_{>b}^{(i-1)}\right]$.

Following Herbst and Schorfheide [2016] the algorithm's hyper-parameters s and c_1 are specified as:

$$s \sim U(0, \bar{s}) \tag{A.97}$$

with $\bar{s} = 2$ and $c_1 = 1$, where s is drawn for every MCMC iteration $i = 1, 2, ..., N$ and every randomly selected parameter block b from the uniform distribution U.

A.7 Pre-estimation and choice of Priors

A.7.1 Hybridized genetic Nelder-Mead pre-estimation algorithm

The hybridized genetic Nelder-Mead algorithm proposed by Chelouah and Siarry [2003] consists of two steps and is constructed for global optimization of multiminima target functions. The steps are called diversification and intensification steps, where the diversification step goes through the global search space in finding the minimal value of the objective function - the function to be minimized. Because of the objective function's minimization in most of the implementations we formulate this section in terms of minimization. Maximizing the objective function - in our case the (multimodal) posterior - is done by only changing the objective function's sign. After the global diversification step there is the intensification (or exploiting) step, with a more local search around the solution determined in the first step. Our diversification step uses the heuristic Differential Evolution (DE) algorithm proposed by Storn and Price [1997]. The intensification step uses the Nelder-Mead algorithm. In more detail the algorithmic procedures of both steps are as follows:

1. Diversification

The diversification step implies mutation, crossover and selection. Starting with a generation G , where the generation consists of NP populations

$$x_{1,G}, x_{2,G}, ..., x_{NP,G} \tag{A.98}$$

Every population is a $D \times 1$ vector $\boldsymbol{x}_{i,G}^T = [x_{1,i,G}, x_{2,i,G}, ..., x_{D,i,G}]$ with $i = 1, 2, ..., NP$ of possible solutions of the objective function f. NP determines the number of populations of every generation and is fixed during the minimization process.

i. Mutation

A mutant $\boldsymbol{v}_{i,G+1}$ with $i = 1, 2, ..., NP$ as a potential candidate for the next generation $G+1$ is created from three randomly selected populations of G:

$$\boldsymbol{v}_{i,G+1} = \boldsymbol{x}_{r_1,G} + F\left(\boldsymbol{x}_{r_2,G} - \boldsymbol{x}_{r_3,G}\right) \tag{A.99}$$

where $F \in [0, 2]$ is a parameter controlling the amplification of the populations difference $(\boldsymbol{x}_{r_2,G} - \boldsymbol{x}_{r_3,G})$in the population's mutation. The indices r_1, r_2, r_3 with $r_1 \neq r_2 \neq r_3$ are selected by random from the index set $\{1, 2, ..., NP\}$. From the mutation it is required that $NP > 4$.

ii. Crossover

In the crossover a trial vector $\boldsymbol{u}_{i,G+1}$ is build for the next generation $G + 1$, where the $j = 1, 2, ..., D$ elements of $\boldsymbol{u}_{i,G+1}$ are chosen according to:

$$\boldsymbol{u}_{i,G+1} = \begin{cases} v_{j,i,G+1} & if \quad randb(j) \leq CR \ \vee \ j = rnbr(i) \\ x_{j,i,G+1} & if \quad randb(j) > CR \ \wedge \ j \neq rnbr(i) \end{cases} \tag{A.100}$$

where $randb(j)$ is the probability of the j-th draw from the uniform distribution. $CR \in [0, 1]$ defines the crossover ratio. $rnbr(i)$ is a randomly chosen index from the set $\{1, 2, ..., NP\}$.

iii. Selection

In the trial and error scheme of the genetic DE the trial vector $\boldsymbol{u}_{i,G+1}$ is evaluated and selected by comparing the fit of $\boldsymbol{u}_{i,G+1}$ with $\boldsymbol{x}_{i,G}$ of generation G. If:

$$f(\boldsymbol{u}_{i,G+1}) < f(\boldsymbol{x}_{i,G}) \tag{A.101}$$

then with $\boldsymbol{x}_{i,G} = \boldsymbol{u}_{i,G}$ the trial vector $\boldsymbol{u}_{i,G}$ becomes a member of the next generation $G+1$. The DE algorithm breaks if a maximal number of generations G_{max} is reached. We set the parameters NP, F and CR of the DE algorithm with $NP = 10D$, $F = 0.8$ and $CR = 0.5$.

2. Intensification (Exploit)

Initialized by $n + 1$ test vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{n+1}$ building a simplex, where $x1 = x_{opt,G_max}$ with the solution $_{opt,G_max}$ from the diversification step, the Nelder-Mead intensification runs through the following seven steps (see Nelder and Mead [1965] or in textbook Arora[2015]):

i. Sorting

The $n + 1$ vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{n+1}$ of the simplex are sorted according:

$$f(\boldsymbol{x}_1) \leq f(\boldsymbol{x}_2) \leq ... \leq f(\boldsymbol{x}_{n+1}) \tag{A.102}$$

where $\boldsymbol{x}_1$ is the best and $\boldsymbol{x}_{n+1}$ is the worst vector.

ii. Centroid

The centroid $\bar{\boldsymbol{x}}$ of the simplex is calculated as the mean of the vectors without the worst $n+1$ vector $\boldsymbol{x}_{n+1}$:

$$\bar{\boldsymbol{x}} = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{x}_i \tag{A.103}$$

iii. Reflection

The reflection vector is calculated by:

$$\boldsymbol{x}_r = \bar{\boldsymbol{x}} + \alpha \left(\bar{\boldsymbol{x}} - \boldsymbol{x}_{n+1} \right) \tag{A.104}$$

with $\alpha > 0$. $\boldsymbol{x}_r$ replaces $\boldsymbol{x}_{n+1}$ if $f(\boldsymbol{x}_1) \leq f(\boldsymbol{x}_r) < f(\boldsymbol{x}_{n+1})$ and i.) and ii.) are passed again. If $f(\boldsymbol{x}_r) < f(\boldsymbol{x}_1)$ the expansion in iv. is done. If $f(\boldsymbol{x}_r) \geq f(\boldsymbol{x}_n)$ which means $\boldsymbol{x}_r$ is at least the second worst vector, the contraction in v. is done.

iv. Expansion

Expansion is defined by:

$$\boldsymbol{x}_e = \bar{\boldsymbol{x}} + \gamma \left(\boldsymbol{x}_r - \bar{\boldsymbol{x}} \right) \tag{A.105}$$

with $\gamma > 1$. If $f(\boldsymbol{x}_e) < f(\boldsymbol{x}_r)$ then $\boldsymbol{x}_1 = \boldsymbol{x}_e$, else we set $\boldsymbol{x}_1 = \boldsymbol{x}_r$.

v. Contraction

The contraction is done according to:

$$\boldsymbol{x}_c = \bar{\boldsymbol{x}} + \rho \left(\boldsymbol{x}_{n+1} - \bar{\boldsymbol{x}} \right) \tag{A.106}$$

with $0 < \rho \leq 0.5$. If $f(\boldsymbol{x}_c) < f(\boldsymbol{x}_{n+1})$ then $\boldsymbol{x}_{n+1}$ 1 is replaced by the contracted vector $\boldsymbol{x}_c$ and the order in i.) is determined again.

vi. Shrinkage

Shrinkage is done by replacing all $j = 2, 3, ..., n+1$ vectors without the best $\boldsymbol{x}_1$ by:

$$\boldsymbol{x}_j = \boldsymbol{x}_1 + \sigma \left(\boldsymbol{x}_j - \boldsymbol{x}_1 \right) \tag{A.107}$$

After the shrinkage the intensification step starts again at i.).

We set the reflection, expansion, contraction and shrinkage parameters α, γ, ρ and σ with $\alpha = 1, \gamma = 2, \rho = 0.5$ and $\sigma = 0.5$.

A.7.2 Prior distributions

SW DSGE Prior Distributions		
Structural Parameters	$\psi \sim beta(0.50, 0.15)$	$\sigma_g \sim InvGamma(0.10, 2.00)$
$\bar{\gamma} \sim N(0.40, 0.10)$	**Monetary Policy**	$\sigma_i \sim InvGamma(0.10, 2.00)$
$\bar{\pi} \sim Gamma(0.62, 0.10)$	$\rho \sim beta(0.75, 0.10)$	$\sigma_r \sim InvGamma(0.10, 2.00)$
$L \sim N(0.00, 2.00)$	$r_\pi \sim N(1.50, 0.25)$	$\sigma_p \sim InvGamma(0.10, 2.00)$
Preferences	$r_y \sim N(0.12, 0.05)$	$\sigma_w \sim InvGamma(0.10, 2.00)$
$(\beta^{-1} - 1)100 \sim Gamma(0.25, 0.10)$	$r_{\Delta y} \sim N(0.12, 0.05)$	**Measurement Errors**
$\lambda \sim \beta(0.70, 0.10)$	**Shock AR Coefficients**	$y \sim InvGamma(0.10, 2.00)$
$\sigma_c \sim N(1.50, 0.37)$	$\rho_a \sim beta(0.50, 0.20)$	$c \sim InvGamma(0.10, 2.00)$
Price and Wage Setting	$\rho_b \sim beta(0.50, 0.20)$	$i \sim InvGamma(0.10, 2.00)$
$\xi_p \sim beta(0.50, 0.10)$	$\rho_g \sim beta(0.50, 0.20)$	$l \sim InvGamma(0.10, 2.00)$
$\iota_p \sim beta(0.50, 0.15)$	$\rho_i \sim beta(0.50, 0.20)$	$\pi \sim InvGamma(0.10, 2.00)$
$\xi_w \sim beta(0.50, 0.10)$	$\rho_r \sim beta(0.50, 0.20)$	$w \sim InvGamma(0.10, 2.00)$
$\iota_w \sim beta(0.50, 0.15)$	$\rho_p \sim beta(0.50, 0.20)$	$r \sim InvGamma(0.10, 2.00)$
Production	$\rho_w \sim beta(0.50, 0.20)$	
$\varphi \sim N(4.00, 1.50)$	**Shock Standard Deviations**	
$\alpha \sim N(0.30, 0.05)$	$\sigma_a \sim InvGamma(0.10, 2.00)$	
$\Phi \sim N(1.25, 0.12)$	$\sigma_b \sim InvGamma(0.10, 2.00)$	

Table A.2: Prior distributions of the SW-DSGE-ATSM (We deviate from the correct parametrization of the distributions in reporting in parentheses the distribution's mean and standard deviation respectively. For the inverse gamma distribution we report the scale and shape parameters)

SW DSGE ATSM Prior Distributions				
Monetary Policy	$\lambda_{1,2,15} \sim N(-0.07, 0.01)$	$\lambda_{1,5,15} \sim N(-0.06, 0.01)$	$\lambda_{1,8,50} \sim N(0.60, 0.12)$	$\psi_{ll,3,2} \sim N(-0.15, 0.03)$
$\delta_1 \sim N(-0.20, 0.04)$	$\lambda_{1,2,16} \sim N(0.62, 0.12)$	$\lambda_{1,5,16} \sim N(-0.12, 0.02)$	$\lambda_{1,8,51} \sim N(-0.10, 0.02)$	$\psi_{ll,3,3} \sim N(0.59, 0.12)$
$\delta_2 \sim N(0.06, 0.01)$	$\lambda_{1,2,17} \sim N(0.04, 0.01)$	$\lambda_{1,5,17} \sim N(0.20, 0.04)$	$\lambda_{1,8,52} \sim N(0.11, 0.02)$	**Measurement Errors**
$\delta_3 \sim N(-0.20, 0.04)$	$\lambda_{1,2,18} \sim N(-0.18, 0.04)$	$\lambda_{1,5,18} \sim N(0.12, 0.02)$	$\lambda_{1,9,50} \sim N(-0.51, 0.10)$	$\sigma_{12M} \sim InvGamma(0.01, 0.001)$
Market Prices of Risk	$\lambda_{1,2,19} \sim N(0.03, 0.01)$	$\lambda_{1,5,19} \sim N(-0.22, 0.04)$	$\lambda_{1,9,51} \sim N(0.34, 0.07)$	$\sigma_{24M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,1,1} \sim N(0.01, 0.02)$	$\lambda_{1,2,20} \sim N(0.18, 0.04)$	$\lambda_{1,5,20} \sim N(0.07, 0.01)$	$\lambda_{1,9,52} \sim N(0.05, 0.01)$	$\sigma_{36M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,2,1} \sim N(-0.09, 0.02)$	$\lambda_{1,2,21} \sim N(0.28, 0.06)$	$\lambda_{1,5,21} \sim N(0.71, 0.14)$	$\lambda_{1,10,50} \sim N(0.23, 0.05)$	$\sigma_{48M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,3,1} \sim N(0.61, 0.12)$	$\lambda_{1,3,15} \sim N(-0.01, 0.00)$	$\lambda_{1,6,15} \sim N(0.19, 0.04)$	$\lambda_{1,10,51} \sim N(-0.47, 0.09)$	$\sigma_{60M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,4,1} \sim N(-0.18, 0.04)$	$\lambda_{1,3,16} \sim N(-0.06, 0.01)$	$\lambda_{1,6,16} \sim N(-1.20, 0.24)$	$\lambda_{1,10,52} \sim N(0.33, 0.07)$	$\sigma_{72M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,5,1} \sim N(-0.12, 0.02)$	$\lambda_{1,3,17} \sim N(-0.12, 0.02)$	$\lambda_{1,6,17} \sim N(-0.49, 0.10)$	Ψ_{lm}	$\sigma_{84M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,6,1} \sim N(-0.27, 0.05)$	$\lambda_{1,3,18} \sim N(-0.20, 0.04)$	$\lambda_{1,6,18} \sim N(-0.74, 0.15)$	$\psi_{lm,1,1} \sim N(0.09, 0.02)$	$\sigma_{96M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,7,1} \sim N(-0.25, 0.05)$	$\lambda_{1,3,19} \sim N(0.15, 0.03)$	$\lambda_{1,6,19} \sim N(0.10, 0.02)$	$\psi_{lm,1,2} \sim N(-0.13, 0.03)$	$\sigma_{108M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,8,1} \sim N(-0.02, 0.01)$	$\lambda_{1,3,20} \sim N(0.08, 0.02)$	$\lambda_{1,6,20} \sim N(-0.22, 0.04)$	$\psi_{lm,2,1} \sim N(-0.01, 0.00)$	$\sigma_{120M} \sim InvGamma(0.10, 2.00)$
$\lambda_{0,9,1} \sim N(-0.26, 0.05)$	$\lambda_{1,3,21} \sim N(1.42, 0.28)$	$\lambda_{1,6,21} \sim N(-0.42, 0.08)$	$\psi_{lm,2,2} \sim N(-0.270.05)$	
$\lambda_{0,10,1} \sim N(-0.10, 0.02)$	$\lambda_{1,4,15} \sim N(0.68, 0.14)$	$\lambda_{1,7,15} \sim N(0.29, 0.06)$	$\psi_{lm,3,1} \sim N(-0.67, 0.13)$	
$\lambda_{1,1,15} \sim N(0.59, 0.12)$	$\lambda_{1,4,16} \sim N(0.49, 0.10)$	$\lambda_{1,7,16} \sim N(0.00, 0.01)$	$\psi_{lm,3,2} \sim N(-0.19, 0.04)$	
$\lambda_{1,1,16} \sim N(-0.29, 0.06)$	$\lambda_{1,4,17} \sim N(0.12, 0.02)$	$\lambda_{1,7,17} \sim N(0.20, 0.04)$	Ψ_{ll}	
$\lambda_{1,1,17} \sim N(1.15, 0.23)$	$\lambda_{1,4,18} \sim N(-0.50, 0.10)$	$\lambda_{1,7,18} \sim N(-0.07, 0.01)$	$\psi_{ll,1,1} \sim N(0.96, 0.19)$	
$\lambda_{1,1,18} \sim N(-0.79, 0.16)$	$\lambda_{1,4,19} \sim N(-0.33, 0.07)$	$\lambda_{1,7,19} \sim N(0.26, 0.05)$	$\psi_{ll,2,1} \sim N(-0.56, 0.11)$	
$\lambda_{1,1,19} \sim N(0.13, 0.03)$	$\lambda_{1,4,20} \sim N(0.18, 0.04)$	$\lambda_{1,7,20} \sim N(-0.03, 0.01)$	$\psi_{ll,3,1} \sim N(-0.19, 0.04)$	
$\lambda_{1,1,21} \sim N(0.11, 0.02)$	$\lambda_{1,4,21} \sim N(-0.05, 0.01)$	$\lambda_{1,7,21} \sim N(0.34, 0.07)$	$\psi_{ll,2,2} \sim N(0.91, 0.18)$	

Table A.3: Prior distributions of the SW-DSGE-ATSM (We deviate from the correct parametrization of the distributions in reporting in parentheses the distribution's mean and standard deviation respectively. For the inverse gamma distribution we report the scale and shape parameters)

A.8 Small-scale New-Keynesian DSGE term structure model

To evaluate our results from a DSGE modeling perspective we use as an alternative to our used large-scale approach the small-scale New-Keynesian DSGE proposed by Beakert, Cho and Moreno [2010], who integrated the term structure in a way similar to our approach. Beakert, Cho and Moreno (BCM) describe their small scale New-Keynesian macroeconomy by the following 5 equations:

$$\pi_t = \delta\, \mathbb{E}_t \left[\pi_{t+1}\right] + (1-\delta)\, \pi_{t-1} + \kappa \left(y_t - y_t^n\right) + \epsilon_{AS,t} \tag{A.108}$$

$$y_t = \alpha_{IS} + \mu\, \mathbb{E}_t \left[y_{t+1}\right] + (1-\mu)\, y_{t-1} + \phi \left(r_t - \mathbb{E}_t \left[\pi_{t+1}\right]\right) + \epsilon_{IS,t} \tag{A.109}$$

$$r_t = \alpha_{MP} + \rho r_{t-1} + (1-\rho) \left[\beta \left(\mathbb{E}_t \left[\pi_{t+1}\right] - \tilde{\pi}_t\right) + \gamma \left(y_t - \tilde{y}_t\right)\right] + \epsilon_{MP,t} \tag{A.110}$$

$$\tilde{y}_t = \lambda \tilde{y}_{t-1} + \epsilon_{MU,t} \tag{A.111}$$

$$\tilde{\pi}_t = \vartheta_1\, \mathbb{E}_t \left[\tilde{\pi}_{t+1}\right] + \vartheta_2 \tilde{\pi}_{t-1} + \vartheta_3 \pi_t + \epsilon_{LTMP,t} \tag{A.112}$$

with $\phi = 1/\left(\sigma + \xi\right), \mu = \sigma\phi, \vartheta_1 = d/(1+d\omega), \vartheta_2 = \omega/(1+d\omega)$ and $\vartheta_3 = 1 - \vartheta_1 - \vartheta_2$. For the formulation of the BCM New-Keynesian economy in state-space form and the application of Sim's QZ solution algorithm on the system, we extend the five BCM equations by five additional equations:

$$\pi_t = E_{t-1} \left[\pi_t\right] + \eta_{\pi,t} \tag{A.113}$$

$$y_t = E_{t-1} \left[y_t\right] + \eta_{y,t} \tag{A.114}$$

$$\tilde{\pi}_t = E_{t-1} \left[\tilde{\pi}_t\right] \tag{A.115}$$

$$\pi_{t-1} = \pi_{t-1} \tag{A.116}$$

$$y_{t-1} = y_{t-1} \tag{A.117}$$

We transform the system into the canonical rational expectations form:

$$\boldsymbol{\Gamma}_0 \boldsymbol{s}_t = \boldsymbol{\Gamma}_1 \boldsymbol{s}_{t-1} + \boldsymbol{\Psi} \boldsymbol{\epsilon}_t + \boldsymbol{\Pi} \boldsymbol{\eta}_t \tag{A.118}$$

where we define the state vector $\boldsymbol{s}_t$ for our implementation of the BCM New-Keynesian DSGE as $\boldsymbol{s}^T = [\pi_t, y_t, r_t \tilde{y}_t, \tilde{\pi}_t, \mathbb{E}_t[\pi_{t+1}], \mathbb{E}_t[y_{t+1}], \mathbb{E}_t[\tilde{\pi}_{t+1}], \pi_{t-1}, y_{t-1}]$. The vectors of exogenous shocks $\boldsymbol{\epsilon}_t$ and expectation errors $\boldsymbol{\eta}_t$ are $\boldsymbol{\epsilon}_t^T = [\epsilon_{AS,t}, \epsilon_{IS,t}, \epsilon_{MP,t}, \epsilon_{MU,t}, \epsilon_{LTMP,t}]$ and $\boldsymbol{\eta}_t^T = [\pi_t - \mathbb{E}_{t-1}[\pi_t], y_t - \mathbb{E}_{t-1}[y_t], \tilde{\pi}_t - \mathbb{E}_{t-1}[\tilde{\pi}_t]]$. The matrices $\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}, \boldsymbol{\Pi}$ and the constant vector

$\boldsymbol{c}$ are specified for our implementation of the small-scale BMC model as:

$$\boldsymbol{\Gamma}_0 = \begin{bmatrix} 1 & -\kappa & 0 & \kappa & 0 & -\delta & 0 & 0 & 0 & 0 \\ 0 & 1 & \phi & 0 & -\phi & -\mu & 0 & 0 & 0 & 0 \\ 0 & (\rho-1)\,\gamma & 1 & (1-\rho)\gamma & (1-\rho)\beta & (\rho-1)\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\vartheta_3 & 0 & 0 & 0 & 1 & 0 & 0 & -\vartheta_1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{\Pi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} (1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\mu) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \vartheta_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{\Psi} = \begin{bmatrix} \sigma_{AS} & 0 & 0 & 0 & 0 \\ 0 & \sigma_{IS} & 0 & 0 & 0 \\ 0 & 0 & \sigma_{MP} & 0 & 0 \\ 0 & 0 & 0 & \sigma_{MU} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{LTMP} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\mu}^T = [0, \alpha_{IS}, \alpha_{MP}, 0, 0, 0, 0, 0, 0, 0]$$

As in section 2.2.2.2 by applying the QZ algorithm we get the solution:

$$\boldsymbol{s}_t = \boldsymbol{\theta}_c + \boldsymbol{\Theta}_0 \boldsymbol{s}_{t-1} + \boldsymbol{\Theta}_1 \boldsymbol{\epsilon}_t \tag{A.119}$$

To combine the macroeconomic state variable's equilibrium transition with the term structure dynamics BCM use an arbitrage-free recursive pricing scheme similar to our used recursive bond pricing scheme outlined in section 2.2.3. As in section 2.2.3 the risk adjustment is done by regarding the Duffie and Kan [1996] pricing kernel:

$$M_{t+1} = exp\left(-r_t - \frac{1}{2}\boldsymbol{\lambda}_t^T \boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t^T \boldsymbol{\epsilon}_{t+1}\right) \tag{A.120}$$

in discounting future expected bond prices. BCM define time invariant market prices of risk $\boldsymbol{\lambda}_t = \boldsymbol{\lambda}$ with:

$$\boldsymbol{\lambda}^T = [1, \sigma, \mathbf{0}_{1\times 8}]\, \boldsymbol{\Theta}_1 - [0, (\sigma - \eta), \mathbf{0}_{1\times 8}] \tag{A.121}$$

The recursive pricing with respect to the constant market prices of risk in the BCM model is defined in its affine form as:

$$y(t,T) = -\frac{a_\tau}{\tau} - \frac{\boldsymbol{b}_\tau^T}{\tau} \boldsymbol{s}_t \tag{A.122}$$

with the maturity dependent constant a_τ and loadings $\boldsymbol{b}_\tau$ recursively defined as:

$$a_\tau = a_{\tau-1} + \boldsymbol{b}_{\tau-1}^T \boldsymbol{\theta}_c + \frac{1}{2} \boldsymbol{b}_{\tau-1}^T \boldsymbol{\Theta}_1 \boldsymbol{\Theta}_1^T \boldsymbol{b}_{\tau-1} - \boldsymbol{\lambda}_1^T \boldsymbol{\Theta}_1^T \boldsymbol{b}_{\tau-1} \tag{A.123}$$

$$\boldsymbol{b}_\tau = -\boldsymbol{\delta}_3 + \boldsymbol{b}_{\tau-1}^T \boldsymbol{\Theta}_0 \tag{A.124}$$

where $\boldsymbol{\delta}_3$ is an 10×1 indicator vector of zeros except $\delta_3 = 1$. In their model implementation BCM combine the macro economy with two term spread variables $s_{1,t} = y(t, 36M) - r_t$ and $s_{2,t} = y(t, 60M)$. In our implementation of the BCM DSGE model, we directly combine the observed rates with time to maturities $\tau = 12, 24, 36, 48, 60, 96, 120$ month with the macroeconomic state variables in formulating the 10 dimensional state-space system:

$$\boldsymbol{y}_t = \boldsymbol{c} + \mathbf{M}\boldsymbol{s}_t + \boldsymbol{\vartheta}_t \tag{A.125}$$

where the vector of measurements in our BCM model implementation $\boldsymbol{y}_t$ is
$\boldsymbol{y}_t^T = [ln(INF_t), ln(\Delta GDP_t), ECB_t, y(t,12), y(t,24), y(t,36), ..., y(t,120)]$ with GDP and GDP deflator based inflation expressed in logarithmic differences as in our SW-DSGE-ATSM setting. The vector of measurement constants $\boldsymbol{c}^T = [0, 0, 0, -a_12, -a_24, ..., -a_120]$ contains the maturity dependent constants calculated according to A.123 and the measurement's 10×10 coefficient matrix M is specified as:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ b_{12,1} & b_{12,2} & b_{12,3} & b_{12,4} & b_{12,5} & b_{12,6} & b_{12,7} & b_{12,8} & b_{12,9} & b_{12,10} \\ & \vdots & & & \vdots & & & & \vdots & \\ & \vdots & & & \vdots & & & & \vdots & \\ b_{120,1} & b_{120,2} & b_{120,3} & b_{120,4} & b_{120,5} & b_{120,6} & b_{120,7} & b_{120,8} & b_{120,9} & b_{120,10} \end{bmatrix} \tag{A.126}$$

$\boldsymbol{\vartheta}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma})$ is the Gaussian measurement error with diagonal covariance $\boldsymbol{\Sigma}$ .Estimation of the BCM model is done by a Gibbs sampling MCMC procedure using the Random-Block-Random-Walk Metropolis Hastings (RB-RW-MH) algorithm for drawing the DSGE- and term-structure parameters. The number of the RB-RW-MH's randomly chosen block clustering is set to $N_b = 3$. For parameter estimation and statistical inference we use 100000 iterations, where we cut the first 50000 as parameter draws of the burn-in phase of the MCMC procedure.

A.9 SW-DSGE-ATSM parameter estimates

A.9.1 German SW-DSGE-ATSM estimates

SW-DSGE Structural Parameters

$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c
0.381	0.849	0.929	0.103	0.719	2.914
(0.108)	(0.355)	(0.388)	(0.002)	(0.004)	(0.02)

β	φ	ψ	ι_p	Φ	ξ_p
1.000	0.416	0.961	0.774	2.014	0.271
(0.001)	(0.013)	(0.003)	(0.006)	(0.021)	(0.005)

ι_w	ξ_w	σ_l	ρ	r_π	r_y
0.514	0.797	0.336	0.942	3.881	0.524
(0.021)	(0.010)	(0.017)	(0.018)	(0.007)	(0.014)

$r_{\Delta y}$	$\delta_{f,1}$	$\delta_{f,2}$	$\delta_{f,3}$
0.612	-0.202	0.087	-0.198
(0.005)	(0,000)	(0.005)	(0.001)

SW-DSGE Disturbance Parameters

ρ_g	ρ_b	ρ_i	ρ_a	ρ_p	ρ_w	ρ_r
0.999	0.966	0.607	0.999	0.629	0.602	0.916
(0.001)	(0.011)	(0.032)	(0.002)	(0.029)	(0.006)	(0.008)

σ_g	σ_b	σ_i	σ_a	σ_p	σ_w	σ_r
0.027	0.354	0.833	0.375	0.275	0.069	0.263
(0.003)	(0.056)	(0.039)	(0.033)	(0.029)	(0.020)	(0.013)

SW-DSGE RMSEs

y	c	i	w	l	π	r
0.517	1.358	2.394	0.669	0.002	0.019	0.130
(0.173)	(0.295)	(1.92)	(0.212)	(0.028)	(0.042)	(0.038)

ATSM Parameters

$\boldsymbol{\lambda}_0^T$	$\lambda_{0,g}$	$\lambda_{0,b}$	$\lambda_{0,i}$	$\lambda_{0,a}$	$\lambda_{0,p}$	$\lambda_{0,w}$	$\lambda_{0,r}$	λ_{0,f_1}	λ_{0,f_2}	λ_{0,f_3}
	0.015	-0.100	0.608	-0.220	-0.126	-0.269	-0.257	-0.028	-0.268	-0.098
	(0.002)	(0.003)	(0.007)	(0.013)	(0.001)	(0.007)	(0.004)	(0.006)	(0.004)	(0.002)

$\boldsymbol{\lambda}_1$	ε^g	ε^b	ε^i	ε^a	ε^p	ε^w	ε^r
ε^g	0.596	-0.292	1.151	-0.799	0.140	0.005	0.097
	(0.003)	(0.004)	(0.000)	(0.004)	(0.003)	(0.004)	(0.004)
ε^b	-0.066	0.627	0.044	-0.180	0.0180	0.182	0.281
	(0.002)	(0.002)	(0.001)	(0.001)	(0.004)	(0.000)	(0.001)
ε^i	-0.005	-0.080	-0.123	-0.203	0.150	0.081	1.433
	(0.002)	(0.005)	(0.002)	(0.001)	(0.001)	(0.005)	(0.007)
ε^a	0.675	0.491	0.120	-0.501	-0.332	0.182	-0.054
	(0.004)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.003)
ε^p	-0.062	-0.120	0.200	0.116	-0.213	0.094	0.714
	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
ε^w	0.189	-1.216	-0.494	-0.748	0.102	-0.220	-0.420
	(0.001)	(0.004)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
ε^r	0.292	-0.006	0.194	-0.070	0.259	-0.058	0.320
	(0.002)	(0.001)	(0.006)	(0.001)	(0.001)	(0.006)	(0.023)
diag($\boldsymbol{\Sigma}_l$)	0.000	0.000	0.000				
	(0.000)	(0.000)	(0.000)				

$\boldsymbol{\lambda}_1$	ε^{f_1}	ε^{f_2}	ε^{f_3}
ε^{f_1}	0.615	-0.096	0.104
	(0.003)	(0.001)	(0.001)
ε^{f_2}	-0.518	0.338	0.053
	(0.002)	(0.001)	(0.000)
ε^{f_3}	0.226	-0.480	0.335
	(0.031)	(0.089)	(0.003)

$\boldsymbol{\Psi}_{l,m}^T$	f_1	f_2	f_3
y	0.090	-0.006	-0.669
	(0.000)	(0.001)	(0.004)
$\bar{\pi}$	-0.130	-0.273	-0.201
	(0.002)	(0.001)	(0.003)

$\boldsymbol{\Psi}_{l,l}$	f_1	f_2	f_3
f_1	0.941		
	(0.004)		
f_2	-0.565	0.902	
	(0.000)	(0.001)	
f_3	-0.186	-0.151	0.594
	(0.001)	(0.001)	(0.000)

ATSM RMSYE's	12M	24M	36M	60M	120M
	0.010	0.010	0.001	0.000	0.004
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)

SW-DSGE Structural Parameters						**SW-DSGE Disturbance Parameters**						
$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c	ρ_g	ρ_b	ρ_i	ρ_a	ρ_p	ρ_w	ρ_r
0.222	0.168	3.542	0.666	0.918	1.369	0.989	0.977	0.322	0.948	0.221	0.562	0.939
(0.057)	(0.076)	(0.499)	(0.056)	(0.068)	(0.593)	(0.041)	(0.003)	(0.167)	(0.008)	(0.143)	(0.182)	(0.007)
β	φ	ψ	ι_p	Φ	ξ_p	σ_g	σ_b	σ_i	σ_a	σ_p	σ_w	σ_r
0.983	1.661	0.312	0.136	7.537	0.785	0.023	0.799	0.731	0.253	0.024	0.000	0.001
(0.003)	(0.685)	(0.034)	(0.128)	(0.399)	(0.091)	(0.06)	(0.159)	(0.115)	(0.087)	(0.025)	(0.014)	(0.139)
ι_w	ξ_w	σ_l	ρ	r_π	r_y	**SW-DSGE Measurement Errors**						
0.508	0.069	-1.411	0.834	-0.649	-0.211	y	c	i	w	l	π	r
(0.252)	(0.102)	(0.395)	(0.029)	(0.567)	(0.047)	0.12	0.43	2.346	0.689	0.000	0.048	0.010
$r_{\Delta y}$						(0.044)	(0.029)	(0.302)	(0.173)	(0.000)	(0.014)	(0.009)
0.078												
(0.021)												
BCM-DSGE Structural Parameters						**BCM-DSGE Disturbance Parameters**						
α_{IS}	α_{MP}	δ	κ	ρ	β	σ_{AS}	σ_{IS}	σ_{MP}	σ_{MU}	σ_{LTMP}		
-0.112	0.32	0.145	-1.368	0.188	1.000	0.105	1.203	1.510	0.959	0.011		
(0.065)	(0.015)	(0.018)	(0.027)	(0.008)	(0.001)	(0.024)	(0.051)	(0.036)	(0.058)	(0.001)		
λ	γ	ω	d			**BCM-ATSM Parameters**						
0.871	-0.345	1.000	0.552			σ	η					
(0.012)	(0.015)	(0.000)	(0.048)			2.653	3.795					
						(0.037)	(0.044)					
BCM DSGE and ATSM Measurement Errors												
y	π	r	12M	24M	36M	48M	60M	96M	120M			
1.041	0.541	0.360	0.113	0.006	0.001	0.001	0.003	0.014	0.513			
(0.032)	(0.031)	(0.047)	(0.037)	(0.002)	(0.001)	(0.001)	(0.001)	(0.095)	(0.087)			

A.9.2 French SW-DSGE-ATSM estimates

SW-DSGE Structural Parameters

$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c
0.131	0.603	-0.633	0.189	0.594	2.855
(0.018)	(0.376)	(0.435)	(0.016)	(0.017)	(0.012)
β	φ	ψ	ι_p	Φ	ξ_p
0.998	0.431	0.777	0.800	2.119	0.228
(0.001)	(0.011)	(0.029)	(0.010)	(0.008)	(0.015)
ι_w	ξ_w	σ_l	ρ	r_π	r_y
0.587	0.909	0.280	0.854	3.916	0.364
(0.020)	(0.019)	(0.014)	(0.009)	(0.003)	(0.016)
$r_{\Delta y}$	$\delta_{f,1}$	$\delta_{f,2}$	$\delta_{f,3}$		
0.627	-0.200	0.069	-0.201		
(0.005)	(0.001)	(0.006)	(0.001)		

SW-DSGE Disturbance Parameters

ρ_g	ρ_b	ρ_i	ρ_a	ρ_p	ρ_w	ρ_r
0.999	0.996	0.645	0.992	0.622	0.745	0.932
(0.002)	(0.012)	(0.019)	(0.008)	(0.027)	(0.039)	(0.004)
σ_g	σ_b	σ_i	σ_a	σ_p	σ_w	σ_r
0.043	0.386	0.809	0.426	0.174	0.061	0.220
(0.008)	(0.019)	(0.015)	(0.018)	(0.016)	(0.01)	(0.01)

SW-DSGE RMSEs

y	c	i	w	l	π	r
0.110	0.326	0.311	0.245	0.554	0.014	0.007
(0.108)	(0.461)	(0.863)	(0.325)	(0.284)	(0.019)	(0.011)

ATSM Parameters

$\boldsymbol{\lambda}_0^T$	$\lambda_{0,g}$	$\lambda_{0,b}$	$\lambda_{0,i}$	$\lambda_{0,a}$	$\lambda_{0,p}$	$\lambda_{0,w}$	$\lambda_{0,r}$	λ_{0,f_1}	λ_{0,f_2}	λ_{0,f_3}
	0.027	-0.083	0.616	-0.132	-0.094	-0.267	-0.248	-0.042	-0.260	-0.066
	(0.009)	(0.005)	(0.003)	(0.008)	(0.015)	(0.003)	(0.005)	(0.004)	(0.005)	(0.005)

$\boldsymbol{\lambda}_1$	ε^g	ε^b	ε^i	ε^a	ε^p	ε^w	ε^r
ε^g	0.598	-0.280	1.155	-0.774	0.143	-0.014	0.110
	(0.002)	(0.002)	(0.004)	(0.004)	(0.003)	(0.002)	(0.002)
ε^b	-0.062	0.634	0.043	-0.183	0.030	0.173	0.269
	(0.002)	(0.004)	(0.001)	(0.002)	(0.002)	(0.002)	(0.009)
ε^i	-0.003	-0.088	-0.125	-0.202	0.155	0.100	1.434
	(0.003)	(0.003)	(0.002)	(0.004)	(0.001)	(0.005)	(0.005)
ε^a	0.676	0.484	0.131	-0.498	-0.337	0.161	-0.056
	(0.002)	(0.002)	(0.004)	(0.001)	(0.003)	(0.005)	(0.001)
ε^p	-0.082	-0.122	0.220	0.123	-0.217	0.068	0.720
	(0.006)	(0.003)	(0.004)	(0.005)	(0.004)	(0.001)	(0.002)
ε^w	0.184	-1.191	-0.483	-0.758	0.107	-0.209	-0.434
	(0.003)	(0.003)	(0.006)	(0.002)	(0.002)	(0.002)	(0.009)
ε^r	0.296	0.054	0.188	-0.071	0.254	-0.033	0.375
	(0.002)	(0.018)	(0.002)	(0.001)	(0.001)	(0.004)	(0.018)

$\boldsymbol{\lambda}_1$	ε^{f_1}	ε^{f_2}	ε^{f_3}
ε^{f_1}	0.609	-0.096	0.104
	(0.002)	(0.001)	(0.002)
ε^{f_2}	-0.512	0.341	0.069
	(0.002)	(0.001)	(0.004)
ε^{f_3}	0.231	-0.472	0.335
	(0.003)	(0.002)	(0.001)

$\boldsymbol{\Psi}_{l,m}^T$	f_1	f_2	f_3
y	0.091	-0.017	-0.665
	(0.001)	(0.002)	(0.002)
$\bar{\pi}$	-0.123	-0.263	-0.193
	(0.002)	(0.001)	(0.002)

$\boldsymbol{\Psi}_{l,l}$	f_1	f_2	f_3
f_1	0.959		
	(0.001)		
f_2	-0.563	0.935	
	(0.001)	(0.008)	
f_3	-0.200	-0.151	0.600
	(0.005)	(0.001)	(0.002)

diag($\boldsymbol{\Sigma}_l$)	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)

ATSM RMSYE's	12M	24M	36M	60M	120M
	0.007	0.011	0.000	0.000	0.000
	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)

SW-DSGE Structural Parameters

$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c
0.302	0.387	1.86	0.198	0.019	5.928
(0.100)	(0.079)	(0.396)	(0.067)	(0.099)	(2.304)
β	φ	ψ	ι_p	Φ	ξ_p
0.999	2.155	0.778	0.764	0.962	0.414
(0.004)	(2.148)	(0.132)	(0.264)	(0.222)	(0.171)
ι_w	ξ_w	σ_l	ρ	r_π	r_y
0.894	0.538	36.487	0.78	9.489	0.256
(0.301)	(0.101)	(15.235)	(0.104)	(8.21)	(0.93)
$r_{\Delta y}$					
0.176					
(0.416)					

SW-DSGE Disturbance Parameters

ρ_g	ρ_b	ρ_i	ρ_a	ρ_p	ρ_w	ρ_r
0.994	0.336	0.966	0.995	0.909	0.987	0.446
(0.357)	(0.101)	(0.07)	(0.005)	(0.3)	(0.029)	(0.081)
σ_g	σ_b	σ_i	σ_a	σ_p	σ_w	σ_r
0.16	0.064	2.383	0.411	0.022	0.393	0.006
(0.142)	(0.1)	(0.82)	(0.087)	(0.024)	(0.294)	(0.126)

SW-DSGE Measurement Errors

y	c	i	w	l	π	r
0.324	0.284	0.068	0.125	0.000	0.039	0.046
(0.068)	(0.098)	(0.148)	(0.054)	(0.002)	(0.016)	(0.027)

BCM-DSGE Structural Parameters

α_{IS}	α_{MP}	δ	κ	ρ	β
-0.150	0.360	0.148	-1.378	0.188	1.000
(0.010)	(0.010)	(0.006)	(0.005)	(0.003)	(0.001)
λ	γ	ω	d		
0.891	-0.329	1.000	0.551		
(0.004)	(0.008)	(0.001)	(0.007)		

BCM-DSGE Disturbance Parameters

σ_{AS}	σ_{IS}	σ_{MP}	σ_{MU}	σ_{LTMP}
0.195	1.204	1.484	0.927	0.012
(0.012)	(0.006)	(0.006)	(0.010)	(0.000)

BCM-ATSM Parameters

σ	η
2.634	3.815
(0.005)	(0.006)

BCM DSGE and ATSM Measurement Errors

y	π	r	12M	24M	36M	48M	60M	96M	120M
0.993	0.558	0.340	0.127	0.005	0.000	0.001	0.003	0.012	0.523
(0.015)	(0.010)	(0.007)	(0.009)	(0.001)	(0.001)	(0.001)	(0.000)	(0.002)	(0.007)

A.9.3 Italian SW-DSGE-ATSM estimates

SW-DSGE Structural Parameters

$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c
0.192	-0.455	0.878	0.178	0.749	2.951
(0.014)	(0.340)	(0.424)	(0.027)	(0.026)	(0.038)

β	φ	ψ	ι_p	Φ	ξ_p
1.000	0.399	0.377	0.799	2.169	0.313
(0.002)	(0.021)	(0.030)	(0.006)	(0.008)	(0.021)

ι_w	ξ_w	σ_l	ρ	r_π	r_y
0.655	0.906	0.296	0.879	3.903	0.242
(0.022)	(0.021)	(0.011)	(0.016)	(0.006)	(0.030)

$r_{\Delta y}$	$\delta_{f,1}$	$\delta_{f,2}$	$\delta_{f,3}$
0.597	-0.174	0.069	-0.191
(0.011)	(0.001)	(0.003)	(0.001)

SW-DSGE Disturbance Parameters

ρ_g	ρ_b	ρ_i	ρ_a	ρ_p	ρ_w	ρ_r
0.997	0.967	0.584	1.000	0.510	0.777	0.936
(0.001)	(0.013)	(0.038)	(0.001)	(0.054)	(0.023)	(0.005)

σ_g	σ_b	σ_i	σ_a	σ_p	σ_w	σ_r
0.026	0.365	0.946	0.339	0.236	0.077	0.238
(0.002)	(0.020)	(0.052)	(0.039)	(0.035)	(0.008)	(0.016)

SW-DSGE RMSEs

y	c	i	w	l	π	r
0.196	1.125	3.329	1.721	0.001	0.502	0.013
(0.103)	(0.207)	(1.162)	(0.502)	(0.017)	(0.149)	(0.016)

ATSM Parameters

$\boldsymbol{\lambda}_0^T$	$\lambda_{0,g}$	$\lambda_{0,b}$	$\lambda_{0,i}$	$\lambda_{0,a}$	$\lambda_{0,p}$	$\lambda_{0,w}$	$\lambda_{0,r}$	λ_{0,f_1}	λ_{0,f_2}	λ_{0,f_3}
	0.022	-0.067	0.604	-0.183	-0.073	-0.233	-0.187	-0.055	-0.188	-0.119
	(0.005)	(0.015)	(0.005)	(0.011)	(0.020)	(0.002)	(0.007)	(0.010)	(0.008)	(0.005)

$\boldsymbol{\lambda}_1$	ε^g	ε^b	ε^i	ε^a	ε^p	ε^w	ε^r
ε^g	0.495	-0.156	1.133	-0.651	0.185	-0.025	0.118
	(0.001)	(0.002)	(0.006)	(0.004)	(0.007)	(0.003)	(0.002)
ε^b	-0.059	0.672	0.037	-0.200	0.028	0.196	0.346
	(0.003)	(0.003)	(0.001)	(0.004)	(0.001)	(0.004)	(0.01)
ε^i	0.007	-0.051	-0.110	-0.191	0.171	0.083	1.214
	(0.002)	(0.003)	(0.003)	(0.003)	(0.001)	(0.004)	(0.005)
ε^a	0.729	0.636	0.145	-0.435	-0.346	0.159	-0.041
	(0.001)	(0.004)	(0.003)	(0.001)	(0.002)	(0.002)	(0.003)
ε^p	-0.065	-0.106	0.152	0.079	-0.204	0.063	0.786
	(0.002)	(0.007)	(0.008)	(0.006)	(0.003)	(0.001)	(0.003)
ε^w	0.208	-1.118	-0.419	-0.806	0.102	-0.204	-0.402
	(0.006)	(0.006)	(0.005)	(0.003)	(0.001)	(0.004)	(0.007)
ε^r	0.295	-0.018	0.211	-0.071	0.230	0.001	0.306
	(0.003)	(0.012)	(0.009)	(0.003)	(0.001)	(0.005)	(0.012)
diag($\boldsymbol{\Sigma}_l$)	0.000	0.000	0.000				
	(0.000)	(0.000)	(0.000)				

$\boldsymbol{\lambda}_1$	ε^{f_1}	ε^{f_2}	ε^{f_3}
ε^{f_1}	0.590	-0.085	0.085
	(0.007)	(0.001)	(0.002)
ε^{f_2}	-0.538	0.347	0.028
	(0.002)	(0.001)	(0.002)
ε^{f_3}	0.332	-0.458	0.344
	(0.002)	(0.002)	(0.002)

$\boldsymbol{\Psi}_{l,m}^T$	f_1	f_2	f_3
y	0.084	-0.020	-0.734
	(0.000)	(0.004)	(0.003)
$\bar{\pi}$	-0.123	-0.240	-0.223
	(0.001)	(0.001)	(0.002)

$\boldsymbol{\Psi}_{l,l}$	f_1	f_2	f_3
f_1	0.932		
	(0.001)		
f_2	-0.506	0.819	
	(0.001)	(0.002)	
f_3	-0.157	-0.139	0.481
	(0.003)	(0.001)	(0.002)

ATSM RMSYE's	12M	24M	36M	60M	120M
	0.012	0.010	0.000	0.001	0.003
	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)

SW-DSGE Structural Parameters

$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c
-0.156	0.465	2.596	0.513	0.161	6.115
(0.070)	(0.174)	(0.234)	(0.048)	(0.090)	(1.260)
β	φ	ψ	ι_p	Φ	ξ_p
0.975	12.906	0.534	0.501	5.103	0.926
(0.005)	(4.859)	(0.017)	(0.128)	(0.257)	(0.062)
ι_w	ξ_w	σ_l	ρ	r_π	r_y
0.215	0.149	-2.518	0.810	5.769	0.181
(0.286)	(0.024)	(0.284)	(0.073)	(1.425)	(0.138)
$r_{\Delta y}$					
0.270					
(0.080)					

SW-DSGE Disturbance Parameters

ρ_g	ρ_b	ρ_i	ρ_a	ρ_p	ρ_w	ρ_r
0.690	0.971	0.680	0.972	0.300	0.794	0.306
(0.072)	(0.095)	(0.154)	(0.007)	(0.154)	(0.234)	(0.272)
σ_g	σ_b	σ_i	σ_a	σ_p	σ_w	σ_r
0.012	0.706	0.043	0.010	0.080	0.001	0.030
(0.020)	(0.759)	(0.101)	(0.110)	(0.026)	(0.006)	(0.067)

SW-DSGE Measurement Errors

y	c	i	w	l	π	r
0.628	0.490	2.207	1.502	0.000	0.327	0.004
(0.045)	(0.030)	(0.270)	(0.153)	(0.001)	(0.100)	(0.006)

BCM-DSGE Structural Parameters

α_{IS}	α_{MP}	δ	κ	ρ	β
-0.134	0.401	0.216	-1.330	0.188	1.000
(0.033)	(0.029)	(0.011)	(0.021)	(0.004)	(0.005)
λ	γ	ω	d		
0.874	-0.429	1.000	0.619		
(0.008)	(0.02)	(0.000)	(0.015)		

BCM-DSGE Disturbance Parameters

σ_{AS}	σ_{IS}	σ_{MP}	σ_{MU}	σ_{LTMP}
0.175	1.083	1.541	1.165	0.012
(0.046)	(0.030)	(0.018)	(0.036)	(0.001)

BCM-ATSM Parameters

σ	η
2.686	3.988
(0.052)	(0.034)

BCM DSGE and ATSM Measurement Errors

y	π	r	12M	24M	36M	48M	60M	96M	120M
1.120	0.798	0.362	0.172	0.007	0.000	0.001	0.004	0.014	0.381
(0.021)	(0.082)	(0.021)	(0.024)	(0.001)	(0.000)	(0.000)	(0.000)	(0.003)	(0.048)

A.9.4 German MCMC diagnostics SW-DSGE-ATSM parameter histograms

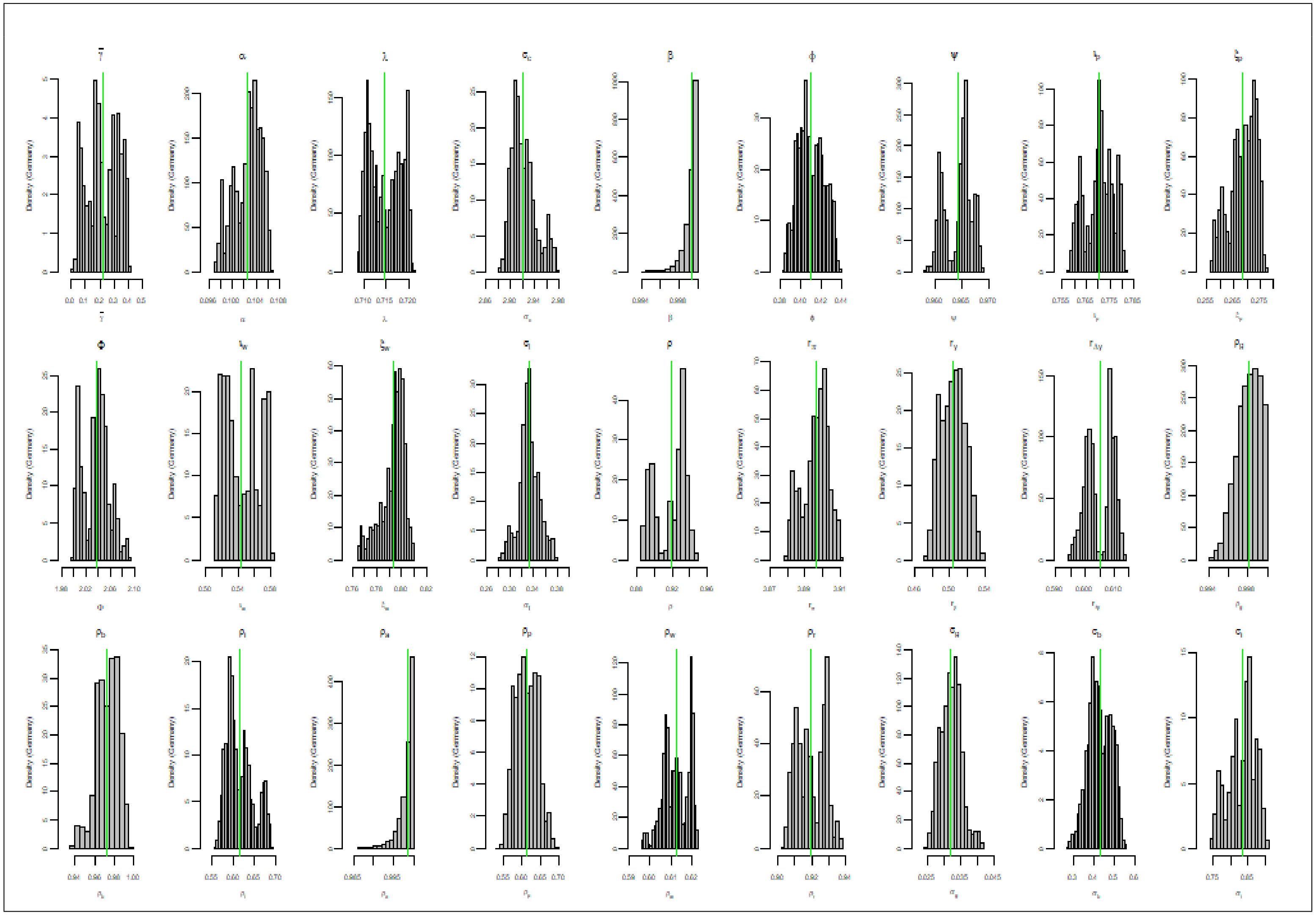


Table A.4: German MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (1/5)

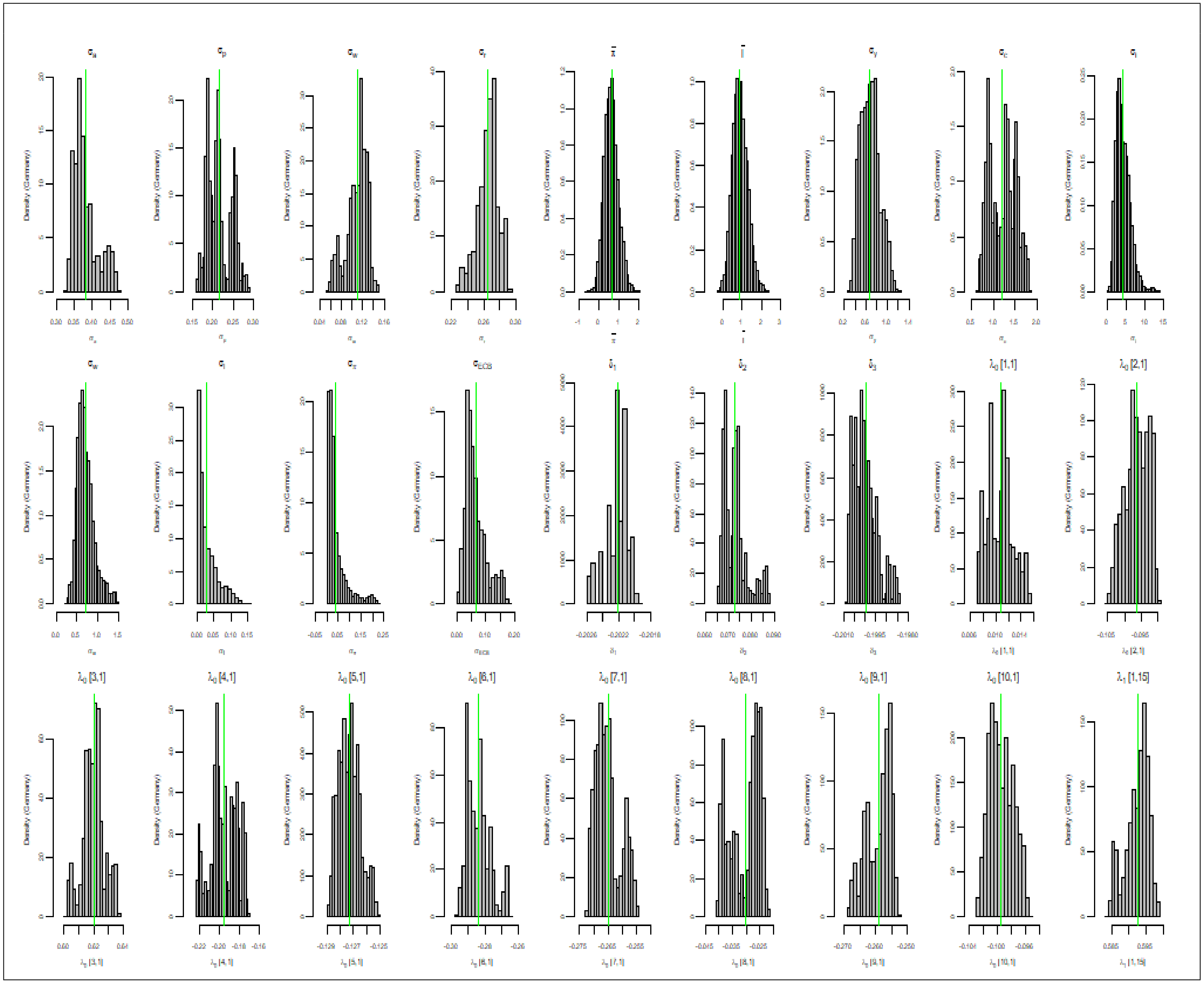


Table A.5: German MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (2/5)

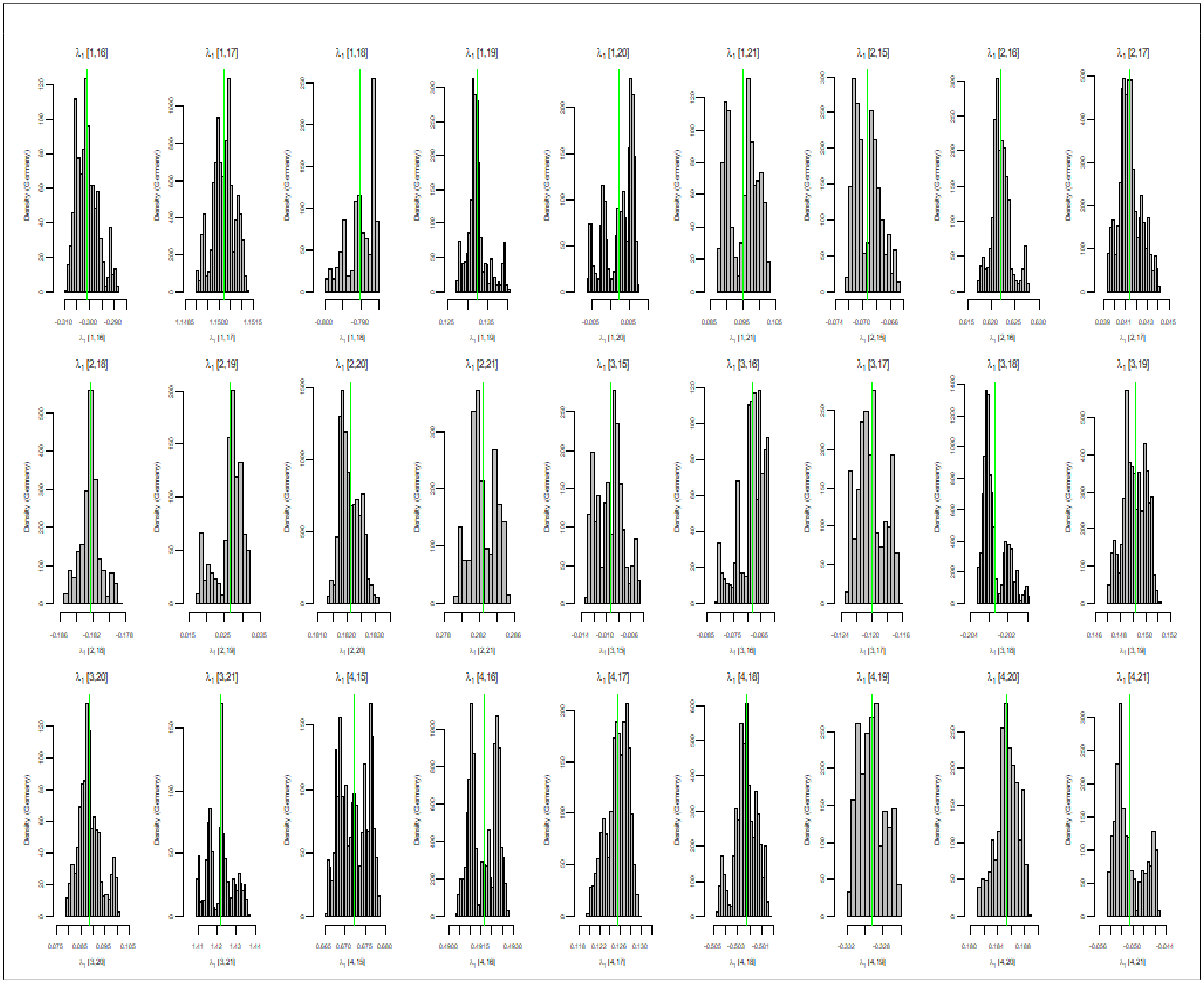


Table A.6: German MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (3/5)

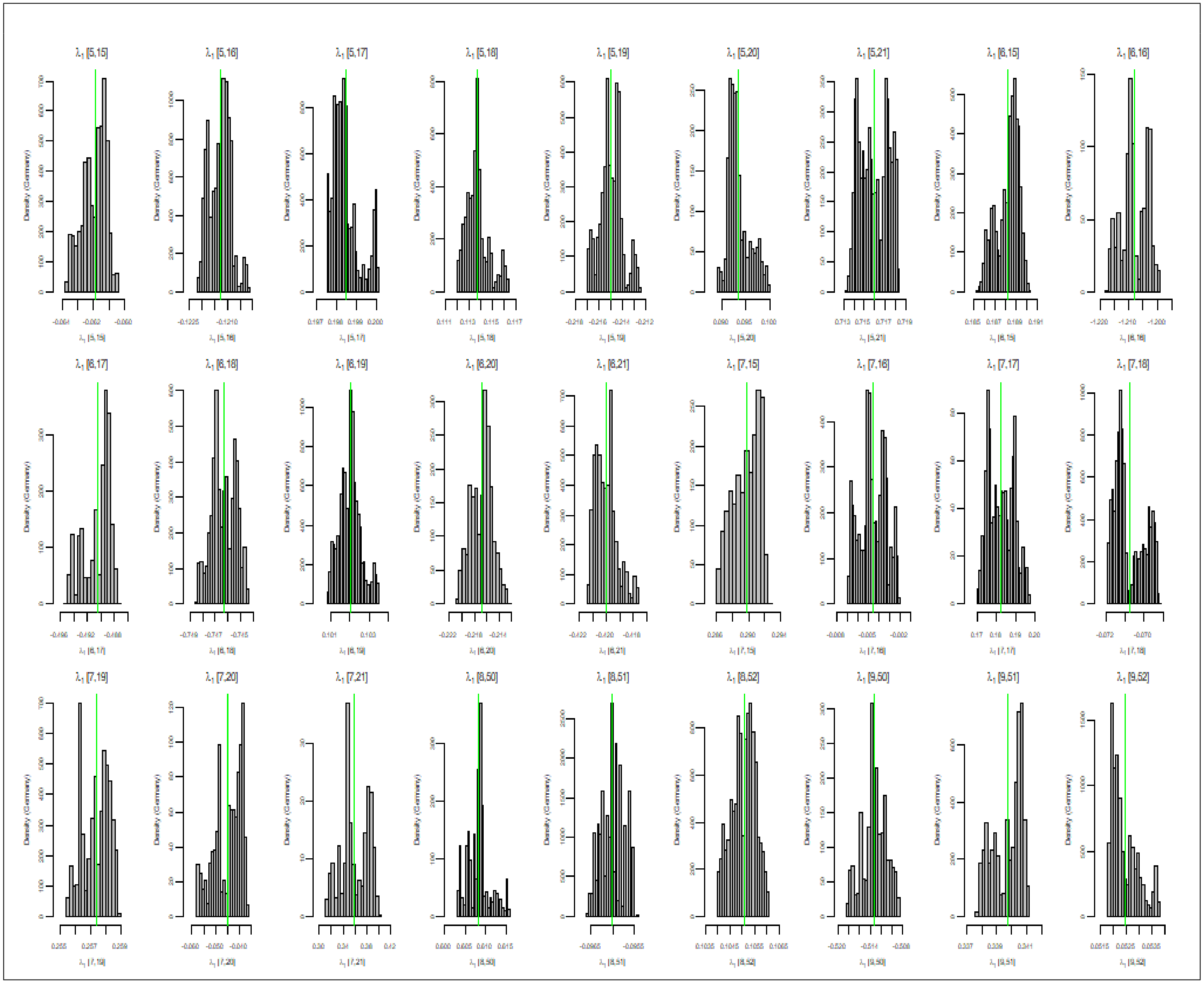


Table A.7: German MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (4/5)

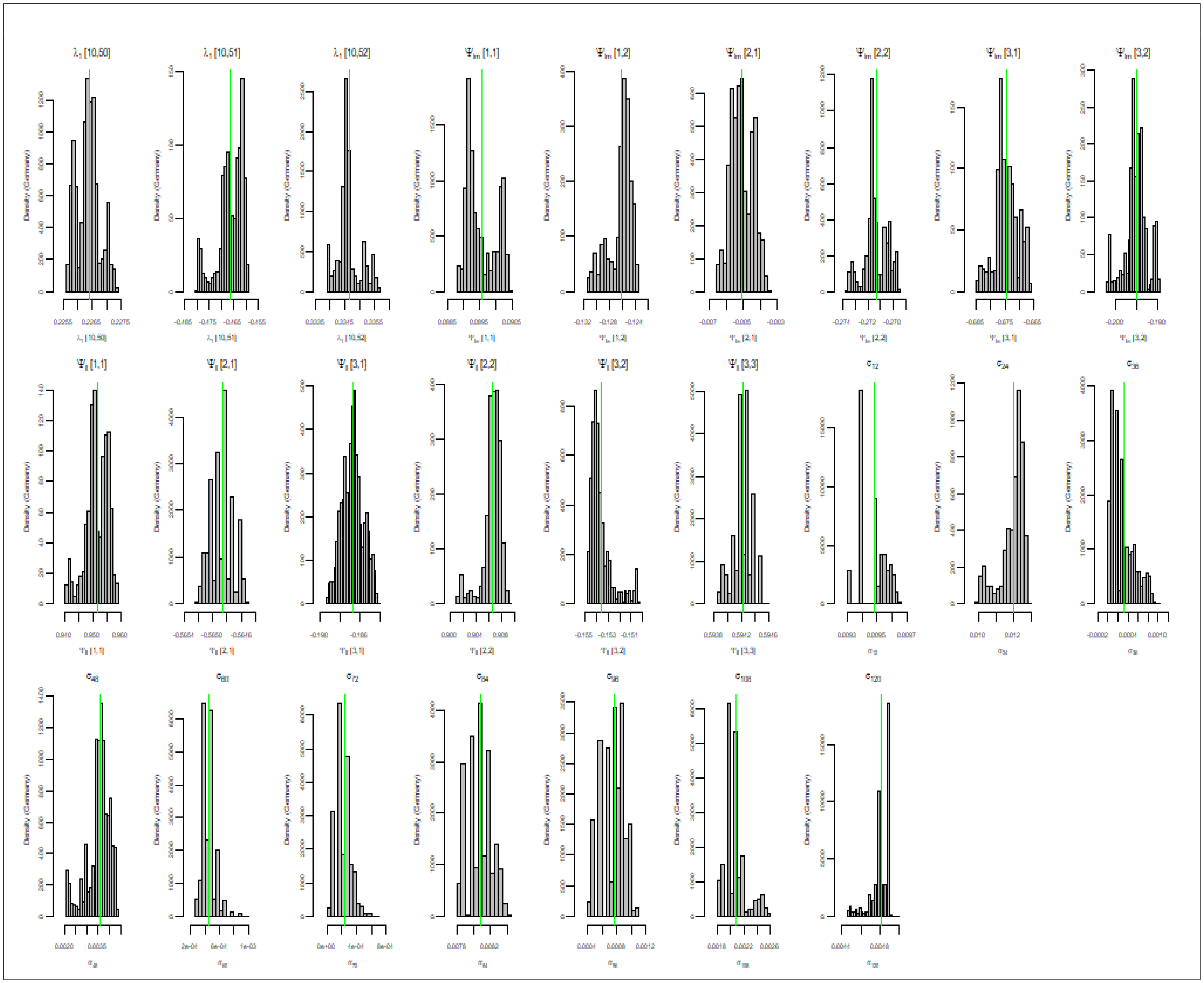


Table A.8: German MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (5/5)

A.9.5 French MCMC diagnostics SW-DSGE-ATSM parameter histograms

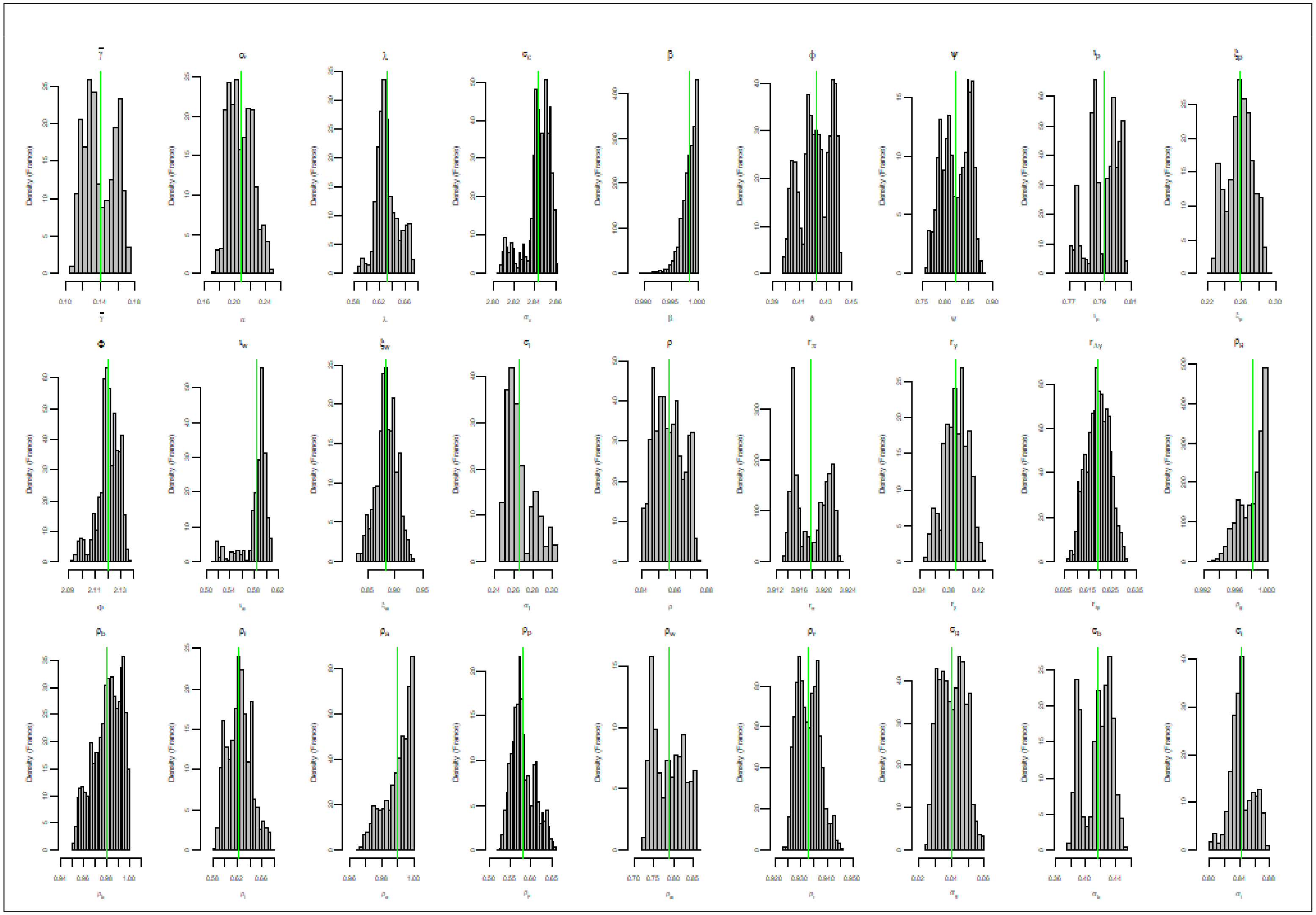


Table A.9: French MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (1/5)

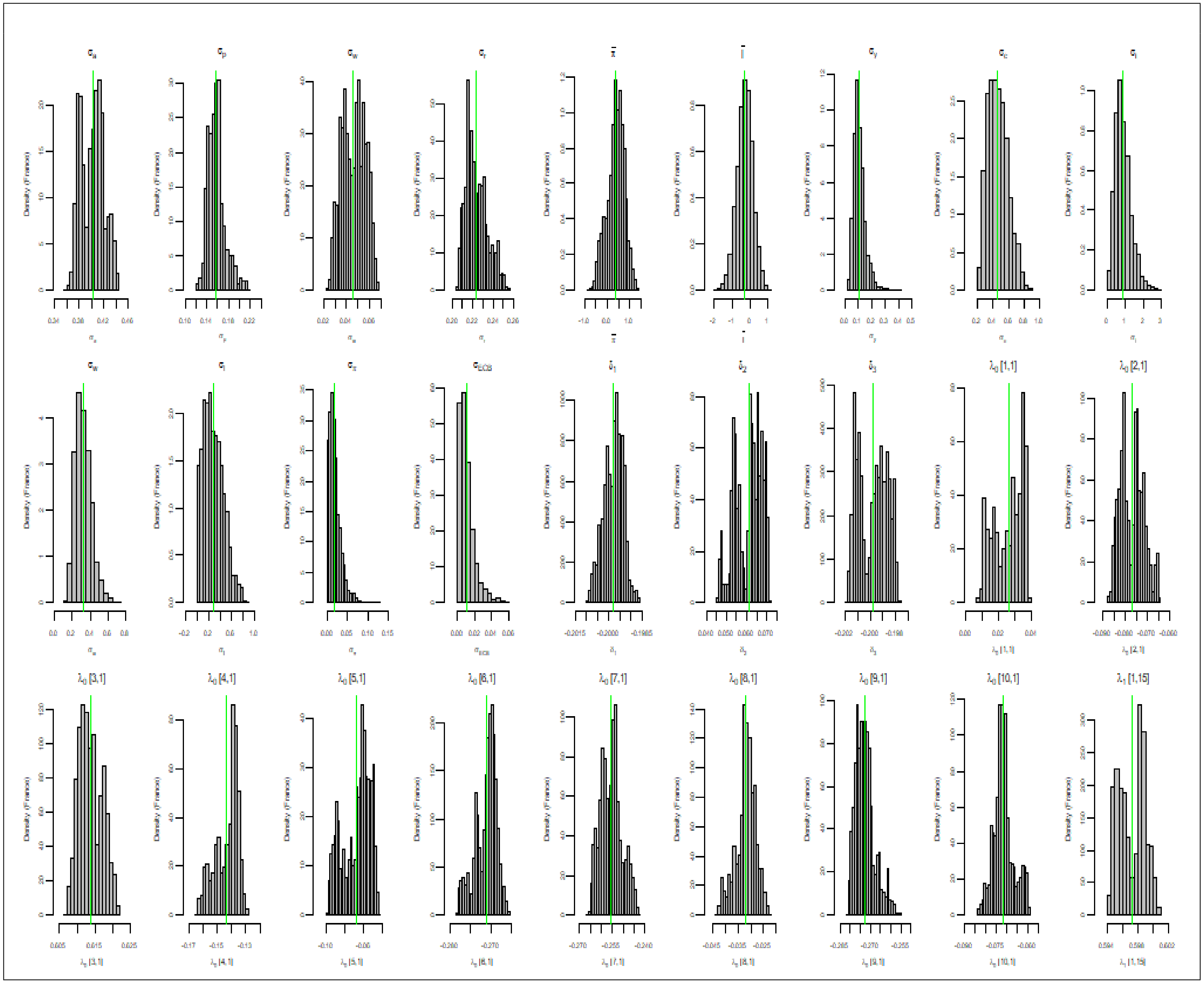


Table A.10: French MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (2/5)

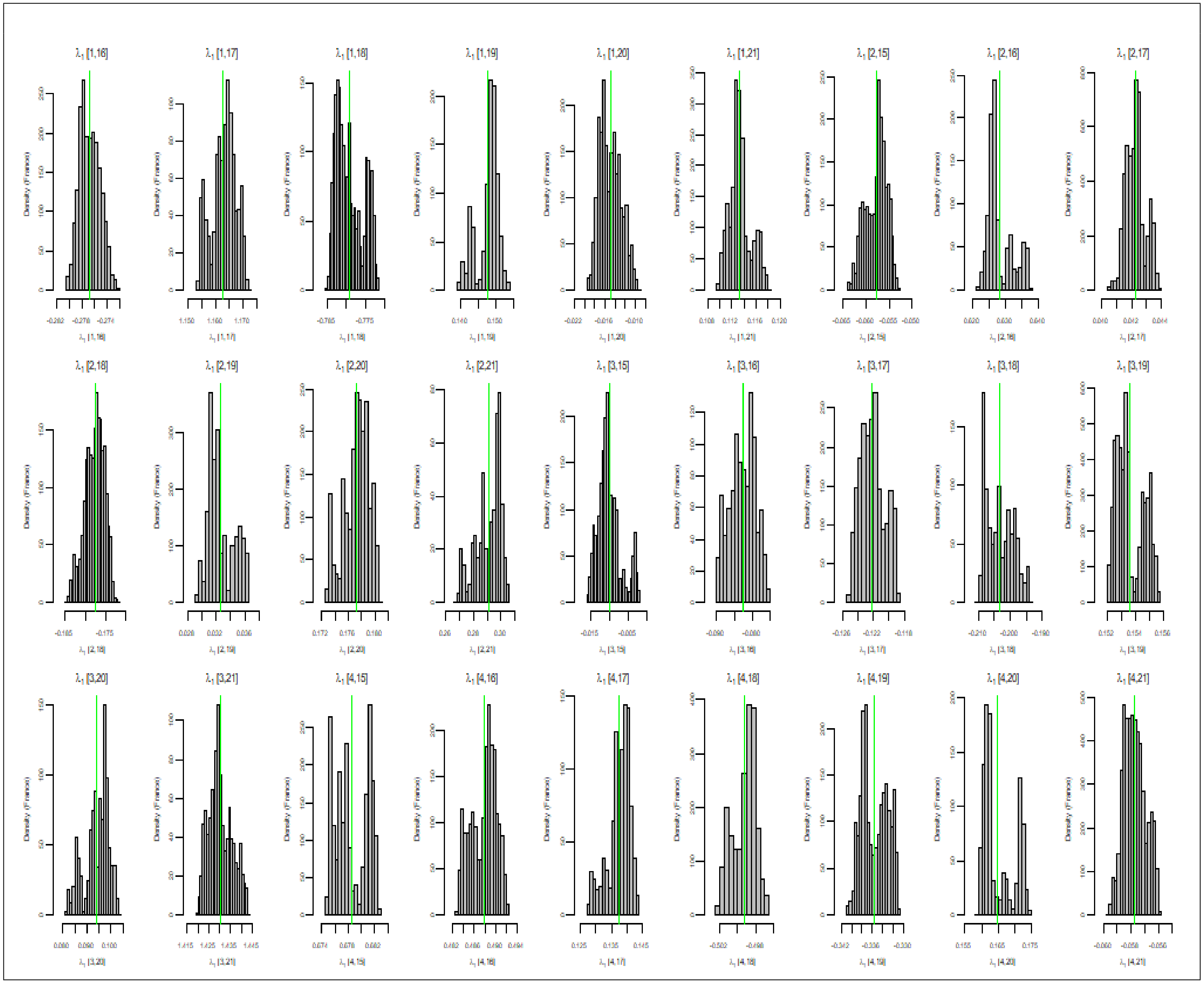


Table A.11: French MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (3/5)

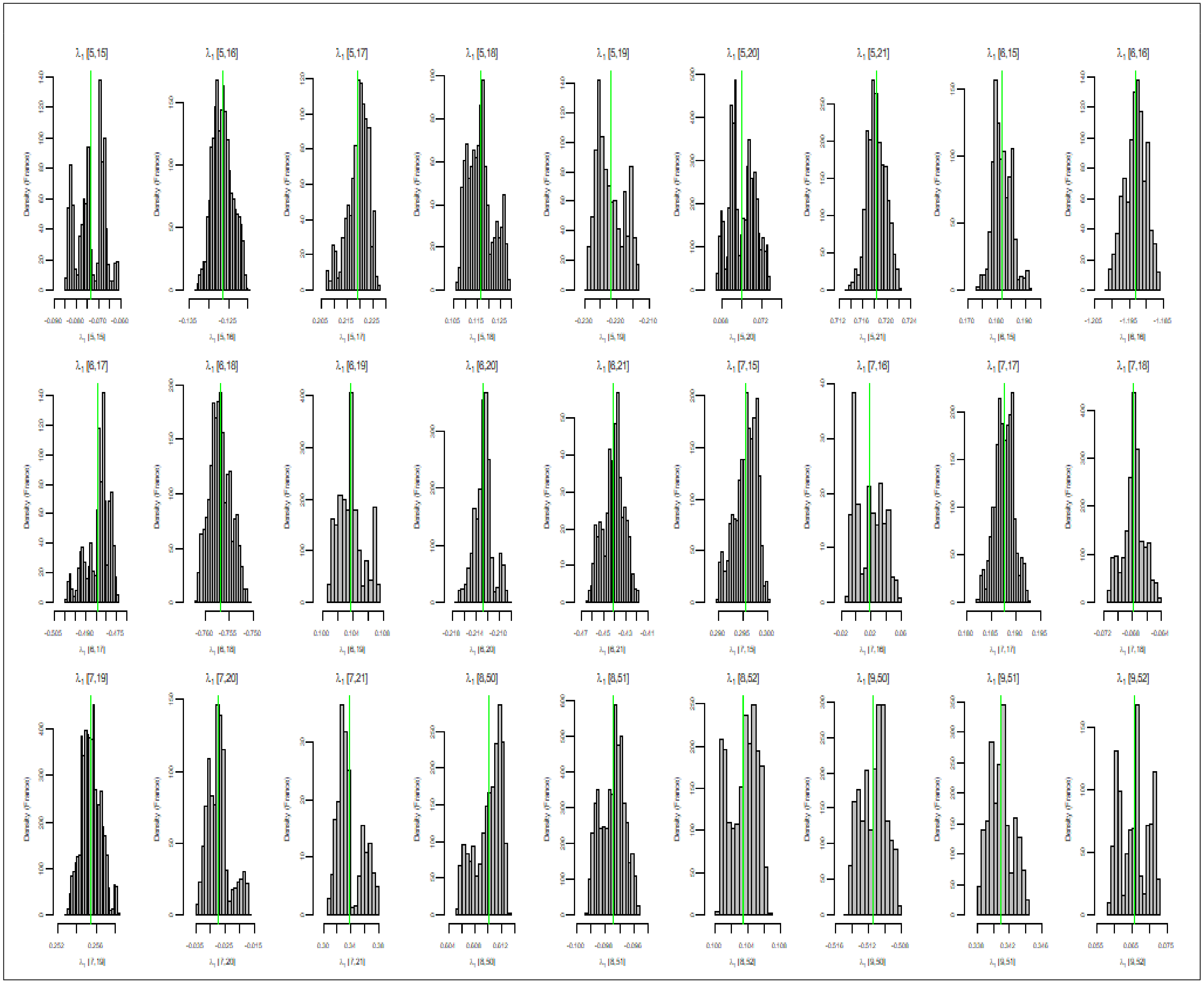


Table A.12: French MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (4/5)

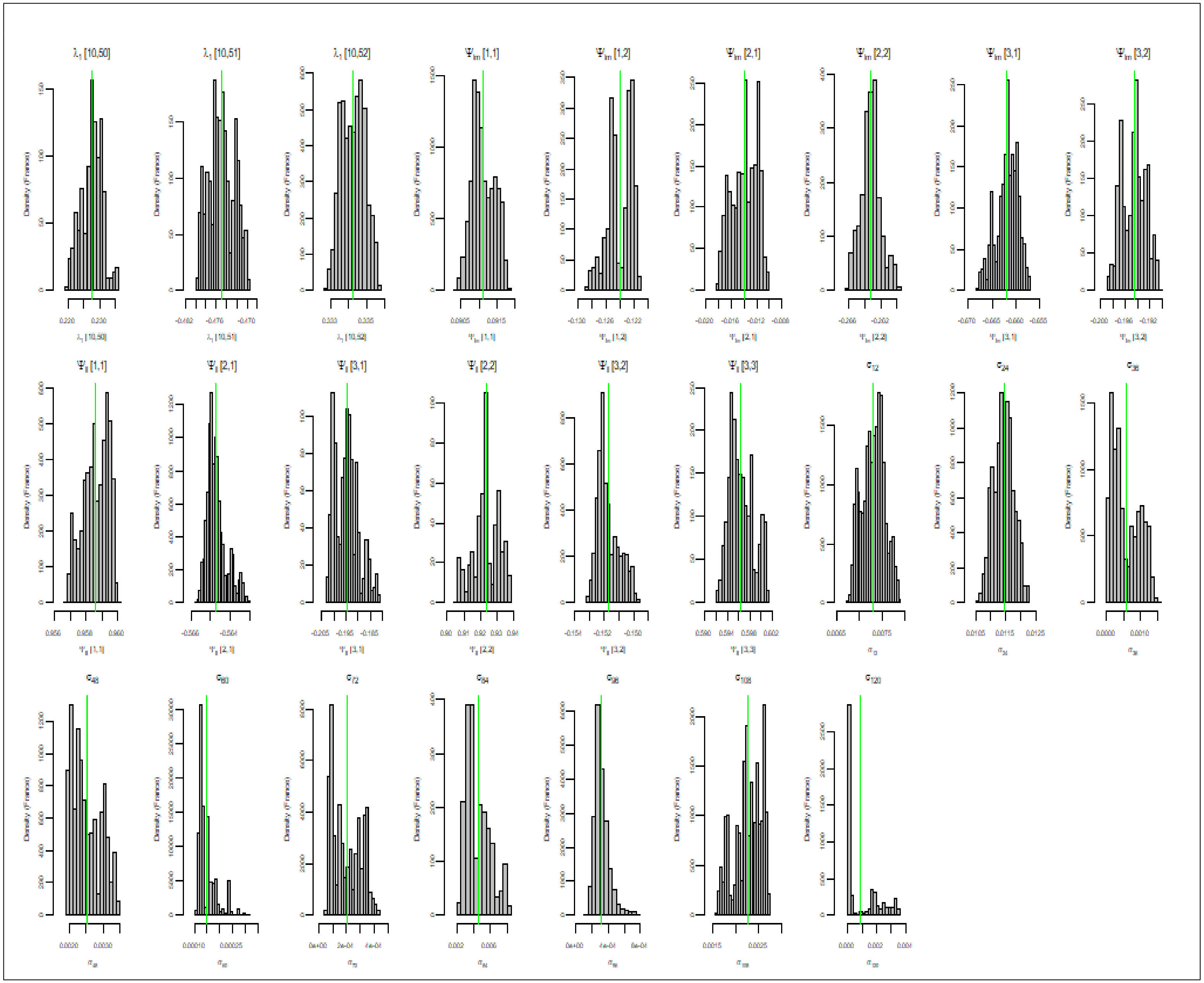


Table A.13: French MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (5/5)

A.9.6 Italian MCMC diagnostics SW-DSGE-ATSM parameter histograms

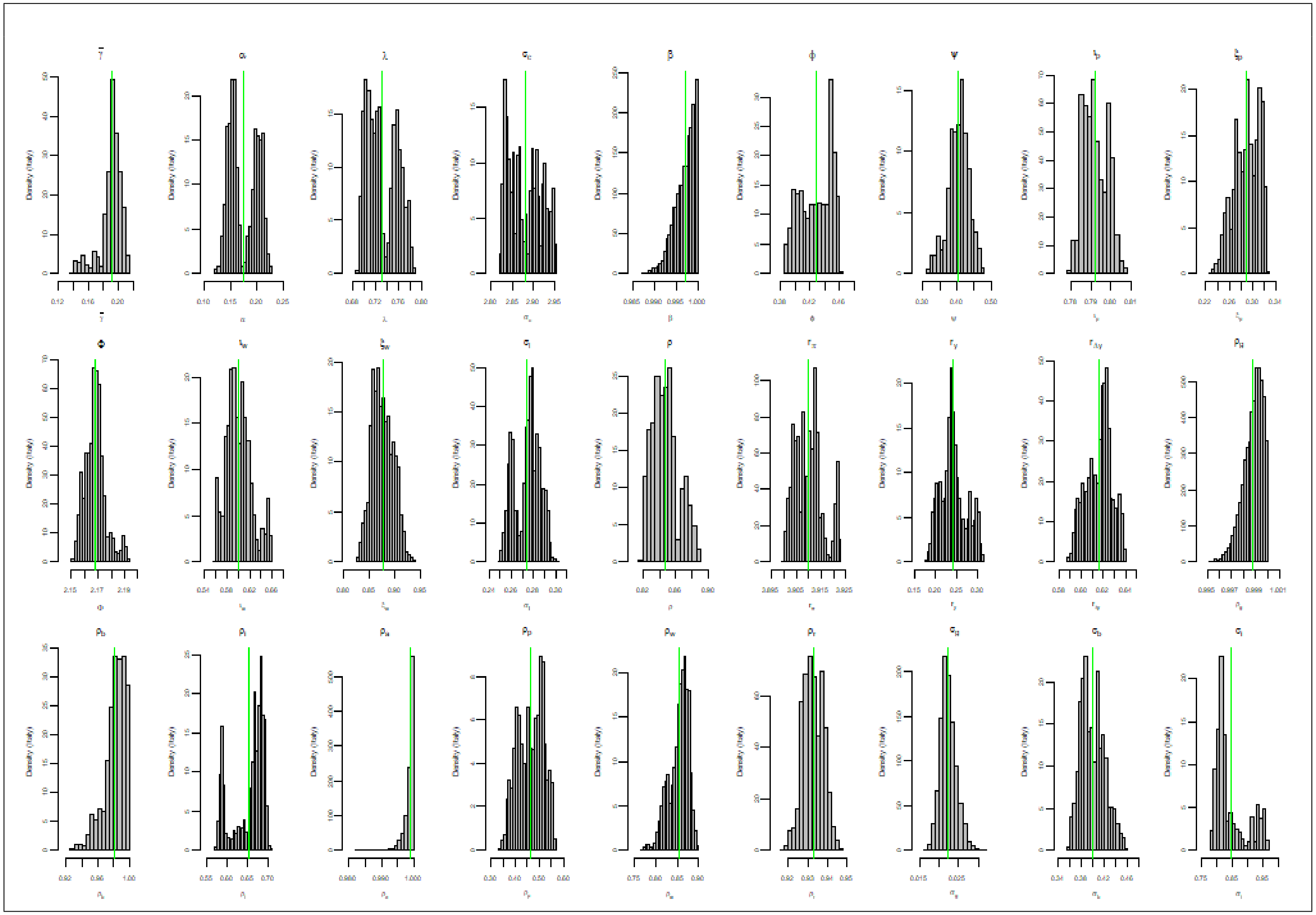


Table A.14: Italian MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (1/5)

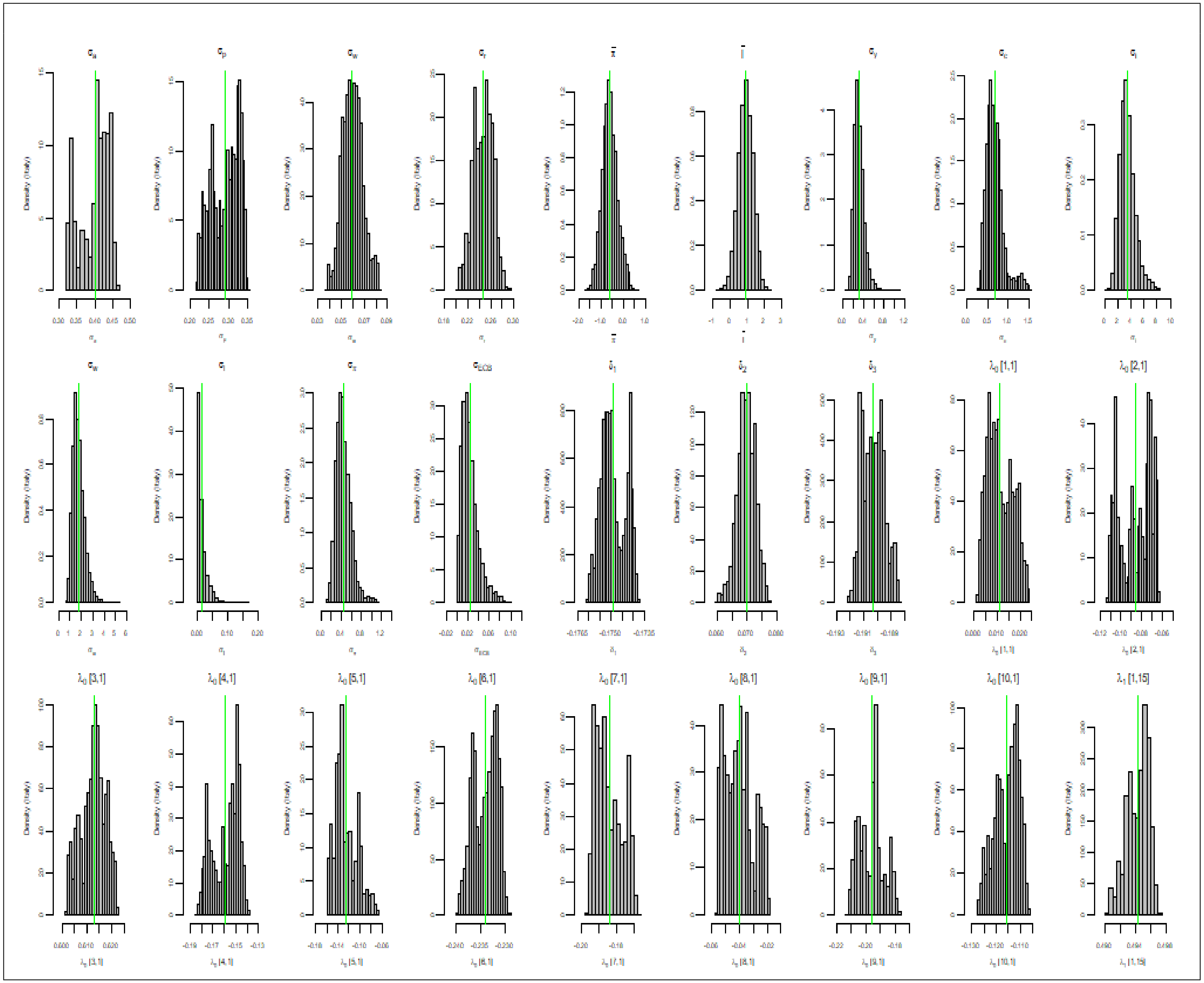


Table A.15: Italian MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (2/5)

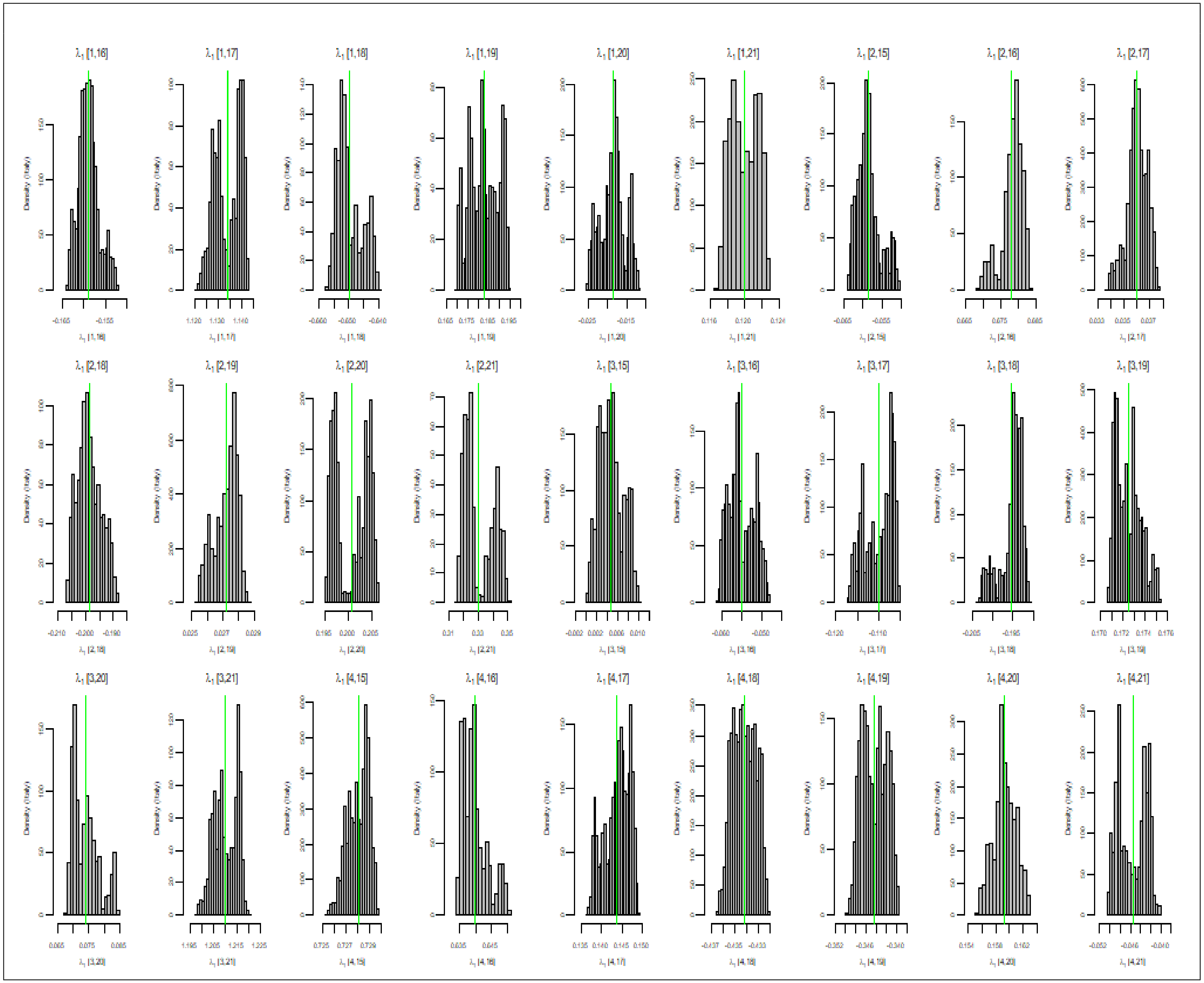


Table A.16: Italian MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (3/5)

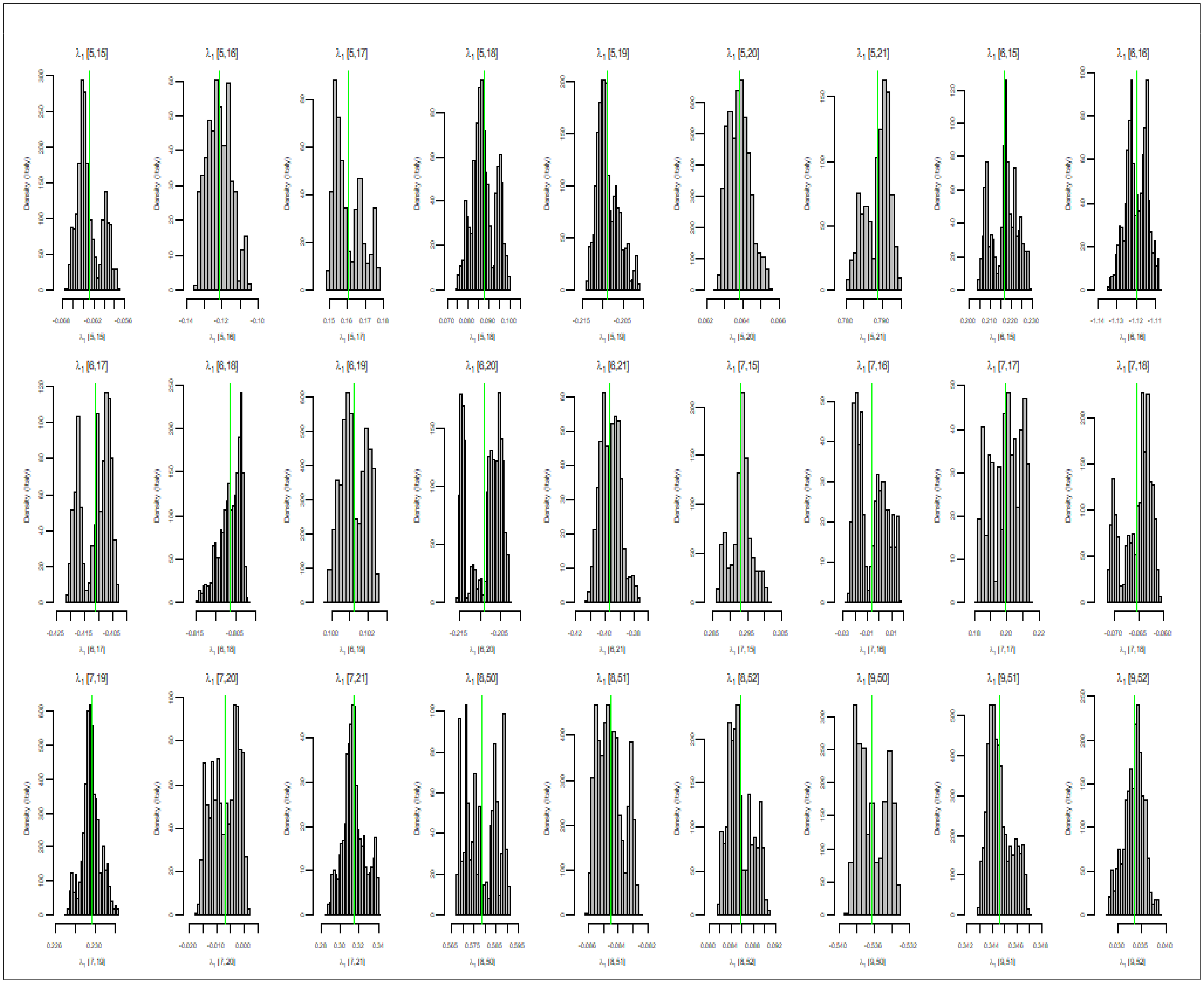


Table A.17: Italian MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (4/5)

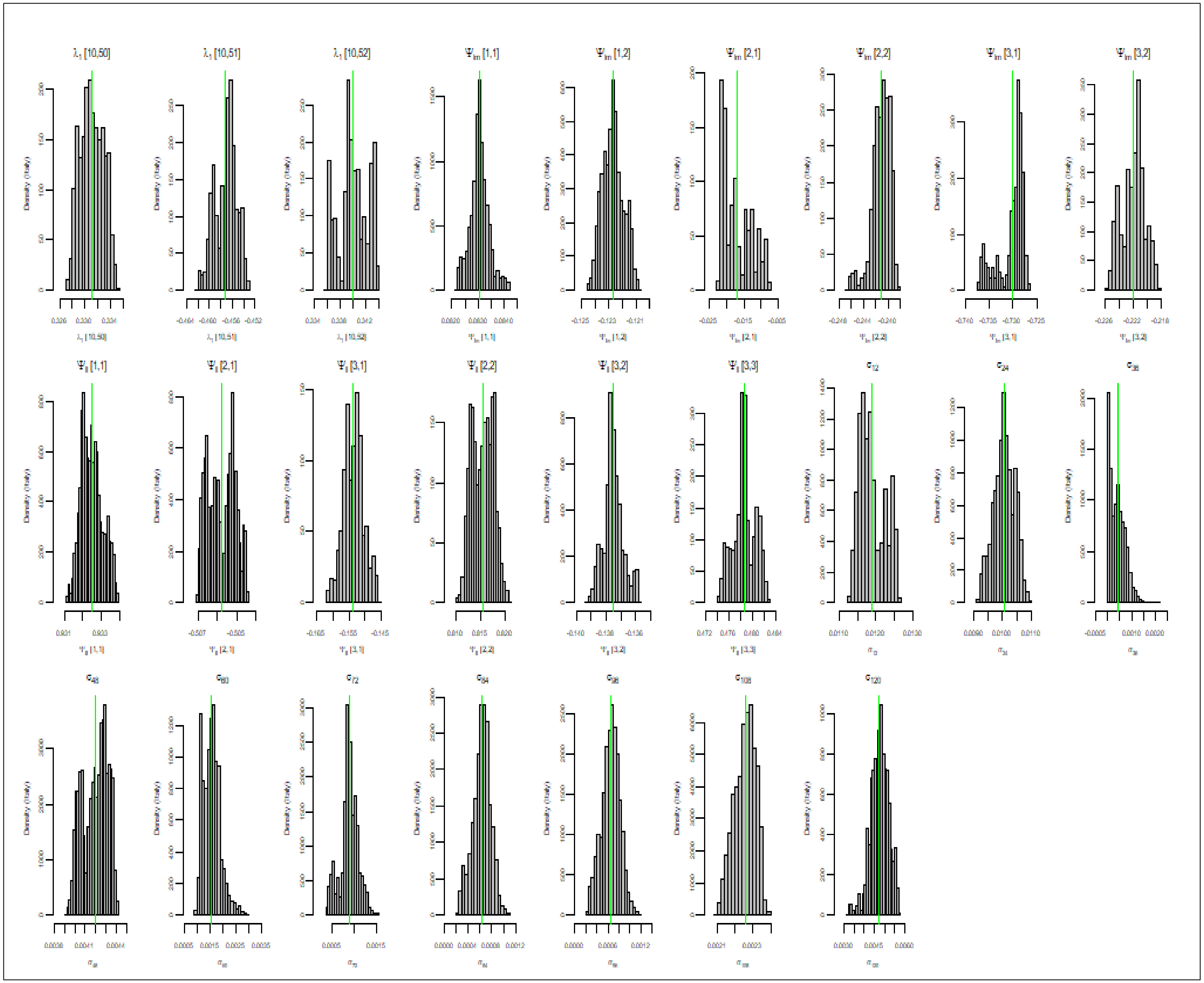


Table A.18: Italian MCMC diagnostic structural SW-DSGE-ATSM parameter distributions from MH drawing (5/5)

A.9.7 Macroeconomic Impulse-Response: France

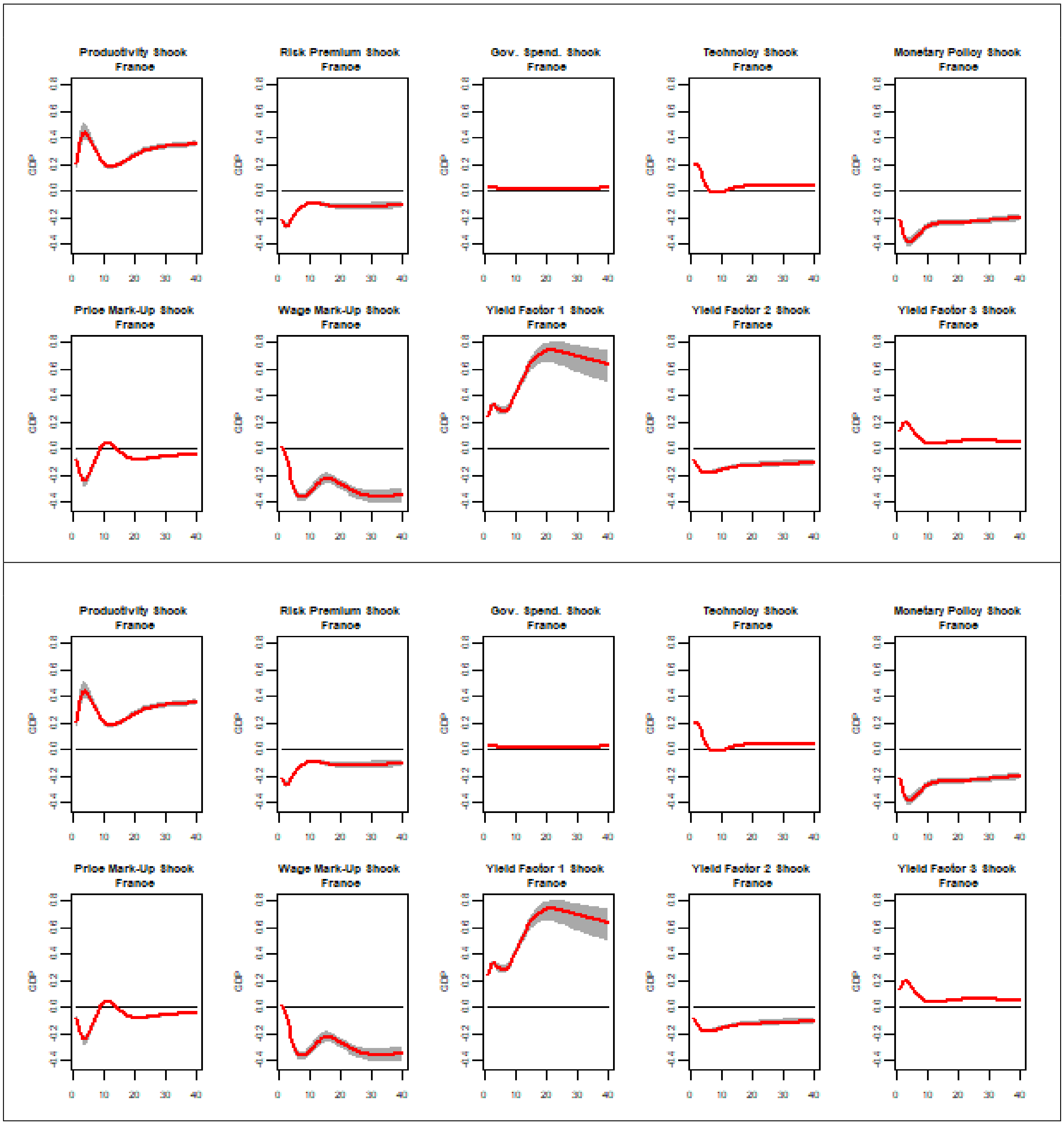


Table A.19: SW-DSGE-ATSM implied responses of GDP and the nominal short term rate to a one standard deviation shock coming from the 10 structural shocks of our SW-DSGE-ATSM for France. For generating the IR's we take the mean of 1000 draws from the models posterior. The shaded areas indicate the [10% , 90%] confidence intervals.

B. Appendix Chapter 3

B.1 Macroeconomic data and data transformation

All of the macroeconomic data used in this paper cover 2005/Q1 to 2014/Q1 and are - except the ECB's monetary policy rate and the world's short term interest rate - from the Area Wide Model (AWM) database. The ECB's monetary policy rate is approximated by the one month EONIA swap rate and the world's short term interest rate by the one month USD-LIBOR. The preparation and transformation of the macroeconomic time series we used for our estimations are as follows (in parenthesis the labels of the macroeconomic variables, with which the variables are listed in the AWM database):

1. EMU's Real Output Growth Rate (YER):

$$\Delta GDP_t^{EMU} = 100 \times \left(ln\left(GDP_t^{EMU}\right) - ln\left(GDP_{t-1}^{EMU}\right)\right)$$

where GDP_t^{EMU} is the quarterly level of the real gross domestic product (in million Euros, reference year in 1995) at quarter t.

2. EMU's Real Consumption Growth Rate (PCR):

$$\Delta CONS_t = 100 \times (ln\,(CONS_t) - ln\,(CONS_{t-1}))$$

where $CONS_t$ are the quarterly real individual consumption expenditures (in million Euros with base year in 1995) at quarter t.

3. EMU's Real Investment Growth Rate (ITR):

$$\Delta INV_t = 100 \times (ln\,(INV_t) - ln\,(INV_{t-1}))$$

where INV_t is the quarterly gross capital formation (in million Euros, reference year in 1995) at quarter t.

4. EMU's Real Government Spending Expenditures (GCR):

$$ln\,(GOV_t) = ln\left(GOV_t^{OBS}\right) - TREND_t^{GOV}$$

where GOV_t^{OBS} are the real quarterly general government final consumption expenditures (in million Euros, reference year in 1995) observed at quarter t. $TREND_t^{GOV}$ is a trend component expressing the assumed NAWM's steady-state growth rate of 2.0 percent per annum.

5. EMU's Real Exports Growth Rate (XTR):

$$\Delta EXPORT_t = 100 \times \left(ln \left(EXPORT_t \right) - ln \left(EXPORT_{t-1} \right) \right)$$

where $EXPORT_t$ is the quarterly export of goods and services (in millions of Euros, reference year in 1995) at quarter t.

6. EMU's Real Imports Growth Rate (MTR):

$$\Delta IMPORT_t = 100 \times \left(ln \left(IMPORT_t \right) - ln \left(IMPORT_{t-1} \right) \right)$$

where $IMPORT_t$ is the quarterly import of goods and services (in millions of Euros, reference year in 1995) at quarter t.

7. EMU's Output Inflation Rate (YED):

$$INF_{Y,t}^{EMU} = 100 \times \left(ln \left(PRICE_{Y,t}^{EMU} \right) - ln \left(PRICE_{Y,t-1}^{EMU} \right) \right)$$

where $PRICE_{Y,t}^{EMU}$ is the observed quarterly index value of the EMU GDP deflator (index base year 1995) at quarter t.

8. EMU's Consumption Inflation Rate (PCD):

$$INF_{C,t} = 100 \times \left(ln \left(PRICE_{C,t} \right) - ln \left(PRICE_{C,t-1} \right) \right)$$

where $PRICE_{C,t}$ is the observed quarterly index value of the EMU individual consumption deflator (index base year 1995) at quarter t.

9. EMU's Import Inflation Rate (MTD):

$$INF_{IM,t} = 100 \times \left(ln \left(PRICE_{IM,t} \right) - ln \left(PRICE_{IM,t-1} \right) \right)$$

where $PRICE_{IM,t}$ is the observed quarterly index value of the EMU import of goods and services deflator (index base year 1995) at quarter t.

10.EMU's Total Employment (LNN):

$$ln \left(LABOR_t \right) = 100 \times \left(ln \left(LABOR_t^{OBS} \right) - ln \left(TREND_t^{LABOR} \right) \right)$$

where $LABOR_t^{OBS}$ is the quarterly total employment (in thousands of persons) observed at quarter t. $TREND_t^{LABOR}$ is a trend component expressing the assumed NAWM's steady-state labor force growth rate of 0.8 percent per annum.

11. EMU's Real Wage Growth Rate (WRN):

$$\Delta WAGE_t = 100 \times \left(ln\left(\frac{WAGE_t}{PRICE_{Y,t}^{EMU}}\right) - ln\left(\frac{WAGE_{t-1}}{PRICE_{Y,t-1}^{EMU}}\right)\right)$$

where $WAGE_t$ is the quarterly observed nominal wage per head at quarter t.

12 .ECB's Monetary Policy Rate:

As already mentioned as an approximation of the ECB's short term monetary policy rate we take the one month EONIA swap rate published by the German Bundesbank where we calculate the quarterly value by averaging over the daily quotes of a quarter.

13. Foreign Exchange Rate (EEN):

$$\Delta FX_t = FX_t^{OBS}\frac{PRICE_{Y,t}^{WORLD}}{PRICE_{Y,t}^{EXPORT}} - \frac{1}{T}\sum_{t=1}^{T}\left(FX_t^{OBS}\frac{PRICE_{Y,t}^{WORLD}}{PRICE_{Y,t}^{EXPORT}}\right)$$

where FX_t^{OBS} is the observed nominal effective exchange rate at quarter t whereas $PRICE_{Y,t}^{EXPORT}$ and $PRICE_{Y,t}^{WORLD}$ are the observed quarterly exports of goods and services deflator and the world GDP deflator observed at quarter t respectively. The real effective exchange rate is adjusted by its mean.

14. World Real Output Growth Rate (YWR):

$$\Delta GDP_t^{WORLD} = 100 \times \left(ln\left(GDP_t^{WORLD}\right) - ln\left(GDP_{t-1}^{WORLD}\right)\right)$$

where GDP_t^{WORLD} is the world GDP (in millions of US-Dollars, reference year 1995) at quarter t.

15. World Output Inflation Rate (YWD):

$$INF_{Y,t}^{WORLD} = 100 \times \left(ln\left(PRICE_{Y,t}^{WORLD}\right) - ln\left(PRICE_{Y,t-1}^{WORLD}\right)\right)$$

where $PRICE_{Y,t}^{WORLD}$ is the observed quarterly index value of the world GDP deflator (index base year 1995) at quarter t.

16. World Short Term Interest Rate:

As a proxy of the world's short term interest rate we take the one month USD-LIBOR queried from the FED St. Louis economic database (FRED mnemonic: USD1MTD156N) where we calculate the quarterly value by averaging over the daily quotes of a quarter.

17. EMU's Competitors' Export Price Inflation Rate (XTD):

$$INF_{EXPORT,t} = 100 \times \left(ln \left(\frac{PRICE_t^{EXPORT}}{PRICE_{Y,t}^{WORLD}} \right) - ln \left(\frac{WAGE_{t-1}^{EXPORT}}{PRICE_{Y,t-1}^{WORLD}} \right) - TREND_{Y,t}^{EXPORT} \right)$$

where $PRICE_{Y,t}^{EXPORT}$ and $PRICE_{Y,t}^{WORLD}$ are the observed quarterly exports of goods and services deflator and the world GDP deflator observed at quarter t respectively. $TREND_t^{EXPORT}$ is a linear trend component.

18. Oil Price (POILU):

$$PRICE_t^{OIL} = 100 \frac{PRICE_t^{OIL,OBS}}{PRICE_{Y,t}^{WORLD}} - TREND_{Y,t}^{OIL}$$

where $PRICE_t^{OIL,OBS}$ is the quarterly observed oil price (UK Brent in US dollars per barrel) at quarter t . $PRICE_{Y,t}^{WORLD}$ is the observed world GDP deflator and $TREND_t^{OIL}$ is a linear trend component.

B.2 Estimation of the EMU yield factors and parameters

Due to the independent modeling of the level and slope factors in3.4 - 3.6 the EMU wide level and slope factors $\mathbf{L}^T = [L_1, L_2, ..., L_T]$ and $\mathbf{S}^T = [S_1, S_2, ..., S_T]$ and the factor related parameters $\boldsymbol{\theta}_i^k = \left[\phi^k \alpha_i^k, \beta_i^k, \psi_i^k \sigma_i^k\right]$ with $i = 1, 2, ..., N$ and $k = l, s$ can be estimated separately by two independent runs of a Markov Chain Monte Carlo (MCMC) algorithm as worked out by Diebold, Li and Yue [2008]. Defining $\mathbf{F} = [\mathbf{L}, \mathbf{S}]^T$ and $\hat{\boldsymbol{f}} = \left[\hat{\boldsymbol{l}}, \hat{\boldsymbol{s}}\right]^T$ with $\hat{\boldsymbol{l}} = \left[\hat{\boldsymbol{l}}_1, \hat{\boldsymbol{l}}_2, ..., \hat{\boldsymbol{l}}_N\right]^T$ and $\hat{\boldsymbol{s}} = [\hat{\boldsymbol{s}}_1, \hat{\boldsymbol{s}}_2, ..., \hat{\boldsymbol{s}}_N]^T$ as vectors of the EMU wide and country specific factors with $\hat{\boldsymbol{l}}_i^T = \left[\hat{l}_{i,1}, \hat{l}_{i,2}, ..., \hat{l}_{i,T}\right]$ and $\hat{\boldsymbol{s}}_i^T = [\hat{s}_{i,1}, \hat{s}_{i,2}, ..., \hat{s}_{i,T}]$ for $i = 1, 2, ..., N$, both runs of the MCMC procedure are used to approximate the joint marginal posterior distributions $p\left(\boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k, \mathbf{F}^k\right)$ with $k = l, s$ where $\mathbf{F}^k$ is the EMU wide level or slope factor. By Bayes theorem $p\left(\boldsymbol{\theta}_i^k, \mathbf{F}^k \hat{\boldsymbol{f}}^k\right)$ can be written as:

$$\begin{aligned} p\left(\boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k, \mathbf{F}^k, \hat{\boldsymbol{f}}^k\right) &= p\left(\hat{\boldsymbol{f}}^k | \boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k, \mathbf{F}^k\right) \\ &\quad \times p\left(\boldsymbol{F}^k | \boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k\right) p\left(\boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k\right) \quad k = l, s \end{aligned} \tag{B.1}$$

with $p\left(\boldsymbol{\theta}_1^k, \boldsymbol{\theta}_2^k, ..., \boldsymbol{\theta}_N^k\right) = \prod_{i=1}^N p\left(\boldsymbol{\theta}_i^k\right)$ due to the independence of $\boldsymbol{l}_i$ and $\boldsymbol{s}_i$ across the countries $i = 1, 2, ..., N$. For every country i $p\left(\boldsymbol{\theta}_i^k\right)$ can be expressed as:

$$p\left(\boldsymbol{\theta}_i^k\right) = p\left(\boldsymbol{\gamma}_i^k | \psi_i^k, \sigma_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k\right) p\left(\sigma_i^k | \boldsymbol{\gamma}_i^k \psi_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k\right) \times p\left(\psi_i^k | \boldsymbol{\gamma}_i^k, \sigma_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k\right) p\left(\phi^k | \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k\right) \tag{B.2}$$

with $\boldsymbol{\gamma}_i^k = \left[\alpha_i^k, \beta_i^k\right]^T$ and $\hat{\boldsymbol{f}}_i^k$ as the k-th factor of the i-th country. Here the prior distributions for $\boldsymbol{\gamma}_i^k$, σ_i^k and ψ_i^k in 3.6 are the posterior distributions of the linear regressions 3.5 and 3.6 formulated in a Bayesian framework. To derive the posterior distribution of $\boldsymbol{\gamma}_i^k$ conditional to $\sigma_i^k, \psi_i^k, \mathbf{F}^k$ and $\hat{\boldsymbol{f}}_i^k$ 3.5 and 3.6 can be combined to:

$$\begin{aligned} \hat{\boldsymbol{f}}_i^k - \psi_i^k L\left[\hat{\boldsymbol{f}}_i^k\right] &= \left[\boldsymbol{e}\left(1 - \psi_i^k\right), \left(\mathbf{F}^k - \psi_i^k L\left[F^k\right]\right)\right] \boldsymbol{\gamma}_i^k + \boldsymbol{u}_i^k \\ \Delta \hat{\boldsymbol{f}}_i^k &= \mathbf{X} \boldsymbol{\gamma}_i^k + \boldsymbol{u}_i^k \end{aligned} \tag{B.3}$$

where $L[*]$ is the lag operator. Then the bivariate (conditional) posterior distribution of the parameter vector $\boldsymbol{\gamma}_i^k$ from the regression B.3 is:

$$\boldsymbol{\gamma}_i^k | \psi_i^k, \sigma_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k \sim N\left(\boldsymbol{\mu}_{\gamma,i}^k, \boldsymbol{\Sigma}_{\gamma,i}^k\right) \tag{B.4}$$

with mean:

$$\boldsymbol{\mu}_{\gamma,i}^k = \left[\left(\tilde{\boldsymbol{\Sigma}}_{\gamma,i}^k\right)^{-1} + \left(\sigma_i^k\right)^{-2} \mathbf{X}^T \mathbf{X}\right]^{-1} \left[\tilde{\boldsymbol{\Sigma}}_{\gamma,i}^k \tilde{\boldsymbol{\mu}}_{\gamma,i}^k + \left(\sigma_i^k\right)^{-2} \mathbf{X}^T \Delta \hat{\boldsymbol{f}}_i^k\right] \tag{B.5}$$

and covariance:

$$\boldsymbol{\Sigma}_{\gamma,i}^k = \left[\left(\tilde{\boldsymbol{\Sigma}}_{\gamma,i}^k\right)^{-1} + \left(\sigma_i^k\right)^{-2} \mathbf{X}^T \mathbf{X}\right]^{-1} \tag{B.6}$$

where$\tilde{\boldsymbol{\mu}}_{\gamma,i}^k = \mathbf{0}$ and $\tilde{\boldsymbol{\Sigma}}_{\gamma,i}^k = \mathbf{I}_{2\times 2}$ are the prior mean and the prior precision matrix. For getting the (conditional) prior $p\left(\psi_i^k | \boldsymbol{\gamma}_i^k, \sigma_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k\right)$ in B.2 the AR[1] process with parameter ψ_i^k in 3.6 is combined with the regression 3.5, so that a linear regression model with parameter ψ_i^k conditional to $\boldsymbol{\gamma}_i^k, \sigma_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k$ can be formulated:

$$\begin{aligned} \hat{\boldsymbol{f}}_i^k - \left[\boldsymbol{e}, \mathbf{F}^k\right] \boldsymbol{\gamma}_i^k &= \psi_i^k \left(L\left[\hat{\boldsymbol{f}}_i^k\right] - \left[\boldsymbol{e}, L\left[\mathbf{F}^k\right] \boldsymbol{\gamma}_i^k\right]\right) + \boldsymbol{u}_i^k \\ \hat{\boldsymbol{\varepsilon}}_i^k &= \psi_i^k L\left[\hat{\boldsymbol{\varepsilon}}_i^k\right] + \boldsymbol{u}_i^k \end{aligned} \tag{B.7}$$

where $\hat{\boldsymbol{\varepsilon}}_i^k$ and $L\left[\hat{\boldsymbol{\epsilon}}_i^k\right]$ collect the current and lagged residuals calculated with the parameters and variables $\boldsymbol{\theta}_i^k, \sigma_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k$. As for the regression in B.3 the (conditional) posterior distribution of the ψ_i^k is:

$$\psi_i^k | \boldsymbol{\gamma}_i^k, \sigma_i^k, \mathbf{F}^k, \hat{\boldsymbol{f}}_i^k \sim N\left(\mu_{\psi,i}^k, \sigma_{\psi,i}^k\right) I\left[\psi_i^k\right] \tag{B.8}$$

with mean:

$$\mu_{\psi,i}^{k} = \left[\left(\tilde{\sigma}_{\psi,i}^{k}\right)^{-1} + \left(\sigma_{i}^{k}\right)^{-2} L\left[\hat{\boldsymbol{\varepsilon}}_{i}\right]^{t} L\left[\hat{\boldsymbol{\varepsilon}}_{i}\right]\right]^{-1} \left[\tilde{\sigma}_{\psi,i}^{k}\tilde{\mu}_{\psi,i}^{k} + \left(\sigma_{i}^{k}\right)^{-2} L\left[\hat{\boldsymbol{\varepsilon}}_{i}\right]^{t} L\left[\hat{\boldsymbol{\varepsilon}}_{i}\right]\right] \tag{B.9}$$

and variance:

$$\sigma_{\psi,i}^{k} = \left[\left(\tilde{\sigma}_{\psi,i}^{k}\right)^{-1} + \left(\sigma_{i}^{k}\right)^{-2} L\left[\hat{\boldsymbol{\varepsilon}}_{i}\right]^{t} L\left[\hat{\boldsymbol{\varepsilon}}_{i}\right]\right]^{-1} \tag{B.10}$$

where $\mu_{\psi,i}^{k} = 0$ and $\sigma_{\psi,i}^{k} = 1$ are the prior mean and the prior precision. $I\left[\psi_{i}^{k}\right]$ is an indicator function with $I\left[\psi_{i}^{k}\right] = 1$ for $\psi_{i}^{k} \leq 1$ and $I\left[\psi_{i}^{k}\right] = 0$ for $\psi_{i}^{k} > 1$ to guarantee stationarity of the AR[1] process in 3.6. The prior for σ_{i}^{k} in B.2 is given by the posterior distribution of σ_{i}^{k} used in the derivation of the joint posteriors $p\left(\boldsymbol{\gamma}_{i}^{k}, \sigma_{i}^{k} | \Delta\hat{\boldsymbol{f}}_{i}^{k}, \mathbf{X}\right)$ and $p\left(\psi_{i}^{k}, \sigma_{i}^{k} | \hat{\boldsymbol{\varepsilon}}_{i}^{k}\right)$ for the (Bayesian) linear regressions in B.3 and B.7. Using the regression in B.3 the (conditional) posterior of σ_{i}^{k} is:

$$\sigma_{i}^{k} | \boldsymbol{\gamma}_{i}^{k}, \psi_{i}^{k}, \mathbf{F}^{k}, \hat{\boldsymbol{f}}_{i}^{k} \sim IG\left(\vartheta, \kappa\right) \tag{B.11}$$

where IG denotes the inverse gamma distribution with scale $\vartheta = \left(\tilde{\vartheta} + T - 1\right)/2$ and shape $\kappa = \left(\tilde{\kappa} + \left[\Delta\tilde{\boldsymbol{f}}_{i}^{k} - \mathbf{X}\boldsymbol{\gamma}_{i}^{k}\right]^{T} \left[\Delta\tilde{\boldsymbol{f}}_{i}^{k} - \mathbf{X}\boldsymbol{\gamma}_{i}^{k}\right]\right)/2$. $\tilde{\vartheta}$ and $\tilde{\kappa}$ are the prior scale and shape parameters. ϕ^{k} is unconditional to the parameters $\boldsymbol{\gamma}_{i}^{k}, \psi_{i}^{k}$ and σ_{i}^{k} and only depends on the EMU wide factor $\mathbf{F}^{k}$. The posterior of ϕ^{k} for the regression in 3.4 is:

$$\phi^{k} | \mathbf{F}^{k} \sim N\left(\mu_{\phi}^{k}, \sigma_{\phi}^{k}\right) I\left[\phi^{k}\right] \tag{B.12}$$

with mean:

$$\mu_{\phi}^{k} = \left[\left(\tilde{\sigma}_{\phi}^{k}\right)^{-1} + L\left[\mathbf{F}^{k}\right]^{T} L\left[\mathbf{F}^{k}\right]\right]^{-1} \left[\left(\tilde{\sigma}_{\phi}^{k}\right)\tilde{\mu}_{\phi}^{k} + L\left[\mathbf{F}^{k}\right]^{T} L\left[\mathbf{F}^{k}\right]\right] \tag{B.13}$$

and variance:

$$\mu_{\phi}^{k} = \left[\left(\tilde{\sigma}_{\phi}^{k}\right)^{-1} + L\left[\mathbf{F}^{k}\right]^{T} L\left[\mathbf{F}^{k}\right]\right]^{-1} \tag{B.14}$$

where $\tilde{\mu}_{\phi}^{k}$ and $\tilde{\sigma}_{\phi}^{k}$ are the prior mean and precision. $I\left[\phi^{k}\right]$ is the indicator function with $I\left[\phi^{k}\right] = 1$ for $\phi^{k} \leq 1$ and $I\left[\phi^{k}\right] = 0$ for $\phi^{k} > 1$. The indicator function guarantees the stationarity of the AR[1] process defined in 3.4.

The (conditional) prior $p\left(\mathbf{F}^{k} | \boldsymbol{\theta}_{1}^{k}, ..., \boldsymbol{\theta}_{N}^{k}, \hat{\boldsymbol{f}}_{1}^{k}, ..., \hat{\boldsymbol{f}}_{N}^{k}\right)$ on the RHS of B.2 for the k-th EMU factor $\mathbf{F}^{k}$ is derived by Carter and Kohn [1994] and Kim and Nelson [1999] where a reformulation of B.3 and 3.4 specify the state space model with measurement equation:

$$\Delta\hat{\boldsymbol{f}}_{i}^{k} = \mathbf{C} + \mathbf{H}\begin{bmatrix} F_{t}^{k} \\ F_{t-1}^{k} \end{bmatrix} + \boldsymbol{u}_{i}^{k} \tag{B.15}$$

derived from B.3 where $\mathbf{C} = \left[\boldsymbol{e}\left(1-\psi_i^k\right), \mathbf{0}\right]\boldsymbol{\gamma}^k$ is the 2×1 intercept and $\mathbf{H} = \left[\left[\mathbf{0}, \mathbf{I}\right]\boldsymbol{\gamma}^k, \left[\mathbf{0}, \boldsymbol{\Lambda}_\psi\right]\boldsymbol{\gamma}^k\right]$ is the 2×2 coefficient matrix. The $N \times 1$ vector $\Delta\hat{\boldsymbol{f}}_t^k = \left[\Delta\hat{f}_{1,t}, \Delta\hat{f}_{2,t}, ..., \Delta\hat{f}_{N,t}\right]^T$ contains the country specific changes of the factors k from B.3 at time t, the $N \times 2$ matrix $\boldsymbol{\gamma}^k = [\boldsymbol{\gamma}_1^k, ...\boldsymbol{\gamma}_2^k, ..., \boldsymbol{\gamma}_N^k]$ collects the vectors $\boldsymbol{\gamma}_i^k = \left[\alpha_i^k, \beta_i^k\right]^T$ for the EMU countries $i = 1, 2, ..., N$ and $\boldsymbol{\Lambda}_\psi = \sum_{i=1}^N I_{N\times N}\boldsymbol{\psi}^k\boldsymbol{e}^T$ is a $N \times N$ diagonal matrix with the elements of $\boldsymbol{\psi}^k = \left[\psi_1^k, ..., \psi_N^k\right]$ on its diagonal and $\mathbb{E}_t\left[\boldsymbol{u}_t^k\boldsymbol{u}_t^{k\,T}\right]$ where $\boldsymbol{\Sigma}^k$ is diagonal with $\left(\sigma_i^k\right)^2$ as the variance of the i-th measurement error. F_t^k is the k-th factor at time t in $\mathbf{F}$. From 3.4 the transition equation of the factors $\mathbf{F}^k$ is derived with:

$$\begin{bmatrix} F_t^k \\ F_{t-1}^k \end{bmatrix} = \mathbf{G}\begin{bmatrix} F_{t-1}^k \\ F_{t-2}^k \end{bmatrix} + \begin{bmatrix} U_t^k \\ 0 \end{bmatrix} \tag{B.16}$$

with $\mathbf{G} = \begin{bmatrix} \phi^k & 0 \\ 0 & 0 \end{bmatrix}$ and $\mathbb{E}_t\left[\begin{bmatrix} U_t^k & 0 \end{bmatrix}\begin{bmatrix} U_t^k \\ 0 \end{bmatrix}\right] = \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. To get the EMU factor series $\mathbf{F}^k$ conditional to the country specific parameters and factors $\boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k, \hat{\boldsymbol{f}}_1^k, ..., \hat{\boldsymbol{f}}_N^k$, the state space model from B.15 and B.16 is extracted with the Kalman filter. From running the Kalman filter over the time series $\Delta\hat{\boldsymbol{f}}_i^k$ the series of updated factors $\hat{\mathbf{F}}_{t|t}^k$ and updated MSE matrices $\mathbf{P}_{t|t}^k$ are generated for $t = 2, 3, ..., T$. With $\hat{\mathbf{F}}_{t|t}^k$ and $\mathbf{P}_{t|t}^k$ building the prior distribution $p\left(\mathbf{F}^k|\boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k, \hat{\boldsymbol{f}}_1^k, ..., \hat{\boldsymbol{f}}_N^k\right)$ is done by iterating backwards, starting at time $T-1$ with $\hat{\mathbf{F}}_{T|T}^k$ and $\mathbf{P}_{T|T}^k$. The distribution of the k-th factor F_t^k at time t then is given by:

$$\mathbf{F}_t^k|F_{1,t+1}^k, \boldsymbol{\theta}_1^k, ..., \boldsymbol{\theta}_N^k, \hat{\boldsymbol{f}}_1^k, ..., \hat{\boldsymbol{f}}_N^k \sim N\left(\mathbf{F}_{t|t,F_{1|t}^k}, \mathbf{P}_{t|t,F_{1|t}^k}^k\right) \tag{B.17}$$

with mean:

$$\mathbf{F}_{t|t,F_{1|t}^k} = \hat{\mathbf{F}}_{t|t}^k + \mathbf{P}_{t|t}^k\mathbf{g}^T\left(\mathbf{g}\mathbf{P}_{t|t}^k\mathbf{g}^T + \mathbf{q}\right)^{-1}\left(F_{1,t+1}^k - \mathbf{g}\hat{\mathbf{F}}_{t|t}^k\right) \tag{B.18}$$

and covariance:

$$\mathbf{P}_{t|t,F_{1|t}^k}^k = \mathbf{P}_{t|t}^k - \mathbf{P}_{t|t}^k\mathbf{g}^T\left(\mathbf{g}\mathbf{P}_{t|t}^k\mathbf{g}^T + \mathbf{q}\right)^{-1}\mathbf{g}\mathbf{P}_{t|t}^k \tag{B.19}$$

$\boldsymbol{g} = \left[\phi^k, 0\right]$ and $\boldsymbol{q} = [1, 0]$ are the first rows of $\boldsymbol{G}$ and $\boldsymbol{Q}$ respectively. $F_{1,t+1}^k$ is the first element of the of foregoing two dimensional $\mathbf{F}_{t+1}^k$ in the backward iteration.

B.3 Determining recession probabilities conditional to $\hat{s}_t$

As in Estrella and Mishkin [1996] and in Estrella und Trubin [2006] we use the Probit model:

$$P\left(I_{GER}\left(t+12\right) = 1|\Delta_y\left(\hat{s}_{GER,t}\right)\right) = F\left(\alpha + \beta\Delta_y\left(\hat{s}_{GER,t}\right)\right) \tag{B.20}$$

for estimating the recession probability $P\left(I_{GER}\left(t+12\right)=1|\Delta_y\left(\hat{s}_{GER,t}\right)\right)$ between 09/1972 and 02/2014. $I_{GER}\left(t+12\right)$ is the (binary) OECD recession indicator (where $I_{GER}\left(t+12\right)=1$ indicates a recession) for Germany and $\hat{s}_{GER,t}$ is the German slope factor extracted by estimating the reduced two factor independent DNS on German term structure data between 09/1972 and 02/2014 provided by the German Bundesbank. $\Delta_y\left(\hat{s}_{GER,t}\right)=c\hat{s}_{GER,t}$ is the term term spread between the 120 and 12 month yields $y(t,120)$ and $y(t,12)$ where c is a constant with:

$$c=\frac{\left(1-exp\left(-120\lambda\right)\right)}{120\lambda}-\frac{\left(1-exp\left(-12\lambda\right)\right)}{12\lambda} \tag{B.21}$$

determined by the maturity dependent DNS weightings of the slope factor.

$$F\left(z\right)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}exp\left(-\frac{x^2}{2}dx\right) \tag{B.22}$$

is the (standardized) Gaussian cumulative probability function. With the T observations between 09/1972 and 02/2014, estimation of the Probit's parameters $[\alpha,\beta]$ is done by MLE with:

$$\begin{aligned}\left[\hat{\alpha},\hat{\beta}\right]=\arg\max_{[\alpha,\beta]}\sum_{t=1}^{T}&(I_{GER}\left(t+12\right)ln\left(F\left(\alpha+\beta\Delta_y\left(\hat{s}_{GER,t}\right)\right)\right)\\&+\left(1-I_{GER}\left(t+12\right)\right)ln\left(1-F\left(\alpha+\beta\Delta_y\left(\hat{s}_{GER,t}\right)\right)\right)\end{aligned} \tag{B.23}$$

B.4 Implementation and estimation of the NAWM

B.4.1 Market clearing, aggregate constraint and relative prices

The log-linearized market clearing in the capital market implies that the effective capital utilization equals the capital served in the production process of the NAWM economy:

$$\hat{u}_t=\hat{k}_t=\hat{k}_t^s \tag{B.24}$$

Log-linearized market clearing in the markets for final private and public consumption goods as well as for the investment goods implies:

$$\hat{q}_t^c=\hat{c}_t \tag{B.25}$$

$$\hat{q}_t^g=\hat{g}_t \tag{B.26}$$

$$\hat{q}_t^i=\hat{i}_t+r_kp_i^{-1}g_z^{-1}\frac{k}{q^i}\hat{u}_t \tag{B.27}$$

where the last term on the LHS of the market clearing in the investment goods market is from the adjustment costs in the variation of the intensity of capital's effective utilization $\hat{u}_t$.

EMU wide aggregation in nominal and real terms is given by:

$$\begin{aligned}\hat{p}_{y,t} + \hat{y}_t = & \frac{p_c c}{p_y y}\left(\hat{p}_{c,t} + \hat{c}_t\right) + \frac{p_i i}{p_y y}\left(\hat{p}_{i,t} + \hat{i}_t\right) + \frac{p_i k g_z^{-1}}{p_y y}\gamma_{u,1}\hat{u}_t \\ & + \frac{p_g g}{p_y y}\left(\hat{p}_{g,t} + \hat{g}_t\right) + \frac{p_x x}{p_y y}\left(\hat{p}_{x,t} + \hat{x}_t\right) \\ & - \left[\frac{p_{im} im^c}{p_y y}\left(\hat{p}_{im,t} + \hat{im}_t^c - \hat{\Gamma}_{im^c,t}\right) + \frac{p_{im} im^i}{p_y y}\left(\hat{p}_{im,t} + \hat{im}_t^i - \hat{\Gamma}_{im^i,t}\right)\right]\end{aligned} \tag{B.28}$$

$$\hat{y}_t = \frac{h}{y}\hat{h}_t + \frac{x}{y}\hat{x}_t \tag{B.29}$$

The NAWM's prices of domestic intermediate goods, imports, exports, investment goods and aggregated output are expressed in units of prices of consumption goods:

$$\hat{p}_{h,t} = \hat{p}_{h,t-1} + \hat{\pi}_{h,t} - \hat{\pi}_{c,t} \tag{B.30}$$

$$\hat{p}_{im,t} = \hat{p}_{im,t-1} + \hat{\pi}_{im,t} - \hat{\pi}_{c,t} \tag{B.31}$$

$$\hat{p}_{x,t} = \hat{p}_{x,t-1} + \hat{\pi}_{x,t} - \hat{\pi}_{c,t} \tag{B.32}$$

$$\hat{p}_{i,t} = \hat{p}_{i,t-1} + \hat{\pi}_{i,t} - \hat{\pi}_{c,t} \tag{B.33}$$

$$\hat{p}_{y,t} = \hat{p}_{y,t-1} + \hat{\pi}_{y,t} - \hat{\pi}_{c,t} \tag{B.34}$$

The price index for consumption goods $\hat{p}_{c,t}$ is restricted to:

$$\hat{p}_{c,t} = 0 \tag{B.35}$$

B.4.2 NAWM's canonical rational expectations form

The NAWM's matrices $\mathbf{\Gamma}_0, \mathbf{\Gamma}_1, \mathbf{\Psi}$ and $\mathbf{\Pi}$ the constant vector $\boldsymbol{\gamma}$ of the linear canonical rational expectations form in 3.61 are row-wise specified and implemented as follows (where all other elements in the rows are set to zero):

$$\gamma[1,1] = -\frac{1}{(1+\tau^c)}\hat{\bar{\tau}}^c$$

$$\mathbf{\Gamma}_0[1,1] = 1, \mathbf{\Gamma}_0[1,2] = \frac{1}{(1-\kappa g_z^{-1})}, \mathbf{\Gamma}_0[1,81] = -1, \mathbf{\Gamma}_0[1,82] = \frac{\kappa g_z^{-1}}{(1-\kappa g_z^{-1})}$$

$$\mathbf{\Gamma}_1[1,2] = \frac{\kappa g_z^{-1}}{(1-\kappa g_z^{-1})}$$

$$\mathbf{\Gamma}_0[2,3] = -1, \mathbf{\Gamma}_0[2,4] = 1, \mathbf{\Gamma}_0[2,12] = \gamma_i g_z^2\left(1+\beta\right), \mathbf{\Gamma}_0[2,65] = -\gamma_i g_z^2 \beta$$

$$\mathbf{\Gamma}_0[2,77] = -\gamma_i g_z^2, \mathbf{\Gamma}_0[2,82] = \gamma_i g_z^2, \mathbf{\Gamma}_0[2,83] = -1$$

$$\mathbf{\Gamma}_1[2,12] = -\gamma_i g_z^2$$

$$\gamma[3,1] = \frac{\beta\delta p_i}{g_z}\frac{\beta\left(1-\tau^k\right)r_k}{g_z\left(1-\tau^k\right)}\bar{E}\hat{\tau}^k$$

$$\mathbf{\Gamma}_0[3,1] = 1, \mathbf{\Gamma}_0[3,3] = 1, \mathbf{\Gamma}_0[3,61] = -1, \mathbf{\Gamma}_0[3,63] = -\frac{\beta\left(1-\delta\right)}{g_z}, \mathbf{\Gamma}_0[3,64] = -\frac{\beta\delta p_i\tau^k}{g_z}$$

$$\mathbf{\Gamma}_0[3,66] = -\frac{\beta\left(1-\tau^k\right)r_k}{g_z}, \mathbf{\Gamma}_0[3,77] = 1$$

$$\mathbf{\Gamma}_0[4,4] = -1, \mathbf{\Gamma}_0[4,13] = 1, \mathbf{\Gamma}_0[4,15] = -\frac{\gamma_{u,2}}{\gamma_{u,1}},$$

$$\mathbf{\Gamma}_0[5,1] = -1, \mathbf{\Gamma}_0[5,14] = 1, \mathbf{\Gamma}_0[5,61] = 1, \mathbf{\Gamma}_0[5,67] = -1, \mathbf{\Gamma}_0[5,77] = -1, \mathbf{\Gamma}_0[5,84] = 1$$

$$\mathbf{\Gamma}_0[6,1] = -1, \mathbf{\Gamma}_0[6,24] = -1, \mathbf{\Gamma}_0[6,25] = -\gamma_{b^*}, \mathbf{\Gamma}_0[6,61] = 1, \mathbf{\Gamma}_0[6,67] = -1$$

$$\mathbf{\Gamma}_0[6,69] = 1, \mathbf{\Gamma}_0[6,75] = 1, \mathbf{\Gamma}_0[6,77] = -1, \mathbf{\Gamma}_0[6,85] = -1, \mathbf{\Gamma}_0[6,97] = -1, \mathbf{\Gamma}_0[6,99] = 1$$

$$\mathbf{\Gamma}_0[7,33] = 1$$

$$\mathbf{\Gamma}_1[7,12] = 1-\left(1-\delta\right)g_z^{-1}, \mathbf{\Gamma}_1[7,33] = \left(1-\delta\right)g_z^{-1}, \mathbf{\Gamma}_1[7,82] = -\left(1-\delta\right)g_z^{-1}$$

$$\mathbf{\Gamma}_1[7,83] = 1-\left(1-\delta\right)g_z^{-1}$$

$$\mathbf{\Gamma}_0[8,3] = -\frac{1}{\gamma_i g_z^2\left(1+\beta\right)}, \mathbf{\Gamma}_0[8,4] = \frac{1}{\gamma_i g_z^2\left(1+\beta\right)}, \mathbf{\Gamma}_0[8,12] = 1, \mathbf{\Gamma}_0[8,65] = -\frac{\beta}{\left(1+\beta\right)}$$

$$\mathbf{\Gamma}_0[8,77] = -\frac{1}{\beta\left(1+\beta\right)}, \mathbf{\Gamma}_0[8,82] = \frac{1}{\left(1+\beta\right)}, \mathbf{\Gamma}_0[8,83] = -\frac{1}{\gamma_i g_z^2\left(1+\beta\right)}$$

$$\mathbf{\Gamma}_1[8,12] = \frac{1}{(1+\beta)}$$

$$\gamma[9,1] = \frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})(1+\tau^c)}\left(\bar{E}\hat{\tau}^c - \hat{\bar{\tau}}^c\right)$$

$$\mathbf{\Gamma}_0[9,2] = 1, \mathbf{\Gamma}_0[9,14] = \frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})}, \mathbf{\Gamma}_0[9,62] = -\frac{1}{(1+\kappa g_z^{-1})}$$

$$\mathbf{\Gamma}_0[9,67] = -\frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})}, \mathbf{\Gamma}_0[9,77] = \frac{1}{(1+\kappa g_z^{-1})}, \mathbf{\Gamma}_0[9,81] = -\frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})},$$

$$\mathbf{\Gamma}_0[9,82] = -\frac{\kappa g_z^{-1}}{(1+\kappa g_z^{-1})}, \mathbf{\Gamma}_0[9,84] = \frac{(1-\kappa g_z^{-1})}{(1+\kappa g_z^{-1})}$$

$$\mathbf{\Gamma}_1[9,2] = \frac{\kappa g_z^{-1}}{(1+\kappa g_z^{-1})}$$

$$\mathbf{\Gamma}_0[10,14] = 1, \mathbf{\Gamma}_0[10,24] = 1, \mathbf{\Gamma}_0[10,25] = \gamma_{b^*}, \mathbf{\Gamma}_0[10,69] = -1, \mathbf{\Gamma}_0[10,75] = -1$$

$$\mathbf{\Gamma}_0[10,84] = 1, \mathbf{\Gamma}_0[10,85] = 1, \mathbf{\Gamma}_0[10,97] = 1, \mathbf{\Gamma}_0[10,99] = -1$$

$$\mathbf{\Gamma}_0[11,16] = \frac{(1+\beta\chi_w)}{(1+\beta)}, \mathbf{\Gamma}_0[11,17] = -\frac{(1-\chi_w)}{(1+\beta)}, \mathbf{\Gamma}_0[11,35] = 1$$

$$\mathbf{\Gamma}_0[11,36] = \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\,\xi_w\left(1+\frac{\varphi_w}{(\varphi_w-1)}\zeta\right)}, \mathbf{\Gamma}_0[11,37] = -\frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\,\xi_w\left(1+\frac{\varphi_w}{(\varphi_w-1)}\zeta\right)}$$

$$\mathbf{\Gamma}_0[11,67] = -\frac{\beta}{(1+\beta)}, \mathbf{\Gamma}_0[11,68] = \frac{\beta(1-\chi_w)}{(1+\beta)}, \mathbf{\Gamma}_0[11,74] = -\frac{\beta}{(1+\beta)}$$

$$\mathbf{\Gamma}_0[11,86] = -\frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\,\xi_w\left(1+\frac{\varphi_w}{(\varphi_w-1)}\zeta\right)}$$

$$\boldsymbol{\Gamma}_1[11,16]=\frac{\chi_w}{(1+\beta)},\boldsymbol{\Gamma}_1[11,35]=\frac{1}{(1+\beta)}$$

$$\gamma[12,1]=-\frac{\left(\hat{\hat{\tau}}^N-\hat{\hat{\tau}}^{w_h}\right)}{(1-\hat{\tau}^N-\hat{\tau}^{w_h})}$$

$$\boldsymbol{\Gamma}_0[12,35]=-1,\boldsymbol{\Gamma}_0[12,36]=1$$

$$\boldsymbol{\Gamma}_0[13,1]=1,\boldsymbol{\Gamma}_0[13,37]=1,\boldsymbol{\Gamma}_0[13,38]=-\zeta,\boldsymbol{\Gamma}_0[13,87]=-1$$

$$\boldsymbol{\Gamma}_0[14,34]=-\left(1+\psi y^{-1}\right)\alpha,\boldsymbol{\Gamma}_0[14,41]=-\left(1+\psi y^{-1}\right)(1-\alpha),\boldsymbol{\Gamma}_0[14,42]=1$$

$$\boldsymbol{\Gamma}_0[14,82]=-\left(1+\psi y^{-1}\right),\boldsymbol{\Gamma}_0[14,95]=-\left(1+\psi y^{-1}\right)$$

$$\gamma[15,1]=\frac{1}{(1+\tau^{w_f})}\hat{\hat{\tau}}^{w_f}$$

$$\boldsymbol{\Gamma}_0[15,13]=1,\boldsymbol{\Gamma}_0[15,34]=1,\boldsymbol{\Gamma}_0[15,35]=-1,\boldsymbol{\Gamma}_0[15,41]=-1,\boldsymbol{\Gamma}_0[15,82]=-1$$

$$\gamma[16,1]=\frac{(1-\alpha)}{(1+\tau^{w_f})}\hat{\hat{\tau}}^{w_f}$$

$$\boldsymbol{\Gamma}_0[16,13]=-\alpha,\boldsymbol{\Gamma}_0[16,35]=(1+\alpha),\boldsymbol{\Gamma}_0[16,38]=1,\boldsymbol{\Gamma}_0[16,95]=1$$

$$\boldsymbol{\Gamma}_0[17,18]=1,\boldsymbol{\Gamma}_0[17,19]=\frac{(\chi_h+\beta\chi_h)}{(1+\beta\chi_h)}-1,\boldsymbol{\Gamma}_0[17,39]=-\frac{(1-\beta\xi_h)(1-\xi_h)}{\xi_h(1+\beta\chi_h)}$$

$$\boldsymbol{\Gamma}_0[17,70]=-\frac{\beta}{(1+\beta\chi_h)},\boldsymbol{\Gamma}_0[17,71]=\frac{\beta(1-\chi_h)}{(1+\beta\chi_h)},\boldsymbol{\Gamma}_0[17,88]=-\frac{(1-\beta\xi_h)(1-\xi_h)}{\xi_h(1+\beta\chi_h)}$$

$$\boldsymbol{\Gamma}_1[17,18]=\frac{\chi_h}{(1+\beta\chi_h)}$$

$$\boldsymbol{\Gamma}_0[18,5]=1,\boldsymbol{\Gamma}_0[18,38]=-1,\boldsymbol{\Gamma}_0[18,39]=1$$

$$\mathbf{\Gamma}_0[19,19]=\frac{(\chi_x+\beta\chi_x)}{(1+\beta\chi_x)}-1,\mathbf{\Gamma}_0[19,20]=1,\mathbf{\Gamma}_0[19,40]=-\frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x(1+\beta\chi_x)}$$

$$\mathbf{\Gamma}_0[19,71]=\frac{\beta(1-\chi_x)}{(1+\beta\chi_x)},\mathbf{\Gamma}_0[19,72]=-\frac{\beta}{(1+\beta\chi_x)},\Gamma_0[19,89]=-\frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x(1+\beta\chi_x)}$$

$$\mathbf{\Gamma}_1[19,20]=\frac{\chi_x}{(1+\beta\chi_x)}$$

$$\mathbf{\Gamma}_0[20,6]=1,\mathbf{\Gamma}_0[20,38]=-1,\mathbf{\Gamma}_0[20,40]=1$$

$$\mathbf{\Gamma}_0[21,19]=\frac{(\chi^*+\beta^*\chi^*)}{(1+\beta^*\chi^*)}-1,\mathbf{\Gamma}_0[21,21]=1,\mathbf{\Gamma}_0[21,71]=\frac{\beta^*(1-\chi^*)}{(1+\beta^*\chi^*)}$$

$$\mathbf{\Gamma}_0[21,73]=-\frac{\beta^*}{(1+\beta^*\chi^*)},\mathbf{\Gamma}_0[21,79]=-\frac{(1-\beta^*\xi^*)(1-\xi^*)}{\xi^*(1+\beta^*\chi^*)},\mathbf{\Gamma}_0[21,80]=-\frac{(1-\beta^*\xi^*)(1-\xi^*)}{\xi^*(1+\beta^*\chi^*)}$$

$$\mathbf{\Gamma}_1[21,21]=\frac{\chi^*}{(1+\beta^*\chi^*)}$$

$$\mathbf{\Gamma}_0[22,7]=-1,\mathbf{\Gamma}_0[22,8]=1,\mathbf{\Gamma}_0[22,24]=-1,\mathbf{\Gamma}_0[22,79]=1,\mathbf{\Gamma}_0[22,100]=-\omega^*$$

$$\mathbf{\Gamma}_0[23,44]=1,\mathbf{\Gamma}_0[23,47]=-v_c^{\frac{1}{\mu_c}}\left(\frac{h^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)},\mathbf{\Gamma}_0[23,51]=(1-v_c)^{\frac{1}{\mu_c}}\left(\frac{im^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)}$$

$$\mathbf{\Gamma}_0[23,54]=-\frac{1}{(\mu_c-1)}\left[v_c^{\frac{1}{\mu_c}}\left(\frac{h^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)}-\frac{v_c}{(1-v_c)}(1-v_c)^{\frac{1}{\mu_c}}\left(\frac{im^c}{q^c}\right)^{\left(1-\frac{1}{\mu_c}\right)}\right]$$

$$\mathbf{\Gamma}_0[24,5]=\mu_c,\mathbf{\Gamma}_0[24,9]=-\mu_c,\mathbf{\Gamma}_0[24,44]=-1,\mathbf{\Gamma}_0[24,47]=1,\mathbf{\Gamma}_0[24,54]=-1$$

$$\mathbf{\Gamma}_0[25,8]=\mu_c,\mathbf{\Gamma}_0[25,9]=-\mu_c,\mathbf{\Gamma}_0[25,44]=-1,\mathbf{\Gamma}_0[25,51]=1,\mathbf{\Gamma}_0[25,54]=\frac{v_c}{(1-v_c)}$$

$$\mathbf{\Gamma}_0[25,56]=-\mu_c,\mathbf{\Gamma}_0[26,44]=-\gamma_{im^c},\mathbf{\Gamma}_0[26,51]=\gamma_{im^c},\mathbf{\Gamma}_0[26,56]=1,\mathbf{\Gamma}_0[26,91]=-1$$

$$\mathbf{\Gamma}_1[26,44] = -\gamma_{im^c}, \mathbf{\Gamma}_1[26,51] = \gamma_{im^c}$$

$$\mathbf{\Gamma}_1[27,5] = -v_c \left(\frac{p_h}{p_c}\right)^{(1-\mu_c)}, \mathbf{\Gamma}_1[27,8] = -(1-v_c)\left(\frac{p_{im}}{p_c}\right)^{(1-\mu_c)}, \mathbf{\Gamma}_1[27,9] = 1$$

$$\mathbf{\Gamma}_1[27,54] = -\frac{v_c}{(1-\mu_c)}\left[\left(\frac{p_h}{p_c}\right)^{(1-\mu_c)} - \left(\frac{p_{im}}{p_c}\right)^{(1-\mu_c)}\right], \mathbf{\Gamma}_1[27,56] = (1-v_c)\left(\frac{p_{im}}{p_c}\right)^{(1-\mu_c)}$$

$$\mathbf{\Gamma}_0[28,45] = 1, \mathbf{\Gamma}_0[28,48] = -v_i^{\frac{1}{\mu_i}}\left(\frac{h^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)}, \mathbf{\Gamma}_0[28,52] = -\left(1-v_i\right)^{\frac{1}{\mu_i}}\left(\frac{im^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)}$$

$$\mathbf{\Gamma}_0[28,55] = -\frac{1}{(\mu_i-1)}\left[v_i^{\frac{1}{\mu_i}}\left(\frac{h^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)} - \frac{v_i}{(1-v_i)}\left(1-v_i\right)^{\frac{1}{\mu_i}}\left(\frac{im^i}{q^i}\right)^{\left(1-\frac{1}{\mu_i}\right)}\right]$$

$$\mathbf{\Gamma}_0[29,4] = -\mu_i, \mathbf{\Gamma}_0[29,5] = \mu_i, \mathbf{\Gamma}_0[29,45] = -1, \mathbf{\Gamma}_0[29,48] = 1, \mathbf{\Gamma}_0[29,55] = -1$$

$$\mathbf{\Gamma}_0[30,4] = -\mu_i, \mathbf{\Gamma}_0[30,8] = \mu_i, \mathbf{\Gamma}_0[30,45] = -1, \mathbf{\Gamma}_0[30,52] = 1, \mathbf{\Gamma}_0[30,55] = \frac{v_i}{(1-v_i)}$$

$$\mathbf{\Gamma}_0[30,57] = -\mu_i, \mathbf{\Gamma}_0[31,45] = -\gamma_{im^i}, \mathbf{\Gamma}_0[31,52] = \gamma_{im^i}, \mathbf{\Gamma}_0[31,57] = 1, \mathbf{\Gamma}_0[31,91] = -1$$

$$\mathbf{\Gamma}_1[31,45] = -\gamma_{im^i}, \mathbf{\Gamma}_1[31,52] = \gamma_{im^i}$$

$$\mathbf{\Gamma}_0[32,4] = 1, \mathbf{\Gamma}_0[32,5] = -v_i\left(\frac{p_h}{q_i}\right)^{(1-\mu_i)}, \mathbf{\Gamma}_0[32,8] = -(1-v_i)\left(\frac{p_{im}}{p_i}\right)^{(1-\mu_i)}$$

$$\mathbf{\Gamma}_0[32,55] = -\frac{v_i}{(1-\mu_i)}\left[\left(\frac{p_h}{q_i}\right)^{(1-\mu_i)} - \left(\frac{p_{im}}{q_i}\right)^{(1-\mu_i)})\right], \mathbf{\Gamma}_0[32,57] = (1-v_i)\left(\frac{p_{im}}{p_i}\right)^{(1-\mu_i)}$$

$$\mathbf{\Gamma}_0[33,46] = 1, \mathbf{\Gamma}_0[33,49] = -1$$

$$\mathbf{\Gamma}_0[34,5] = -1, \mathbf{\Gamma}_0[34,49] = 1$$

$$\mathbf{\Gamma}_0[35,47] = -\frac{h^c}{h}, \mathbf{\Gamma}_0[35,48] = -\frac{h^i}{h}, \mathbf{\Gamma}_0[35,49] = -\frac{h^g}{h}, \mathbf{\Gamma}_0[35,50] = 1$$

$$\mathbf{\Gamma}_0[36,51] = -\frac{im^c}{im}, \mathbf{\Gamma}_0[36,52] = -\frac{im^i}{im}, \mathbf{\Gamma}_0[36,53] = 1$$

$$\mathbf{\Gamma}_0[37,6] = \mu^*, \mathbf{\Gamma}_0[37,7] = -\mu^*, \mathbf{\Gamma}_0[37,11] = -\mu^*, \mathbf{\Gamma}_0[37,24] = -\mu^*, \mathbf{\Gamma}_0[37,43] = 1$$

$$\mathbf{\Gamma}_0[37,58] = -\mu^*, \mathbf{\Gamma}_0[37,59] = -1, \mathbf{\Gamma}_0[37,96] = -1, \mathbf{\Gamma}_0[37,98] = -1$$

$$\mathbf{\Gamma}_0[38,43] = \gamma^*, \mathbf{\Gamma}_0[38,58] = 1, \mathbf{\Gamma}_0[38,59] = -\gamma^*, \mathbf{\Gamma}_0[38,94] = -1, \mathbf{\Gamma}_0[38,98] = -\gamma^*$$

$$\mathbf{\Gamma}_1[38,43] = \gamma^*, \mathbf{\Gamma}_1[38,59] = -\gamma^*, \mathbf{\Gamma}_1[38,98] = -\gamma^*$$

$$\mathbf{\Gamma}_0[39,14] = 1, \mathbf{\Gamma}_0[39,16] = -\phi_{\Delta\pi}, \mathbf{\Gamma}_0[39,19] = (1-\phi_r)(\phi_\pi - 1)$$

$$\mathbf{\Gamma}_0[39,42] = -(1-\phi_r)\phi_y + \phi_{\Delta y}$$

$$\mathbf{\Gamma}_1[39,14] = \phi_r, \mathbf{\Gamma}_1[39,16] = (1-\phi_r)\phi_\pi - \phi_{\Delta\pi}, \mathbf{\Gamma}_1[39,42] = -\phi_{\Delta y}$$

$$\mathbf{\Gamma}_0[40,19] = 1, \mathbf{\Gamma}_0[40,93] = -1$$

$$\mathbf{\Gamma}_1[40,19] = \rho_\pi$$

$$\gamma[41,1] = \frac{p_c c}{p_y y}\hat{\bar{\tau}}^c + \frac{wN}{p_y y}\left(\hat{\bar{\tau}}^N + \hat{\bar{\tau}}^{w_h} + \hat{\bar{\tau}}^{w_f}\right) + \frac{r_k k g_z^{-1}}{p_y y}\hat{\bar{\tau}}^k$$

$$\mathbf{\Gamma}_0[41,2] = -\frac{p_c c}{p_y y}\tau^c, \mathbf{\Gamma}_0[41,7] = \frac{p_c c}{p_y y}\tau^c + \frac{wN}{p_y y}\left(\tau^N + \tau^{w_h} + \tau^{w_f}\right) + \frac{r_k k g_z^{-1}}{p_y y}\tau^k$$

$$\mathbf{\Gamma}_0[41,9] = -\frac{p_c c}{p_y y}\tau^c, \mathbf{\Gamma}_0[41,13] = -\frac{r_k k g_z^{-1}}{p_y y}\tau^k, \mathbf{\Gamma}_0[41,15] = -\frac{r_k k g_z^{-1}}{p_y y}\tau^k, \mathbf{\Gamma}_0[41,26] = 1$$

$$\mathbf{\Gamma}_0[41,33] = -\frac{r_k k g_z^{-1}}{p_y y}\tau^k, \mathbf{\Gamma}_0[41,35] = -\frac{wN}{p_y y}\left(\tau^N + \tau^{w_h} + \tau^{w_f}\right)$$

$$\mathbf{\Gamma}_0[41,41] = -\frac{wN}{p_y y}\left(\tau^N + \tau^{w_h} + \tau^{w_f}\right), \mathbf{\Gamma}_0[41,42] = \frac{p_c c}{p_y y}\tau^c + \frac{wN}{p_y y}\left(\tau^N + \tau^{w_h} + \tau^{w_f}\right) + \frac{r_k k g_z^{-1}}{p_y y}\tau^k$$

$$\mathbf{\Gamma}_0[41,82] = \frac{r_k k g_z^{-1}}{p_y y}\tau^k$$

$$\mathbf{\Gamma}_0[42,7] = s_g, \mathbf{\Gamma}_0[42,10] = -s_g, \mathbf{\Gamma}_0[42,26] = 1, \mathbf{\Gamma}_0[42,42] = s_g, \mathbf{\Gamma}_0[42,102] = -s_g$$

$$\mathbf{\Gamma}_0[43,15] = 1, \mathbf{\Gamma}_0[43,33] = 1, \mathbf{\Gamma}_0[43,34] = -1$$

$$\mathbf{\Gamma}_0[44,2] = -1, \mathbf{\Gamma}_0[44,44] = 1$$

$$\mathbf{\Gamma}_0[45,12] = -1, \mathbf{\Gamma}_0[45,15] = -\frac{r_k k}{p_i g_z q_i}$$

$$\mathbf{\Gamma}_0[46,46] = 1, \mathbf{\Gamma}_0[46,102] = -1$$

$$\mathbf{\Gamma}_0[47,42] = 1, \mathbf{\Gamma}_0[47,43] = -\frac{x}{y}, \mathbf{\Gamma}_0[47,50] = -\frac{h}{q}$$

$$\mathbf{\Gamma}_0[48,2] = -\frac{p_c c}{p_y y}, \mathbf{\Gamma}_0[48,4] = -\frac{p_i i}{p_y y}, \mathbf{\Gamma}_0[48,6] = -\frac{p_x x}{p_y y}, \mathbf{\Gamma}_0[48,7] = 1$$

$$\mathbf{\Gamma}_0[48,8] = \frac{p_{im} im^c + p_{im} im^i}{p_y y}, \mathbf{\Gamma}_0[48,9] = -\frac{p_c c}{p_y y}, \mathbf{\Gamma}_0[48,10] = -\frac{p_g g}{p_y y}, \mathbf{\Gamma}_0[48,12] = -\frac{p_i i}{p_y y}$$

$$\mathbf{\Gamma}_0[48,15] = -\frac{p_i k g_z^{-1}}{p_y y}\gamma_{u,1}, \mathbf{\Gamma}_0[48,42] = 1, \mathbf{\Gamma}_0[48,43] = -\frac{p_x x}{p_y y}, \mathbf{\Gamma}_0[48,51] = \frac{p_{im} im^c}{p_y y}$$

$$\mathbf{\Gamma}_0[48,52] = \frac{p_{im} im^c}{p_y y}, \mathbf{\Gamma}_0[48,56] = -\frac{p_{im} im^c}{p_y y}, \mathbf{\Gamma}_0[48,57] = -\frac{p_{im} im^i}{p_y y}, \mathbf{\Gamma}_0[48,102] = -\frac{p_g g}{p_y y}$$

$$\mathbf{\Gamma}_0[49,7] = -\frac{1}{\phi}\left(1 + \frac{\psi}{y}\right), \mathbf{\Gamma}_0[49,27] = 1, \mathbf{\Gamma}_0[49,38] = \frac{1}{\phi}\left(1 + \frac{\psi}{y}\right), \mathbf{\Gamma}_0[49,42] = -\frac{(h + x + \psi)}{\phi y}$$

$$\mathbf{\Gamma}_0[49,43] = \frac{1}{\phi}\frac{x}{y}, \mathbf{\Gamma}_0[49,50] = \frac{1}{\phi}\frac{h}{y}$$

$$\mathbf{\Gamma}_0[50,80] = \frac{1}{R^*}$$

$$\mathbf{\Gamma}_1[50,6] = \frac{p_x x}{s p_y}, \mathbf{\Gamma}_1[50,7] = \frac{p_{im} im - p_x x}{s p_y}, \mathbf{\Gamma}_1[50,8] = -\frac{p_{im} im}{s p_y}, \mathbf{\Gamma}_1[50,24] = \frac{p_{im} im - p_x x}{s p_y}$$

$$\mathbf{\Gamma}_1[50,43] = \frac{p_x x}{s p_y}, \mathbf{\Gamma}_1[50,53] = -\frac{p_{im} im}{s p_y}, \mathbf{\Gamma}_1[50,59] = \frac{p_{im} im - p_x x}{s p_y}, \mathbf{\Gamma}_1[50,80] = g_z^{-1} \bar{\Pi}^{-1}$$

$$\mathbf{\Gamma}_0[51,25] = 1, \mathbf{\Gamma}_0[51,80] = -s\frac{z}{y}$$

$$\mathbf{\Gamma}_0[52,29] = 1, \mathbf{\Gamma}_0[52,30] = -1, \mathbf{\Gamma}_0[52,31] = 1$$

$$\mathbf{\Gamma}_0[53,6] = -s_x, \mathbf{\Gamma}_0[53,7] = s_x, \mathbf{\Gamma}_0[53,30] = 1, \mathbf{\Gamma}_0[53,42] = s_x, \mathbf{\Gamma}_0[53,43] = -s_x$$

$$\mathbf{\Gamma}_0[54,7] = s_{im}, \mathbf{\Gamma}_0[54,8] = -s_{im}, \mathbf{\Gamma}_0[54,31] = 1, \mathbf{\Gamma}_0[54,42] = s_{im}, \mathbf{\Gamma}_0[54,53] = -s_{im}$$

$$\mathbf{\Gamma}_0[55,6] = 1, \mathbf{\Gamma}_0[55,8] = -1, \mathbf{\Gamma}_0[55,32] = 1$$

$$\mathbf{\Gamma}_0[56,5] = 1, \mathbf{\Gamma}_0[56,16] = 1, \mathbf{\Gamma}_0[56,18] = -1$$

$$\mathbf{\Gamma}_1[56,5] = 1$$

$$\mathbf{\Gamma}_0[57,6] = 1, \mathbf{\Gamma}_0[57,16] = 1, \mathbf{\Gamma}_0[57,20] = -1$$

$$\mathbf{\Gamma}_1[57,6] = 1$$

$$\mathbf{\Gamma}_0[58,7] = 1, \mathbf{\Gamma}_0[58,16] = 1, \mathbf{\Gamma}_0[58,22] = -1$$

$$\mathbf{\Gamma}_1[58,7] = 1$$

$$\mathbf{\Gamma}_0[59,9]=1$$

$$\mathbf{\Gamma}_0[60,4]=1,\mathbf{\Gamma}_0[60,16]=1,\mathbf{\Gamma}_0[60,23]=-1$$

$$\mathbf{\Gamma}_1[60,4]=1$$

$$\mathbf{\Gamma}_0[61,8]=1,\mathbf{\Gamma}_0[61,16]=1,\mathbf{\Gamma}_0[61,21]=-1$$

$$\mathbf{\Gamma}_1[61,8]=1$$

$$\mathbf{\Gamma}_0[62,41]=-\frac{(1-\beta\xi_E)(1-\xi_E)}{(1+\beta)\xi_E},\mathbf{\Gamma}_0[62,60]=1+\frac{(1-\beta\xi_E)(1-\xi_E)}{(1+\beta)\xi_E},\mathbf{\Gamma}_0[62,78]=-\frac{\beta}{(1+\beta)}$$

$$\mathbf{\Gamma}_1[62,60]=\frac{1}{(1+\beta)}$$

$$\mathbf{\Gamma}_0[63,1]=1,\mathbf{\Gamma}_1[63,61]=1,\mathbf{\Pi}[1,1]=1$$

$$\mathbf{\Gamma}_0[64,2]=1,\mathbf{\Gamma}_1[64,62]=1,\mathbf{\Pi}[2,2]=1$$

$$\mathbf{\Gamma}_0[65,3]=1,\mathbf{\Gamma}_1[65,63]=1,\mathbf{\Pi}[3,3]=1$$

$$\mathbf{\Gamma}_0[66,4]=1,\mathbf{\Gamma}_1[66,64]=1,\mathbf{\Pi}[4,4]=1$$

$$\mathbf{\Gamma}_0[67,12]=1,\mathbf{\Gamma}_1[67,65]=1,\mathbf{\Pi}[5,5]=1$$

$$\mathbf{\Gamma}_0[68,13]=1,\mathbf{\Gamma}_1[68,66]=1,\mathbf{\Pi}[6,6]=1$$

$$\mathbf{\Gamma}_0[69,16]=1,\mathbf{\Gamma}_1[69,67]=1,\mathbf{\Pi}[7,7]=1$$

$$\mathbf{\Gamma}_0[70,17]=1,\mathbf{\Gamma}_1[70,68]=1,\mathbf{\Pi}[8,8]=1$$

$$\mathbf{\Gamma}_0[71,22]=1,\mathbf{\Gamma}_1[71,69]=1,\mathbf{\Pi}[9,9]=1$$

$$\mathbf{\Gamma}_0[72,18]=1,\mathbf{\Gamma}_1[72,70]=1,\mathbf{\Pi}[10,10]=1$$

$$\mathbf{\Gamma}_0[73,19]=1,\mathbf{\Gamma}_1[73,71]=1,\mathbf{\Pi}[11,11]=1$$

$$\mathbf{\Gamma}_0[74,20]=1,\mathbf{\Gamma}_1[74,72]=1,\mathbf{\Pi}[12,12]=1$$

$$\mathbf{\Gamma}_0[75,21]=1,\mathbf{\Gamma}_1[75,73]=1,\mathbf{\Pi}[13,13]=1$$

$$\mathbf{\Gamma}_0[76,35]=1,\mathbf{\Gamma}_1[76,74]=1,\mathbf{\Pi}[14,14]=1$$

$$\mathbf{\Gamma}_0[77,24]=1,\mathbf{\Gamma}_1[77,75]=1,\mathbf{\Pi}[15,15]=1$$

$$\mathbf{\Gamma}_0[78,81]=1,\mathbf{\Gamma}_1[78,76]=1,\mathbf{\Pi}[16,16]=1$$

$$\mathbf{\Gamma}_0[79,82]=1,\mathbf{\Gamma}_1[79,77]=1,\mathbf{\Pi}[17,17]=1$$

$$\mathbf{\Gamma}_0[80,60]=1,\mathbf{\Gamma}_1[80,78]=1,\mathbf{\Pi}[18,18]=1$$

$$\mathbf{\Gamma}_0[81,101]=1,\mathbf{\Gamma}_0[81,113]=-1,\mathbf{\Gamma}_1[81,97]=-1,\mathbf{\Pi}[19,19]=1$$

$$\mathbf{\Gamma}_0[82,81]=1,\mathbf{\Gamma}_1[82,81]=\rho_c,\mathbf{\Psi}[1,1]=\sigma_c$$

$$\mathbf{\Gamma}_0[83,82]=1,\mathbf{\Gamma}_1[83,82]=\rho_g,\mathbf{\Psi}[2,2]=\sigma_g$$

$$\mathbf{\Gamma}_0[84,83]=1,\mathbf{\Gamma}_1[84,83]=\rho_i,\mathbf{\Psi}[3,3]=\sigma_i$$

$$\mathbf{\Gamma}_0[85,84]=1,\mathbf{\Gamma}_1[85,84]=\rho_{RP},\mathbf{\Psi}[4,4]=\sigma_{RP}$$

$$\mathbf{\Gamma}_0[86,85]=1,\mathbf{\Gamma}_1[86,85]=\rho_{RP^*},\mathbf{\Psi}[5,5]=\sigma_{RP^*}$$

$$\mathbf{\Gamma}_0[87,86]=1,\mathbf{\Gamma}_1[87,86]=\rho_{\phi^w},\mathbf{\Psi}[6,6]=\sigma_{\phi^w}$$

$$\mathbf{\Gamma}_0[88,87]=1,\mathbf{\Gamma}_1[88,87]=\rho_N,\mathbf{\Psi}[7,7]=\sigma_N$$

$\mathbf{\Gamma}_0[89, 88] = 1, \mathbf{\Gamma}_1[89, 88] = \rho_{\phi^h}, \mathbf{\Psi}[8, 8] = \sigma_{\phi^h}$

$\mathbf{\Gamma}_0[90, 89] = 1, \mathbf{\Gamma}_1[90, 89] = \rho_{\phi^x}, \mathbf{\Psi}[9, 9] = \sigma_{\phi^x}$

$\mathbf{\Gamma}_0[91, 90] = 1, \mathbf{\Gamma}_1[91, 90] = \rho_{\phi^*}, \mathbf{\Psi}[10, 10] = \sigma_{\phi^*}$

$\mathbf{\Gamma}_0[92, 91] = 1, \mathbf{\Gamma}_1[92, 91] = \rho_{im}, \mathbf{\Psi}[11, 11] = \sigma_{im}$

$\mathbf{\Gamma}_0[93, 92] = 1, \mathbf{\Gamma}_1[93, 92] = \rho_r, \mathbf{\Psi}[12, 12] = \sigma_r$

$\mathbf{\Gamma}_0[94, 93] = 1, \mathbf{\Gamma}_1[94, 93] = \rho_{\bar{\pi}}, \mathbf{\Psi}[13, 13] = \sigma_{\bar{\pi}}$

$\mathbf{\Gamma}_0[95, 94] = 1, \mathbf{\Gamma}_1[95, 94] = \rho_x, \mathbf{\Psi}[14, 14] = \sigma_x$

$\mathbf{\Gamma}_0[96, 95] = 1, \mathbf{\Gamma}_1[96, 95] = \rho, \mathbf{\Psi}[15, 15] = \sigma$

$\mathbf{\Gamma}_0[97, 96] = 1, \mathbf{\Gamma}_1[97, 96] = \rho_{v^*}, \mathbf{\Psi}[16, 16] = \sigma_{v^*}$

$\gamma[98, 1] = \boldsymbol{c}[2, 1]$

$\mathbf{\Gamma}_0[98, 96] = \mathbf{A}[2, 2], \mathbf{\Gamma}_0[98, 101] = \mathbf{A}[2, 1]$

$\mathbf{\Gamma}_1[98, 11] = \mathbf{B}[2, 5], \mathbf{\Gamma}_1[98, 98] = \mathbf{B}[2, 2], \mathbf{\Gamma}_1[98, 99] = \mathbf{B}[2, 3], \mathbf{\Gamma}_1[98, 100] = \mathbf{B}[2, 4]$

$\mathbf{\Gamma}_1[98, 101] = \mathbf{B}[2, 1]$

$\gamma[99, 1] = \mathbf{c}[3, 1]$

$\mathbf{\Gamma}_0[99, 98] = \mathbf{A}[3, 2], \mathbf{\Gamma}_0[99, 99] = \mathbf{A}[3, 3], \mathbf{\Gamma}_0[99, 101] = \mathbf{A}[3, 1]$

$\mathbf{\Gamma}_1[99, 11] = \mathbf{B}[3, 5], \mathbf{\Gamma}_1[99, 98] = \mathbf{B}[3, 2], \mathbf{\Gamma}_1[99, 99] = \mathbf{B}[3, 3], \mathbf{\Gamma}_1[99, 100] = \mathbf{B}[3, 4]$

$\mathbf{\Gamma}_1[99, 101] = \mathbf{B}[3, 1]$

$\gamma[100,1] = \mathbf{c}[4,1]$

$\mathbf{\Gamma}_0[100,98] = \mathbf{A}[4,2], \mathbf{\Gamma}_0[100,99] = \mathbf{A}[4,3], \mathbf{\Gamma}_0[100,100] = \mathbf{A}[4,4], \mathbf{\Gamma}_0[100,101] = \mathbf{A}[4,1]$

$\mathbf{\Gamma}_1[100,11] = \mathbf{B}[4,5], \mathbf{\Gamma}_1[100,98] = \mathbf{B}[4,2], \mathbf{\Gamma}_1[100,99] = \mathbf{B}[4,3], \mathbf{\Gamma}_1[100,100] = \mathbf{B}[4,4]$

$\mathbf{\Gamma}_1[100,101] = \mathbf{B}[4,1]$

$\gamma[101,1] = \mathbf{c}[1,1]$

$\mathbf{\Gamma}_0[101,101] = \mathbf{A}[1,1]$

$\mathbf{\Gamma}_1[101,11] = \mathbf{B}[1,5], \mathbf{\Gamma}_1[101,98] = \mathbf{B}[1,2], \mathbf{\Gamma}_1[101,99] = \mathbf{B}[1,3], \mathbf{\Gamma}_1[101,100] = \mathbf{B}[1,4]$

$\mathbf{\Gamma}_1[101,101] = \mathbf{B}[1,1]$

$\gamma[102,1] = c, \mathbf{\Gamma}_0[102,102] = 1$

$\mathbf{\Gamma}_1[102,102] = a$

$\mathbf{\Gamma}_0[103,103] = 1, \mathbf{\Gamma}_1[103,42] = 1$

$\mathbf{\Gamma}_0[104,104] = 1, \mathbf{\Gamma}_1[104,2] = 1$

$\mathbf{\Gamma}_0[105,105] = 1, \mathbf{\Gamma}_1[105,12] = 1$

$\mathbf{\Gamma}_0[106,106] = 1, \mathbf{\Gamma}_1[106,43] = 1$

$\mathbf{\Gamma}_0[107,107] = 1, \mathbf{\Gamma}_1[107,53] = 1$

$\mathbf{\Gamma}_0[108,108] = 1, \mathbf{\Gamma}_1[108,11] = 1$

$\mathbf{\Gamma}_0[109,109] = 1, \mathbf{\Gamma}_1[109,8] = 1$

$\mathbf{\Gamma}_0[110,110] = 1, \mathbf{\Gamma}_1[110,35] = 1$

$\mathbf{\Gamma}_0[111,111] = 1, \mathbf{\Gamma}_1[111,98] = 1$

$\mathbf{\Gamma}_0[112,112] = 1, \mathbf{\Gamma}_1[112,101] = 1$

B.5 NAWM priors

Preferences	$\Psi_{\Delta\pi} \sim N(0.30, 0.10)$	$\sigma_N \sim InvGamma(0.10, 2.00)$
$\kappa \sim beta(0.70, 0.05)$	$\Psi_{\Delta y} \sim N(0.063, 0.05)$	$\sigma_N \sim InvGamma(0.10, 2.00)$
$\beta^* \sim N(0.70, 0.05)$	**Employment**	$\sigma_{\varphi^h} \sim InvGamma(0.10, 2.00)$
Wage and Price Setting	$\xi_E \sim beta(0.50, 0.15)$	$\sigma_{\varphi^x} \sim InvGamma(0.10, 2.00)$
$\xi_w \sim beta(0.75, 0.05)$	**Autoregressive Coefficients**	$\sigma_{\varphi^*} \sim InvGamma(0.10, 2.00)$
$\chi_w \sim beta(0.75, 0.10)$	$\rho_c \sim beta(0.75, 0.10)$	$\sigma_\iota \sim InvGamma(0.10, 2.00)$
$\xi_h \sim beta(0.75, 0.05)$	$\rho_{g_y} \sim beta(0.75, 0.10)$	$\sigma_r \sim InvGamma(0.10, 2.00)$
$\chi_h \sim beta(0.75, 0.10)$	$\rho_i \sim beta(0.75, 0.10)$	$\sigma_\pi \sim InvGamma(0.10, 2.00)$
$\xi_x \sim beta(0.75, 0.05)$	$\rho_{RP} \sim beta(0.75, 0.10)$	$\sigma_x \sim InvGamma(0.10, 2.00)$
$\chi_x \sim beta(0.75, 0.10)$	$\rho_{RP^*} \sim beta(0.75, 0.10)$	$\sigma \sim InvGamma(0.10, 2.00)$
$\xi^* \sim beta(0.75, 0.05)$	$\rho_{\varphi^w} \sim beta(0.75, 0.10)$	$\sigma_{v^x} \sim InvGamma(0.10, 2.00)$
$\chi^* \sim beta(0.75, 0.10)$	$\rho_N \sim beta(0.75, 0.10)$	**Measurement Errors**
$\omega^* \sim beta(0.75, 0.10)$	$\rho_{\varphi^h} \sim beta(0.75, 0.10)$	$\Delta GDP^{EMU} \sim InvGamma(0.10, 2.00)$
Final-goods Production	$\rho_{\varphi^x} \sim beta(0.75, 0.10)$	$\Delta CONS \sim InvGamma(0.10, 2.00)$
$\mu_c \sim Gamma(1.50, 0.25)$	$\rho_{\varphi^*} \sim beta(0.75, 0.10)$	$\Delta INV \sim InvGamma(0.10, 2.00)$
$\mu_i \sim Gamma(1.50, 0.25)$	$\rho_\iota \sim beta(0.75, 0.10)$	$GOV \sim InvGamma(0.10, 2.00)$
$\mu^* \sim Gamma(1.50, 0.25)$	$\rho_r \sim beta(0.75, 0.10)$	$\Delta EXPORT \sim InvGamma(0.10, 2.00)$
Adjustment Costs	$\rho_\pi \sim beta(0.75, 0.10)$	$\Delta IMPORT \sim InvGamma(0.10, 2.00)$
$\gamma_i \sim Gamma(4.00, 0.50)$	$\rho_x \sim beta(0.75, 0.10)$	$\Delta INF_Y^{EMU} \sim InvGamma(0.10, 2.00)$
$\gamma_{\iota^c} \sim Gamma(2.50, 1.00)$	$\rho \sim beta(0.75, 0.10)$	$\Delta INF_Y^{EMU} \sim InvGamma(0.10, 2.00)$
$\gamma_{\iota^i} \sim Gamma(2.50, 1.00)$	$\rho_{v^x} \sim beta(0.75, 0.10)$	$\Delta INF_C \sim InvGamma(0.10, 2.00)$
$\gamma^* \sim Gamma(0.10, 0.50)$	**Standard Deviations**	$\Delta INF_\iota \sim InvGamma(0.10, 2.00)$
$\gamma_{u,1} \sim Gamma(1.50, 0.25)$	$\sigma_c \sim InvGamma(0.10, 2.00)$	$LABOR \sim InvGamma(0.10, 2.00)$
Monetary Policy	$\sigma_{g_y} \sim InvGamma(0.10, 2.00)$	$\Delta WAGE \sim InvGamma(0.10, 2.00)$
$\Psi_r \sim beta(0.90, 0.05)$	$\sigma_i \sim InvGamma(0.10, 2.00)$	$ECB \sim InvGamma(0.10, 2.00)$
$\Psi_\pi \sim N(1.70, 0.10)$	$\sigma_{RP} \sim InvGamma(0.10, 2.00)$	$FX \sim InvGamma(0.10, 2.00)$
$\Psi_y \sim N(0.00, 0.40)$	$\sigma_{RP^*} \sim InvGamma(0.10, 2.00)$	
$\Psi_{\bar{\pi}} \sim N(1.00, 0.10)$	$\sigma_{\varphi^w} \sim InvGamma(0.10, 2.00)$	

Table B.1: Marginal prior distributions used for estimating the NAWM's structural parameters. (Note: The inverse Gamma distribution is parameterized by the shape and the scale parameter).

B.6 Estimation results

B.6.1 Country specific independent DNS estimates

Germany

DNS Measurement										DNS Transition			
σ^2_{12M}	σ^2_{24M}	σ^2_{36M}	σ^2_{48M}	σ^2_{60M}	σ^2_{72M}	σ^2_{84M}	σ^2_{96M}	σ^2_{108M}	σ^2_{120M}	γ^l_i	γ^s_i	$\sigma^l_{v,i}$	$\sigma^s_{v,i}$
0.141	0.000	0.032	0.042	0.024	0.0126	0.000	0.011	0.041	0.087	0.993	0.935	0.067	0.361
(0.018)	(0.000)	(0.005)	(0.006)	(0.003)	(0.003)	(0.000)	(0.002)	(0.006)	(0.012)	(0.007)	(0.013)	(0.010)	(0.044)

France

DNS Measurement										DNS Transition			
σ^2_{12M}	σ^2_{24M}	σ^2_{36M}	σ^2_{48M}	σ^2_{60M}	σ^2_{72M}	σ^2_{84M}	σ^2_{96M}	σ^2_{108M}	σ^2_{120M}	γ^l_i	γ^s_i	$\sigma^l_{v,i}$	$\sigma^s_{v,i}$
0.104	0.009	0.017	0.035	0.024	0.007	0.000	0.007	0.022	0.047	0.991	0.995	0.044	0.097
(0.009)	(0.005)	(0.001)	(0.003)	(0.003)	(0.001)	(0.000)	(0.000)	(0.002)	(0.005)	(0.006)	(0.008)	(0.006)	(0.014)

Netherlands

DNS Measurement										DNS Transition			
σ^2_{12M}	σ^2_{24M}	σ^2_{36M}	σ^2_{48M}	σ^2_{60M}	σ^2_{72M}	σ^2_{84M}	σ^2_{96M}	σ^2_{108M}	σ^2_{120M}	γ^l_i	γ^s_i	$\sigma^l_{v,i}$	$\sigma^s_{v,i}$
0.120	0.009	0.016	0.0.25	0.178	0.006	0.000	0.005	0.022	0.028	0.999	0.980	0.006	0.196
(0.000)	(0.002)	(0.003)	(0.004)	(0.003)	(0.001)	(0.000)	(0.001)	(0.003)	(0.003)	(0.000)	(0.013)	(0.008)	(0.038)

Italy

DNS Measurement										DNS Transition			
σ^2_{12M}	σ^2_{24M}	σ^2_{36M}	σ^2_{48M}	σ^2_{60M}	σ^2_{72M}	σ^2_{84M}	σ^2_{96M}	σ^2_{108M}	σ^2_{120M}	γ^l_i	γ^s_i	$\sigma^l_{v,i}$	$\sigma^s_{v,i}$
0.103	0.008	0.003	0.010	0.008	0.003	0.000	0.003	0.006	0.014	0.904	0.969	0.311	0.314
(0.003)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.028)	(0.013)	(0.100)	(0.031)

Spain

DNS Measurement										DNS Transition			
σ^2_{12M}	σ^2_{24M}	σ^2_{36M}	σ^2_{48M}	σ^2_{60M}	σ^2_{72M}	σ^2_{84M}	σ^2_{96M}	σ^2_{108M}	σ^2_{120M}	γ^l_i	γ^s_i	$\sigma^l_{v,i}$	$\sigma^s_{v,i}$
0.075	0.006	0.010	0.019	0.014	0.007	0.000	0.003	0.009	0.015	0.996	0.960	0.216	0.351
(0.004)	(0.003)	(0.000)	(0.002)	(0.002)	(0.005)	(0.000)	(0.000)	(0.001)	(0.001)	(0.009)	(0.014)	(0.062)	(0.047)

Table B.2: Estimated parameters of the reduced independent DNS state space model for the five EMU countries with zero coupon rates of maturities $\tau = 12, 24, 36, ..., 120$ month between 03/2005 and 02/2014. (Standard errors from the diagonal of the Kalman filter's log likelihood's inverse Hessian quoted in parentheses).

B.6.2 NAWM estimation results

B.6.2.1 Q1/2005 - Q1/2014 NAWM estimation

Houshold's Preferences		**Final Goods**		**Adjustment Cost**					**Employment**
κ	β^*	μ_c	μ_i	γ_i	γ_{ι^c}	γ_{ι^i}	γ^*	$\gamma_{u,2}$	ξ_E
0.765	0.832	2.257	1.681	5.278	5.692	0.442	2.807	0.111	0.701
(0.074)	(0.044)	(0.061)	(0.039)	(0.048)	(0.035)	(0.038)	(0.029)	(0.022)	(0.015)

Wage and Price setting			
ξ_w	χ_w	μ^*	
0.869	0.728	1.110	
(0.042)	(0.011)	(0.087)	
		Monetary Policy	
ξ_h	χ_h	ϕ_r	ϕ_y
0.977	0.22	0.668	-0.228
(0.030)	(0.056)	(0.034)	(0.081)
ξ_x	χ_x	ϕ_π	$\phi_{\Delta\pi}$
0.892	0.647	1.807	-0.013
(0.001)	(0.027)	(0.082)	(0.096)
ξ^*	χ^*	$\phi_{\Delta y}$	$\phi_{\bar{\pi}}$
0.254	0.408	0.230	0.830
(0.034)	(0.073)	(0.074)	(0.030)
ω^*			
0.080			
(0.018)			

NAWM Structural Shock Processes

Autoregressive Coefficients							
ρ_c	ρ_{g_z}	ρ_i	ρ_{RP}	ρ_{RP^*}	ρ_{φ^w}	ρ_N	ρ_{φ^h}
0.909	0.792	0.967	0.992	0.984	0.745	0.847	0.574
(0.054)	(0.061)	(0.019)	(0.003)	(0.007)	(0.033)	(0.028)	(0.043)
ρ_{φ_x}	ρ_{φ^*}	ρ_ι	ρ_r	$\rho_{\bar{\pi}}$	ρ_x	ρ	ρ_{v^x}
0.459	0.848	0.721	0.950	0.784	0.757	0.847	0.971
(0.043)	(0.032)	(0.059)	(0.041)	(0.035)	(0.040)	(0.037)	(0.014)

Standard Deviations							
σ_c	σ_{g_z}	σ_i	σ_{RP}	σ_{RP^*}	σ_{φ^w}	σ_N	σ_{φ^h}
0.071	0.006	0.201	0.662	0.557	0.060	0.079	0.196
(0.116)	(0.018)	(0.043)	(0.079)	(0.029)	(0.043)	(0.030)	(0.025)
σ_{φ_x}	σ_{φ^*}	σ_ι	σ_r	$\sigma_{\bar{\pi}}$	σ_x	σ	σ_{v^x}
1.007	1.212	4.785	0.281	0.800	0.089	1.062	8.956
(0.066)	(0.028)	(0.040)	(0.052)	(0.172)	(0.088)	(0.040)	(0.048)

(Squared) Measurement Errors diag($\mathbf{\Sigma}$)

ΔGDP^{EMU}	$\Delta CONS$	ΔINV	GOV	$\Delta EXPORT$	$\Delta IMPORT$	INF_Y^{EMU}	INF_C	INF_ι	$LABOR$	$\Delta WAGE$	ECB	FX
0.379	0.239	0.793	0.350	0.074	0.224	0.109	0.041	0.766	0.008	0.322	0.027	0.186
(0.028)	(0.085)	(0.108)	(0.042)	(0.040)	(0.048)	(0.029)	(0.036)	(0.125)	(0.015)	(0.123)	(0.014)	(0.046)

Table B.3: Q1/2005 - Q1/2014 NAWM estimation

B.6.2.2 Q1/1987 - Q1/2014 NAWM estimation

Houshold's Preferences		Final Goods		Adjustment Cost					Employment
κ	β^*	μ_c	μ_i	γ_i	γ_{ι^c}	γ_{ι^i}	γ^*	$\gamma_{u,2}$	ξ_E
0.802	1.203	1.925	1.597	4.966	5.695	0.454	2.816	0.135	0.493
(0.012)	(0.006)	(0.004)	(0.003)	(0.004)	(0.005)	(0.007)	(0.003)	(0.011)	(0.019)

Wage and Price setting			
ξ_w	χ_w	ξ_h	χ_h
0.635	0.985	0.635	0.318
(0.004)	(0.003)	(0.004)	(0.008)
ξ_x	χ_x	ξ^*	χ^*
0.897	0.428	0.782	0.586
(0.010)	(0.007)	(0.002)	(0.003)
ω^*			
0.079			
(0.004)			

μ^*
0.926
(0.007)

Monetary Policy					
ϕ_r	ϕ_y	ϕ_π	$\phi_{\Delta\pi}$	$\phi_{\Delta y}$	$\phi_{\bar{\pi}}$
0.996	0.063	1.872	0.161	0.229	0.725
(0.003)	(0.010)	(0.011)	(0.003)	(0.009)	(0.004)

NAWM Structural Shock Processes

Autoregressive Coefficients							
ρ_c	ρ_{g_z}	ρ_i	ρ_{RP}	ρ_{RP^*}	ρ_{φ^w}	ρ_N	ρ_{φ^h}
0.996	0.940	0.914	0.997	0.971	0.706	0.665	0.441
(0.002)	(0.002)	(0.004)	(0.003)	(0.003)	(0.005)	(0.005)	(0.008)
ρ_{φ_x}	ρ_{φ^*}	ρ_ι	ρ_r	$\rho_{\bar{\pi}}$	ρ_x	ρ	ρ_{v^x}
0.313	0.978	0.910	0.686	0.908	0.858	0.966	0.783
(0.003)	(0.007)	(0.003)	(0.003)	(0.010)	(0.008)	(0.009)	(0.004)

Standard Deviations							
σ_c	σ_{g_z}	σ_i	σ_{RP}	σ_{RP^*}	σ_{φ^w}	σ_N	σ_{φ^h}
0.066	0.074	0.112	0.460	0.443	0.223	0.161	0.121
(0.003)	(0.005)	(0.006)	(0.003)	(0.004)	(0.003)	(0.002)	(0.004)
σ_{φ_x}	σ_{φ^*}	σ_ι	σ_r	$\sigma_{\bar{\pi}}$	σ_x	σ	σ_{v^x}
1.030	1.335	4.847	0.145	0.119	0.149	1.135	8.977
(0.006)	(0.005)	(0.021)	(0.006)	(0.006)	(0.004)	(0.008)	(0.011)

(Squared) Measurement Errors diag($\mathbf{\Sigma}$)

ΔGDP^{EMU}	$\Delta CONS$	ΔINV	GOV	$\Delta EXPORT$	$\Delta IMPORT$	INF_Y^{EMU}	INF_C	INF_ι	$LABOR$	$\Delta WAGE$	ECB	FX
0.290	0.212	0.587	0.725	0.123	0.123	0.122	0.094	0.499	0.002	0.675	0.205	0.046
(0.005)	(0.003)	(0.003)	(0.003)	(0.008)	(0.006)	(0.006)	(0.003)	(0.005)	(0.001)	(0.003)	(0.004)	(0.005)

Table B.4: Q1/1987 - Q1/2014 NAWM estimation

B.6.2.3 Q1/2005 - Q1/2014 term structure extended NAWM estimation

Houshold's Preferences		Final Goods		Adjustment Cost					Employment
κ	β^*	μ_c	μ_i	γ_i	γ_{ι^c}	γ_{ι^i}	γ^*	$\gamma_{u,2}$	ξ_E
0.752	0.995	2.007	1.650	5.222	5.961	0.190	2.659	0.120	0.738
(0.045)	(0.057)	(0.005)	(0.053)	(0.051)	(0.076)	(0.075)	(0.065)	(0.023)	(0.019)

Wage and Price setting		μ^*	
ξ_w	χ_w	1.106	
0.878	0.645	(0.038)	
(0.013)	(0.034)	**Monetary Policy**	
ξ_h	χ_h	ϕ_r	ϕ_y
0.985	0.315	0.708	-0.131
(0.023)	(0.045)	(0.039)	(0.062)
ξ_x	χ_x	ϕ_π	$\phi_{\Delta\pi}$
0.893	0.681	1.988	0.126
(0.003)	(0.028)	(0.077)	(0.044)
ξ^*	χ^*	$\phi_{\Delta y}$	$\phi_{\bar{\pi}}$
0.738	0.893	0.027	0.689
(0.019)	(0.003)	(0.035)	(0.064)
ω^*		ϕ_L	ϕ_S
0.078		-0.001	-0.183
(0.011)		(0.007)	(0.020)

NAWM Structural Shock Processes							
Autoregressive Coefficients							
ρ_c	ρ_{g_z}	ρ_i	ρ_{RP}	ρ_{RP^*}	ρ_{φ^w}	ρ_N	ρ_{φ^h}
0.712	0.803	0.653	0.997	0.972	0.604	0.978	0.300
(0.047)	(0.042)	(0.051)	(0.002)	(0.007)	(0.038)	(0.031)	(0.024)
ρ_{φ_x}	ρ_{φ^*}	ρ_ι	ρ_r	$\rho_{\bar{\pi}}$	ρ_x	ρ	ρ_{v^x}
0.057	0.709	0.791	0.930	0.951	0.986	0.884	0.700
(0.047)	(0.064)	(0.056)	(0.047)	(0.039)	(0.012)	(0.026)	(0.032)
Standard Deviations							
σ_c	σ_{g_z}	σ_i	σ_{RP}	σ_{RP^*}	σ_{φ^w}	σ_N	σ_{φ^h}
0.046	0.019	0.262	0.498	0.410	0.386	0.196	0.311
(0.043)	(0.019)	(0.031)	(0.043)	(0.064)	(0.086)	(0.038)	(0.050)
σ_{φ_x}	σ_{φ^*}	σ_ι	σ_r	$\sigma_{\bar{\pi}}$	σ_x	σ	σ_{v^x}
1.289	1.219	4.403	0.358	0.404	0.347	1.337	8.884
(0.073)	(0.076)	(0.102)	(0.038)	(0.034)	(0.113)	(0.046)	(0.025)

(Squared) Measurement Errors diag($\mathbf{\Sigma}$)

ΔGDP^{EMU}	$\Delta CONS$	ΔINV	GOV	$\Delta EXPORT$	$\Delta IMPORT$	INF_Y^{EMU}	INF_C	INF_ι	$LABOR$	$\Delta WAGE$	ECB	FX
0.368	0.109	0.868	0.544	0.493	0.088	0.218	0.262	0.786	0.043	0.335	0.024	0.139
(0.052)	(0.044)	(0.125)	(0.068)	(0.119)	(0.035)	(0.058)	(0.053)	(0.123)	(0.031)	(0.093)	(0.041)	(0.040)

Table B.5: Q1/2005 - Q1/2014 term structure extended NAWM estimation

B.6.2.4 In-sample-fit long term Q1/1987 - Q1/2014 baseline NAWM estimation

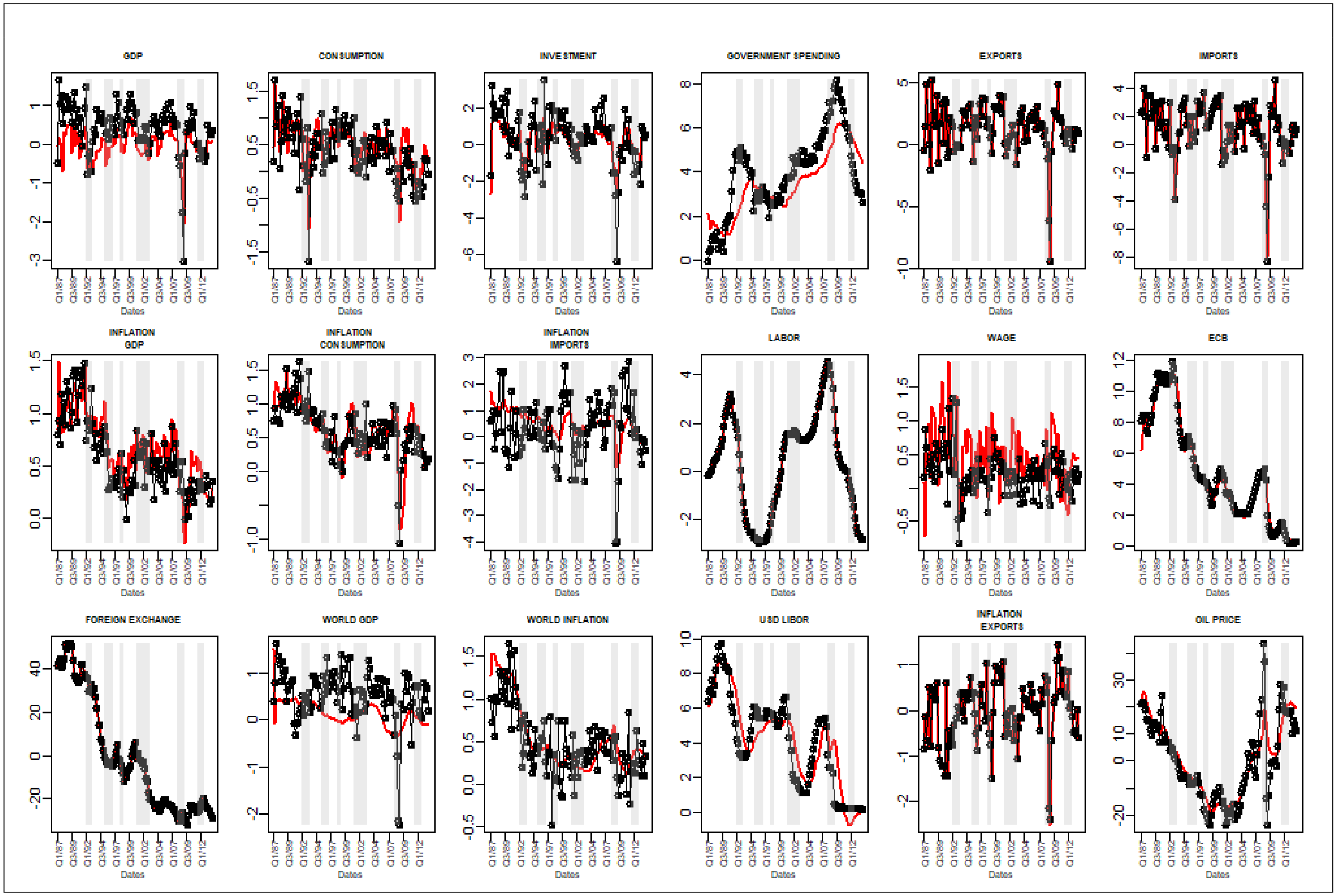


Table B.6: Observed and NAWM implied area wide macroeconomic variables between Q1/1987 and Q1/2014 at the posterior's mean.

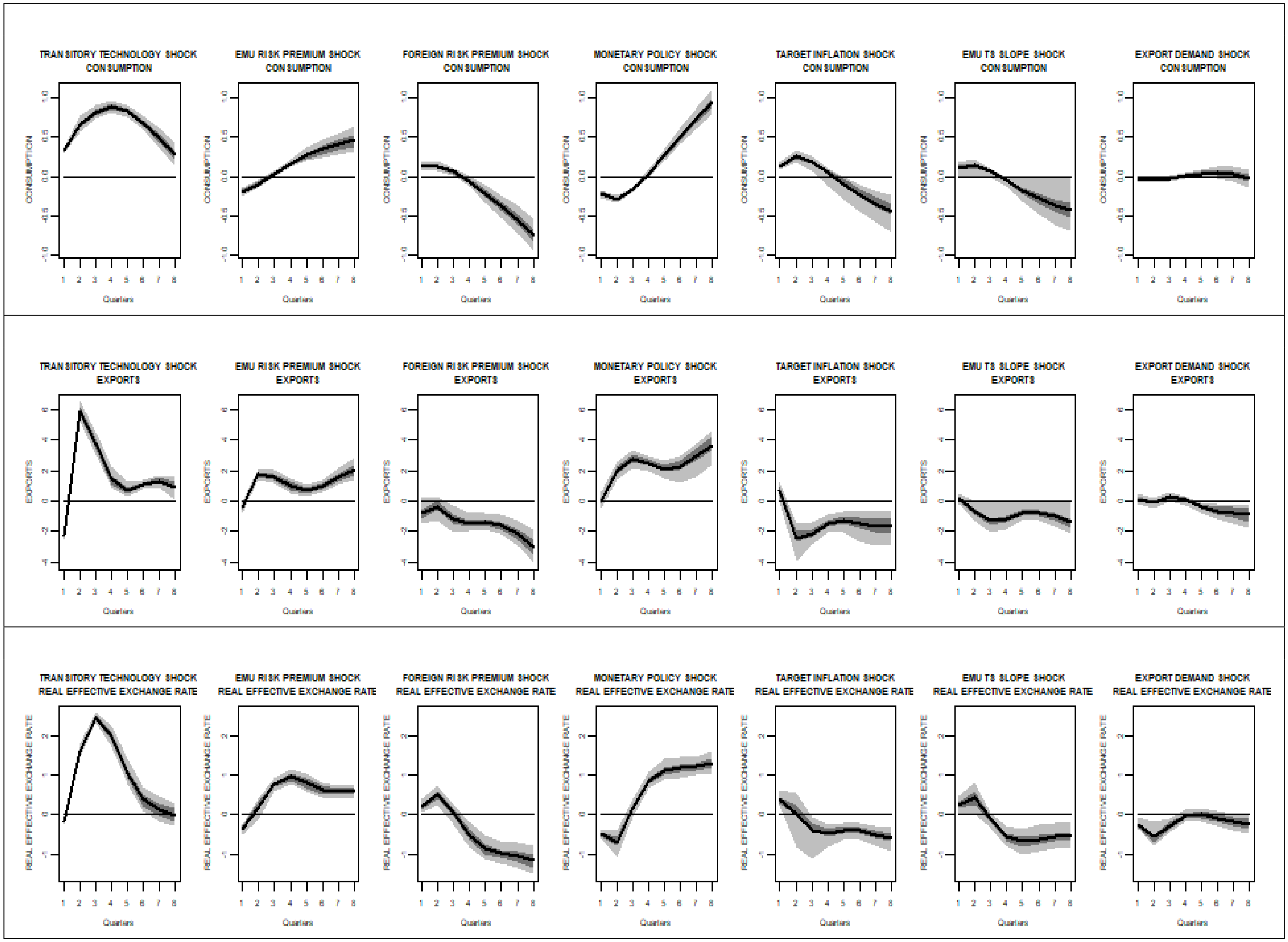


Table B.7: Responses with respect to one standard deviation shocks coming from seven different sources. We show the responses of EMU's consumption c_t, exports x_t and foreign exchange rate s_t. Based on the models posterior distribution we compute 1000 impulse responses and report the mean and the [10% ,90%] and [30% ,70%] confidence intervals.

C. Appendix Chapter 4

C.1 Alternative USV-MF-ATSM implementation

C.1.1 Specification of the alternative USV-MF-ATSM

According to Creal and Wu [2017] we specify the vector of state variables $\boldsymbol{f}_t$ as $\boldsymbol{f}_t = [\boldsymbol{m}_t, \boldsymbol{g}_t, \boldsymbol{h}_t]^T$ where $\boldsymbol{m}_t$ is the $M \times 1$ vector of the $M = 2$ observed macroeconomic variables $ln(INF_t)$ and $ln(\Delta GDP_t)$, $\boldsymbol{g}_t$ is the $G \times 1$ vector of latent yield factors and $\boldsymbol{h}_t$ is the $H \times 1$ vector of stochastic volatility factors. Similar to Creal and Wu in our alternative implementation we set $G = 3$ and $H = 4$, so that the total number of state variables is $N = M + G + H$. $\boldsymbol{\Sigma}_f$ is a $N \times N$ matrix indicating the impact of the Gaussian error $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{I}_{9\times 9})$ with $\boldsymbol{\varepsilon}_t = [\boldsymbol{\varepsilon}_{m,t}, \boldsymbol{\varepsilon}_{g,t}, \boldsymbol{\varepsilon}_{h,t}]^T$ on the ATSM's state-variables $\boldsymbol{f}_t$. To achieve identification the matrices $\boldsymbol{\Sigma}_m, \boldsymbol{\Sigma}_g$ and $\boldsymbol{\Sigma}_h$ are lower triangular where the diagonal elements of $\boldsymbol{\Sigma}_m$ and $\boldsymbol{\Sigma}_g$ are set to 1. Time-varying covariances $\boldsymbol{\Sigma}_{m,t}$ and $\boldsymbol{\Sigma}_{g,t}$ are defined as:

$$\boldsymbol{\Sigma}_{m,t} = \boldsymbol{\Sigma}_m \boldsymbol{\Lambda}_{m,t} \tag{C.1}$$

$$\boldsymbol{\Sigma}_{g,t} = \boldsymbol{\Sigma}_g \boldsymbol{\Lambda}_{g,t} \tag{C.2}$$

where $\boldsymbol{\Lambda}_{m,t}$ and $\boldsymbol{\Lambda}_{g,t}$ are $M \times M$ and $G \times G$ time-varying diagonal matrices with $diag(\boldsymbol{\Lambda}_{m,t}) = \boldsymbol{\gamma}_{m,t}$ and $diag(\boldsymbol{\Lambda}_{g,t}) = \boldsymbol{\gamma}_{g,t}$, where the vector $\boldsymbol{\gamma}_t = [\boldsymbol{\gamma}_{m,t}, \boldsymbol{\gamma}_{g,t}]^T$ is determined by:

$$\boldsymbol{\gamma}_t = exp\left(\left(\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 \boldsymbol{h}_t\right)/2\right) \tag{C.3}$$

$\boldsymbol{\Gamma}_0$ is a $(M+G) \times 1$ vector and the matrix $\boldsymbol{\Gamma}_1$ is $(M+G) \times H$ and transforms the impact of the volatility factors $\boldsymbol{h}_t$ on the covariance matrices of $\boldsymbol{m}_t$ and $\boldsymbol{g}_t$ respectively. Depending on three free parameters $\gamma_{0,4}, \gamma_{1,4,3}, \gamma_{1,4,4}$ $\boldsymbol{\Gamma}_0$ and $\boldsymbol{\Gamma}_1$ are specified as:

$$\boldsymbol{\Gamma}_0 = \begin{bmatrix} 0 \\ 0 \\ \gamma_{0,4} \\ 0 \end{bmatrix} \qquad \boldsymbol{\Gamma}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \gamma_{1,4,3} & \gamma_{1,4,4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

such that the first two volatility factors are macroeconomic volatility factors, whereas the the last two factors in $\boldsymbol{h}_t$ determine the volatility of the yield factors.

For interpreting $\boldsymbol{g}_t$ as $r_t, \mathbb{E}_t[r_{t+n^*}]$ and $TP(t, t+n^*)$, where the current and expected short rate and the term premium are defined by using the arbitrage-free pricing scheme of the (unspanned) stochastic volatility ATSM outlined in 4.2.3.2 as:

$$r_t = a_1 + \boldsymbol{b}_1^T \boldsymbol{f}_t \tag{C.4}$$

$$\begin{aligned}
\mathbb{E}_t[r_{t+n^*}] &= \frac{1}{n^*}\,\mathbb{E}_t[r_1 + ... r_{t+n^*-1}] \\
&= a_1 + \frac{\boldsymbol{b}_1^T}{n^*}\left[(n^*-1)\mathbf{I} + (n^*-2)\boldsymbol{\Psi}_f + (n^*-2)\boldsymbol{\Psi}_f^2 + ... + \boldsymbol{\Psi}_f^{(n^*-2)}\right](\mathbf{I} - \boldsymbol{\Psi}_f)\,\bar{\boldsymbol{\mu}}_t \\
&+ \frac{\boldsymbol{b}_1^T}{n^*}\left[\mathbf{I} + \boldsymbol{\Psi}_f + \boldsymbol{\Psi}_f^2 + ... + \boldsymbol{\Psi}_f^{(n^*-1)}\right]\boldsymbol{f}_t \\
&= c_{\bar{\tau}} + \boldsymbol{d}_{\bar{\tau}}^T \boldsymbol{f}_t
\end{aligned} \tag{C.5}$$

with:

$$c_{\bar{\tau}} = a_1 + \frac{\boldsymbol{b}_1^T}{\bar{\tau}}\left[(n^*-1)\mathbf{I} + (n^*-2)\boldsymbol{\Psi}_f + (n^*-2)\boldsymbol{\Psi}_f^2 + ... + \boldsymbol{\Psi}_f^{(n^*-2)}\right](\mathbf{I} - \boldsymbol{\Psi}_f)\,\bar{\boldsymbol{\mu}}_t \tag{C.6}$$

and

$$\boldsymbol{d}_{\bar{\tau}}^T = \frac{\boldsymbol{b}_1^T}{\bar{\tau}}\left[\mathbf{I} + \boldsymbol{\Psi}_f + \boldsymbol{\Psi}_f^2 + ... + \boldsymbol{\Psi}_f^{(n^*-1)}\right] \tag{C.7}$$

and

$$\begin{aligned}
TP(t, t+n^*) &= y(t, t+\bar{\tau}) - er_t^{\bar{\tau}} \\
&= a_{\bar{\tau}} + \boldsymbol{b}_{\bar{\tau}}^T \boldsymbol{f}_t - \boldsymbol{c}_{\bar{\tau}} - \boldsymbol{d}_{\bar{\tau}}^T \boldsymbol{f}_t \\
&= a_{\bar{\tau}} - \boldsymbol{c}_{\bar{\tau}} + \left(\boldsymbol{b}_{\bar{\tau}}^T - \boldsymbol{d}_{\bar{\tau}}^T\right)\boldsymbol{f}_t
\end{aligned} \tag{C.8}$$

with $\bar{\tau} = n^*$ we apply the rotation of $\boldsymbol{g}_t$ proposed by Creal and Wu by restricting $\delta_0, \boldsymbol{\delta}_1, \bar{\boldsymbol{\mu}}_f, \boldsymbol{\Psi}_f$, $\bar{\boldsymbol{\mu}}_g^Q$ and $\boldsymbol{\Psi}_g^Q$. δ_0 is set to zero and $\boldsymbol{\delta}_1$ is restricted by $\boldsymbol{\delta}_1^T = [1, 0, 0]$. The restrictions of the unconditional means are $\bar{\mu}_{g,1} = \bar{\mu}_{g,2}$ and
$\bar{\mu}_{g,1}^Q = \bar{\mu}_{g,2}^Q + \bar{\mu}_{g,3}^Q - \frac{1}{2\bar{\tau}}\left[\boldsymbol{b}_{g,1}^T \boldsymbol{\Sigma}_g^Q \boldsymbol{\Sigma}_g^{Q\,T} \boldsymbol{b}_{g,1} + 2^2 \boldsymbol{b}_{g,2}^T \boldsymbol{\Sigma}_g^Q \boldsymbol{\Sigma}_g^{Q\,T} \boldsymbol{b}_{g,2} + ... + (\bar{\tau}-1)^2 \boldsymbol{b}_{g,\bar{\tau}-1}^T \boldsymbol{\Sigma}_g^Q \boldsymbol{\Sigma}_g^{Q\,T} \boldsymbol{b}_{g,\bar{\tau}-1}\right]$ respectively. The third term in the restriction of $\bar{\boldsymbol{\mu}}_g^Q$ is Jensen's inequality (JI) term. For restricting $\boldsymbol{\Psi}_f$ we focus on the unique real eigenvalue implementation of Creal and Wu. Using the factorization $\boldsymbol{\Psi}_f = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{-1}$ of $\boldsymbol{\Psi}_f$, where $\mathbf{Q}$ is a $N \times N$ diagonal matrix with the eigenvectors $q_{f,1}, q_{f,2}, ..., q_{f,N}$ of $\boldsymbol{\Psi}_f$ in its columns and $\boldsymbol{\Lambda}$ is a $N \times N$ diagonal matrix with the eigenvalues $\boldsymbol{\lambda}_f^T = [\lambda_{f,1}, \lambda_{f,2}, ..., \lambda_{f,4}]$ of $\boldsymbol{\Psi}_f$ on its diagonal, the eigenvector matrix $\mathbf{Q}$ is

restricted to:

$$
\mathbf{Q} =
$$

$$
\begin{bmatrix}
1 & q_{1,2} & q_{1,3} & q_{1,4} & q_{1,5} & q_{1,6} & q_{1,7} & q_{1,8} & q_{1,9} \\
q_{2,1} & 1 & q_{2,3} & q_{2,4} & q_{2,5} & q_{2,6} & q_{2,7} & q_{2,8} & q_{2,9} \\
q_{3,1} & q_{3,2} & 1 & \frac{\bar{\tau}(1-\lambda_{f,1})}{(1-q_4^{\bar{\tau}}))} & q_{3,5} & q_{3,6} & q_{3,7} & q_{3,8} & q_{3,9} \\
\frac{q_{3,1}(1-\lambda_{f,1}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,1})} & \frac{q_{3,2}(1-\lambda_{f,2}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,2})} & 1 & \frac{q_{3,4}(1-\lambda_{f,4}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,4})} & \frac{q_{3,5}(1-\lambda_{f,5}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,5})} & \frac{q_{3,6}(1-\lambda_{f,6}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,6})} & \frac{q_{3,7}(1-\lambda_{f,7}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,7})} & \frac{q_{3,8}(1-\lambda_{f,8}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,8})} & \frac{q_{3,9}(1-\lambda_{f,9}^{\bar{\tau}})}{\bar{\tau}(1-\lambda_{f,9})} \\
q_{5,1} & q_{5,2} & q_{5,3} & q_{5,4} & 1 & q_{5,6} & q_{5,7} & q_{5,8} & q_{5,9} \\
0 & 0 & 0 & 0 & 0 & 1 & q_{6,7} & q_{6,8} & q_{6,9} \\
0 & 0 & 0 & 0 & 0 & q_{7,6} & 1 & q_{7,8} & q_{7,9} \\
0 & 0 & 0 & 0 & 0 & q_{8,6} & q_{8,7} & 1 & q_{8,9} \\
0 & 0 & 0 & 0 & 0 & q_{9,6} & q_{9,7} & q_{9,8} & 1
\end{bmatrix}
\tag{C.9}
$$

Factorizing the autoregressive matrix $\mathbf{\Psi}_g^Q$ of the diffusion of $\boldsymbol{g}_t$ under the risk neutral measure Q $\mathbf{\Psi}_g^Q = \mathbf{Q}_g^Q \mathbf{\Lambda}_g \left(\mathbf{Q}_g^Q\right)^{-1}$ where $\mathbf{Q}_g^Q$ is a $G \times G$ matrix of eigenvectors $\boldsymbol{q}_{g,1}^Q, \boldsymbol{q}_{g,2}^Q, \boldsymbol{q}_{g,3}^Q$ of $\mathbf{\Psi}_g^Q$ and $\mathbf{\Lambda}_g$ is the $G \times G$ diagonal matrix with the eigenvalues $\boldsymbol{\lambda}_g^{Q\,T} = [\lambda_{g,1}^Q, \lambda_{g,2}^Q, \lambda_{g,3}^Q]$ of $\mathbf{\Psi}_g^Q$ on its diagonal. For $\mathbf{Q}_g^Q$ we use the restriction:

$$
\mathbf{Q}_g^Q = \begin{bmatrix} 1 & 1 & 1 \\ \bar{\lambda}_{g,1}^Q - q_{g,3,1}^Q & \bar{\lambda}_{g,2}^Q - q_{g,3,2}^Q & \bar{\lambda}_{g,3}^Q - q_{g,3,3}^Q \\ q_{g,3,1}^Q & q_{g,3,2}^Q & q_{g,3,3}^Q \end{bmatrix} \tag{C.10}
$$

C.1.2 Bayesian estimation of the alternative USV-MF-ATSM

Analogue to our Bayesian estimation procedure in the ATSM block described in 4.3.2 and to the procedure for estimating the parameters and factors $\{\boldsymbol{g}_t\}_{t=1,2,\ldots,T}$ and $\{\boldsymbol{h}_t\}_{t=1,2,\ldots,T}$ of the USV-MF-ATSM applied by Creal and Wu [2017] we use a MCMC procedure that alternates in every iteration between two state-space models, where the first is conditional on $\{\boldsymbol{h}_t\}_{t=1,2,\ldots,T}$ and the second on $\{\boldsymbol{g}_t\}_{t=1,2,\ldots,T}$. The first state-space model is used for drawing the parameters:

$$
\begin{aligned}
\boldsymbol{\theta}_{USV-MF-ATSM}^T = [&\sigma_{12M}, \sigma_{24M}, \sigma_{36M}, \sigma_{48M}, \sigma_{60M}, \lambda_{f,1}, \lambda_{f,2}, \lambda_{f,3}, \lambda_{f,4}, \lambda_{f,5}, \lambda_{f,6}, \lambda_{f,7}, \lambda_{f,8}, \lambda_{f,9}, \\
&q_{1,2}, q_{1,3}, q_{1,4}, q_{1,5}, q_{1,6}, q_{1,7}, q_{1,8}, q_{1,9}, q_{2,1}, q_{2,3}, q_{2,4}, q_{2,5}, q_{2,6}, q_{2,7}, q_{2,8}, q_{2,9}, \\
&q_{3,1}, q_{3,2}, q_{3,4}, q_{3,5}, q_{3,6}, q_{3,7}, q_{3,8}, q_{3,9}, q_{5,1} q_{3,7}, q_{3,8}, q_{3,9}, q_{5,1}, q_{5,2}, q_{5,3}, q_{5,4}, q_{5,6}, \\
&q_{5,7}, q_{5,8}, q_{5,9}, q_{6,7}, q_{6,8}, q_{6,9} q_{7,6}, q_{7,8}, q_{7,9}, q_{8,6}, q_{8,7}, q_{8,9}, q_{9,6}, q_{9,7}, q_{9,8}, \gamma_{0,4}, \gamma_{1,4,3}, \\
&\gamma_{1,4,4}, \bar{\mu}_{m,1}, \bar{\mu}_{m,2}, \bar{\mu}_{h,1}, \bar{\mu}_{h,2}, , \bar{\mu}_{h,3}, \bar{\mu}_{h,4}, \bar{\mu}_{g,1}, \bar{\mu}_{g,3}, \bar{\mu}_{g,2}^Q, \bar{\mu}_{g,3}^Q, \sigma_{m,2,1}, \sigma_{m,2,2}, \\
&vec(\mathbf{\Sigma}_{gm}), \sigma_{g,2,1}, \sigma_{g,3,1}, \sigma_{g,3,2}, vec(\mathbf{\Sigma}_{hm}), vec(\mathbf{\Sigma}_{hg}), vech(\mathbf{\Sigma}_h), \lambda_{g,1}^Q, \lambda_{g,2}^Q, \lambda_{g,3}^Q, \\
&q_{g,3,1}^Q, q_{g,3,2}^Q, q_{g,3,3}^Q]
\end{aligned}
$$

and the yield factors $\{\boldsymbol{g}_t\}_{t=1,2,\ldots,T}$, whereas the second model is used for drawing the stochastic macroeconomic and term structure volatility factors $\{\boldsymbol{h}_t\}_{t=1,2,\ldots,T}$.

C.1.2.1 First state-space model of the alternative USV-MF-ATSM

Conditional on $\{\boldsymbol{h}_t\}_{t=1,2,...,T}$ the first state-space model for estimating the alternative USV-MF-ATSM is defined with measurement equation:

$$\mathbf{Y}_t = \boldsymbol{c} + \mathbf{H}\boldsymbol{s}_t + \boldsymbol{\varepsilon}_t \tag{C.11}$$

with Gaussian measurement errors $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{R})$ and transition equation:

$$\boldsymbol{s}_{t+1} = \boldsymbol{d} + \mathbf{F}\boldsymbol{s}_t + \boldsymbol{\epsilon}_t \tag{C.12}$$

with the Gaussian $\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \mathbf{Q}_t)$. The measurements $\mathbf{Y}_t$ are $\mathbf{Y}_t^T = [\boldsymbol{y}_t, \boldsymbol{m}_t, \boldsymbol{h}_t]$ where as in 4.3.2.1 the 6×1 vector $\boldsymbol{y}_t$ contains the observed zero-coupon spot rates $\boldsymbol{y}_t^T = [y(t,6), y(t,12), y(t,24), y(t,36), y(t,48)y(t,60)]$ of the $N_\tau = 6$ maturities $\tau = 6, 12, 24, 36, 48, 60$ month and the USV-ATSM's state variables are defined as $\boldsymbol{s}_t^T = [\boldsymbol{m}_t, \boldsymbol{g}_t, \boldsymbol{h}_t, \bar{\boldsymbol{\mu}}_m, \bar{\boldsymbol{\mu}}_g^u, \bar{\boldsymbol{\mu}}_g^{Q,u}]$. Here $\bar{\boldsymbol{\mu}}_g^u$ and $\bar{\boldsymbol{\mu}}_g^{Q,u}$ are the $G \times 1$ unrestricted unconditional means of $\boldsymbol{g}_t$ unter both probability measures. $\boldsymbol{c}, \mathbf{H}$ and the covariance matrix $\mathbf{R}$ of the measurement equation are defined as:

$$\boldsymbol{c} = \begin{bmatrix} \mathbf{0} \\ \mathbf{A}_0 + \mathbf{A}_1\mathbf{M}_0^Q \\ \mathbf{0} \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} \mathbf{I}_{M\times M} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_1\mathbf{M}_1^Q \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{H\times H} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where the $N_\tau \times 1$ vector $\mathbf{A}_0$ and the $N_\tau \times G$ matrix $\mathbf{A}_1$ are from the bond pricing equations 4.41 and 4.42 with $a_{0,i} = \delta_0 - a_{\tau_i-1} - 1/(2\tau_i)\mathbf{B}_{\tau_i-1}^T\boldsymbol{\Sigma}_g^Q\boldsymbol{\Sigma}_g^{Q\,T}\mathbf{B}_{\tau-1}$ and $\boldsymbol{a}_{1,i} = -\mathbf{B}_{\tau_i-1}^T/\tau_i$ with the i-th maturity of $\tau = 6, 12, 24, ..., 60$. $\mathbf{M}_0^Q$ and $\mathbf{M}_1^Q$ are defined as:

$$\mathbf{M}_0^Q = \begin{bmatrix} JI \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with JI as Jensen's inequality term. The transition equation's $\boldsymbol{d}, \mathbf{F}$ and $\mathbf{Q}_t$ are specified as:

$$\boldsymbol{d} = \begin{bmatrix} \bar{\boldsymbol{\Psi}}_{mh}\bar{\boldsymbol{\mu}}_h \\ \bar{\boldsymbol{\Psi}}_{gh}\bar{\boldsymbol{\mu}}_h \\ \bar{\boldsymbol{\Psi}}_h\bar{\boldsymbol{\mu}}_h \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} \boldsymbol{\Psi}_m & \boldsymbol{\Psi}_{mg} & \boldsymbol{\Psi}_{mh} & \bar{\boldsymbol{\Psi}}_{mm} & \bar{\boldsymbol{\Psi}}_{mg} & \mathbf{0} \\ \boldsymbol{\Psi}_{gm} & \boldsymbol{\Psi}_g & \boldsymbol{\Psi}_{gh} & \bar{\boldsymbol{\Psi}}_{mg} & \bar{\boldsymbol{\Psi}}_g & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Psi}_h & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{M\times M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{(G-1)\times(G-1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{(G-1)\times(G-1)} \end{bmatrix}$$

and

$$\mathbf{Q}_t = \begin{bmatrix} \boldsymbol{\Sigma}_m\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_m^T & \boldsymbol{\Sigma}_m\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_{gm}^T & \boldsymbol{\Sigma}_m\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_{hm}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Sigma}_{gm}\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_m^T & \boldsymbol{\Sigma}_{gm}\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_{gm}^T + \boldsymbol{\Sigma}_g\boldsymbol{\Lambda}_{g,t}^2\boldsymbol{\Sigma}_g^T & \boldsymbol{\Sigma}_{gm}\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_{hm}^T + \boldsymbol{\Sigma}_g\boldsymbol{\Lambda}_{g,t}^2\boldsymbol{\Sigma}_{hg}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Sigma}_{hm}\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_m^T & \boldsymbol{\Sigma}_{hm}\boldsymbol{\Lambda}_{m,t}^2\boldsymbol{\Sigma}_{gm}^T + \boldsymbol{\Sigma}_{hg}\boldsymbol{\Lambda}_{g,t}^2\boldsymbol{\Sigma}_g^T & \boldsymbol{\Sigma}_{hm}\boldsymbol{\Sigma}_{hm}^T + \boldsymbol{\Sigma}_{hg}\boldsymbol{\Sigma}_{hg}^T + \boldsymbol{\Sigma}_h\boldsymbol{\Sigma}_h^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where the covariance matrix $\mathbf{Q}_t$ becomes time-varying due to the matrices $\mathbf{\Lambda}_{m,t}$ and $\mathbf{\Lambda}_{g,t}$ determined by $\boldsymbol{h}_t$. The restricted (sub) coefficient matrices $\bar{\mathbf{\Psi}}_m, \bar{\mathbf{\Psi}}_{mg}, \bar{\mathbf{\Psi}}_{mh}, \bar{\mathbf{\Psi}}_{gm}, \bar{\mathbf{\Psi}}_g, \bar{\mathbf{\Psi}}_{gh}, \bar{\mathbf{\Psi}}_h$ in $\mathbf{F}$ are from:

$$\begin{bmatrix} \bar{\mathbf{\Psi}}_m & \bar{\mathbf{\Psi}}_{mg} & \bar{\mathbf{\Psi}}_{mh} \\ \bar{\mathbf{\Psi}}_{gm} & \bar{\mathbf{\Psi}}_g & \bar{\mathbf{\Psi}}_{gh} \\ \mathbf{0} & \mathbf{0} & \bar{\mathbf{\Psi}}_h \end{bmatrix} = [\mathbf{I}_{N\times N} - \mathbf{\Psi}_f]\,\mathbf{L}$$

with $N \times (N-1)$ scaling matrix:

$$\begin{bmatrix} \mathbf{I}_{M\times M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{H\times H} \end{bmatrix}$$

C.1.2.2 Second state-space model of the alternative USV-MF-ATSM

The second state space model is derived from the VAR[1] process expressed in 4.35. The measurement equation of the second state space model is:

$$\boldsymbol{x}_t = \mathbf{G}_t + \mathbf{Z}\boldsymbol{h}_t + \boldsymbol{\varepsilon}_t$$

with the vector of observed variables $\boldsymbol{x}_t^T = [\boldsymbol{m}_t, \boldsymbol{g}_t]$ and

$$\mathbf{G}_t = \begin{bmatrix} \boldsymbol{\mu}_m \\ \boldsymbol{\mu}_g \end{bmatrix} + \begin{bmatrix} \mathbf{\Psi}_m & \mathbf{\Psi}_{mg} \\ \mathbf{\Psi}_{gm} & \mathbf{\Psi}_g \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_{t-1} \\ \boldsymbol{g}_{t-1} \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} \mathbf{\Psi}_{mh} \\ \mathbf{\Psi}_{gh} \end{bmatrix}$$

$$\mathbf{S}_t = \begin{bmatrix} \mathbf{\Sigma}_m \mathbf{\Lambda}_{m,t}^2 \mathbf{\Sigma}_m^T & \mathbf{\Sigma}_m \mathbf{\Lambda}_{m,t}^2 \mathbf{\Sigma}_{gm}^T \\ \mathbf{\Sigma}_{gm} \mathbf{\Lambda}_{m,t}^2 \mathbf{\Sigma}_m^T & \mathbf{\Sigma}_{gm} \mathbf{\Lambda}_{m,t}^2 \mathbf{\Sigma}_{gm}^T + \mathbf{\Sigma}_g \mathbf{\Lambda}_{g,t}^2 \mathbf{\Sigma}_g^T \end{bmatrix}$$

as the measurements time varying constant, the coefficient matrix and the time varying covariance matrix of the Gaussian measurement errors $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{S}_t)$ where the errors are $\boldsymbol{\varepsilon}_t = [\boldsymbol{\varepsilon}_{m,t}, \boldsymbol{\varepsilon}_{g,t}]$.

$$\boldsymbol{h}_{t+1} = \boldsymbol{\mu}_h + \mathbf{\Psi}_h \boldsymbol{h}_t + \boldsymbol{\varepsilon}_{h,t} \tag{C.13}$$

defines the transition equation of the state variable $\boldsymbol{h}_t$ where $\boldsymbol{\varepsilon}_{h,t} \sim N(\mathbf{0}, \mathbf{S}_h)$ with the covariance of $\boldsymbol{h}_t$:

$$\begin{aligned} \mathbf{S}_h = {} & \mathbf{\Sigma}_{hm}\mathbf{\Sigma}_m\mathbf{\Lambda}_{m,t}^2\mathbf{\Sigma}_m^T\mathbf{\Sigma}_{hm}^T + \mathbf{\Sigma}_{hg}\mathbf{\Sigma}_{gm}\mathbf{\Lambda}_{m,t}^2\mathbf{\Sigma}_m^T\mathbf{\Sigma}_{hm}^T + \mathbf{\Sigma}_{hm}\mathbf{\Sigma}_m\mathbf{\Lambda}_{m,t}^2\mathbf{\Sigma}_{gm}^T\mathbf{\Sigma}_{hm}^T \\ & + \mathbf{\Sigma}_{hg}\left(\mathbf{\Sigma}_{gm}\mathbf{\Lambda}_{m,t}^2\mathbf{\Sigma}_{gm}^T + \mathbf{\Sigma}_g\mathbf{\Lambda}_{g,t}^2\mathbf{\Sigma}_g^T\right)\mathbf{\Sigma}_{hg}^T + \mathbf{\Sigma}_h\mathbf{\Sigma}_h^T \end{aligned}$$

C.2 Alternative USV latent- and MF-DNS implementation

Beside our DSGE-USV-ATSM and the USV-MF-ATSM outlined in the previous section we implement (unspanned) stochastic volatility versions of the DNS and the macro-finance (MF-) DNS proposed in their constant volatility version by Diebold and Li [2006] and Diebold, Rudebusch and Aruoba [2006]. To keep the estimation of these stochastic volatility DNS simple we use the reduced two-factor form of the DNS proposed by Diebold, Piazzesi and Rudebusch [2005]. In this Appendix we outline the implementation of the larger MF-DNS. The implementation of the USV-Latent-DNS is analogue. In the USV-MF-DNS we use $(N_l + N_o) \times 1$ term structure spanning factors $\boldsymbol{f}_t^T = [l_t, s_t, \boldsymbol{m}_t]$, where l_t and s_t are the $N_l = 2$ term structure's level and slope factors. $\boldsymbol{m}_t^T = [IP_t, CPI_t, ECB_t]$ are the $N_o = 3$ observed macroeconomic variables where IP_t and CPI_t stands for the monthly observed annual growth rate industrial production and the annual inflation rate both queried from the OECD economic database. ECB_t is the ECB controlled short term rate approximated here by the one month EONIA swap rate. In our specification we use $N_l + N_o$ unspanning volatility factors $\boldsymbol{h}_t$. Estimation of the stochastic volatility DNS is done by a Gibbs sampling MCMC procedure where the procedure alternated between two state-space models - the first conditional to $\{\boldsymbol{h}_t\}_{t=1,2,...,T}$ the second conditional to $\{l_t, s_t\}_{t=1,2,...,T}$

The first state-space model conditional on the volatility factors $\{\boldsymbol{h}_t\}_{t=1,2,...,T}$ is defined as:

$$\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{m}_t \\ ln(\boldsymbol{h}_t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Psi}_y & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_t \\ \boldsymbol{m}_t \\ ln(\boldsymbol{h}_t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_{y,t} \\ \boldsymbol{\eta}_{m,t} \\ \boldsymbol{\eta}_{h,t} \end{bmatrix} \tag{C.14}$$

$$\begin{bmatrix} \boldsymbol{f}_t \\ \boldsymbol{m}_t \\ ln(\boldsymbol{h}_t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{yy} & \boldsymbol{\Phi}_{ym} & \mathbf{0} \\ \boldsymbol{\Phi}_{my} & \boldsymbol{\Phi}_{mm} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Delta}_{hh} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_{t-1} \\ \boldsymbol{m}_{t-1} \\ ln(\boldsymbol{h}_{t-1}) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{y,t} \\ \boldsymbol{\varepsilon}_{m,t} \\ \boldsymbol{\varepsilon}_{h,t} \end{bmatrix} \tag{C.15}$$

with $M \times 2$ coefficient matrix of the measurements

$$\boldsymbol{\Psi}_{yy} = \boldsymbol{e} \left(\sum_{j=1}^{M} \left(\boldsymbol{e}^T - exp\left(-\lambda \boldsymbol{\tau}^T\right)\right) \boldsymbol{\delta}_j \otimes \boldsymbol{\delta}_j \right) \left(\boldsymbol{\delta}_j^T \lambda \boldsymbol{\tau} \right)$$

where $\boldsymbol{e}$ denotes the $M \times 1$ unit vector and $\boldsymbol{\delta}_j$ is an $M \times 1$ vector of zeros and the j-th element equal to 1. M is the number of zero-coupon rates for the M different time to maturities in $\boldsymbol{y}_t^T = [y(t, \tau_1), ..., y(t, \tau_M)]$, where $y(t, \tau_j)$ is the time t zero-coupon rate with time to maturity $\tau_j \geq 0$ for all $j = 1, 2, ..., M$. $\boldsymbol{\tau}^T = [\tau_1, ..., \tau_M]$ is the $M \times 1$ vector of maturities. The matrix $\boldsymbol{\Delta}_{hh}$ in the state-variables transition matrix in C.15 comes from the volatility factors dynamics given by:

$$ln\left(\begin{bmatrix} \boldsymbol{h}_{y,t} \\ \boldsymbol{h}_{m,t} \end{bmatrix}\right) = \begin{bmatrix} \boldsymbol{\Lambda}_y & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_m \end{bmatrix} ln\left(\begin{bmatrix} \boldsymbol{h}_{y,t-1} \\ \boldsymbol{h}_{m,t-1} \end{bmatrix}\right) + \boldsymbol{\vartheta}_t \tag{C.16}$$

where the $N_l \times N_l$ and $N_o \times N_o$ matrices $\boldsymbol{\Lambda}_y$ and $\boldsymbol{\Lambda}_m$ are restricted to keep the stationarity of the log factor dynamics. $\boldsymbol{\vartheta}_t$ is the volatility factors Gaussian error term $\boldsymbol{\vartheta}_{i,t} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\vartheta)$ with time-invariant $N \times N$ covariance matrix.

The error terms of the measurement and transition equation C.14 and C.15 are defined by:

$$\begin{bmatrix} \boldsymbol{\eta}_{y,t} \\ \boldsymbol{\eta}_{m,t} \\ \boldsymbol{\eta}_{h,t} \end{bmatrix} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\eta}) \tag{C.17}$$

where the measurements covariance matrix is specified as:

$$\boldsymbol{\Sigma}_{\eta} = \begin{bmatrix} \boldsymbol{\Sigma}_{\eta,y} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

and

$$\begin{bmatrix} \boldsymbol{\varepsilon}_{y,t} \\ \boldsymbol{\varepsilon}_{m,t} \\ \boldsymbol{\varepsilon}_{h,t} \end{bmatrix} \sim N\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}^{y}_{\varepsilon,t} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}^{m}_{\varepsilon,t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma}^{h}_{\varepsilon} \end{bmatrix} \right) \tag{C.18}$$

Both error vectors are orthogonal to each other. The measurements sigma matrix $\boldsymbol{\Sigma}_{\eta,y}$ and the time varying matrices $\boldsymbol{\Sigma}^{y}_{\varepsilon,t}$ and $\boldsymbol{\Sigma}^{m}_{\varepsilon,t}$ are diagonal matrices, where the diagonal elements of the last two matrices contain the time varying log normal distributed volatility factors $\boldsymbol{h}_t^T = \left[h_t^l, h_t^s, \boldsymbol{h}_t^m\right]$.

The second state space model conditional to the term structure factors $\{l_t, s_t\}_{t=1,2,\ldots,T}$ is defined by the measurement equation:

$$\begin{bmatrix} \boldsymbol{f}_t \\ \boldsymbol{m}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{yy} & \boldsymbol{\Phi}_{ym} \\ \boldsymbol{\Phi}_{my} & \boldsymbol{\Phi}_{mm} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_{t-1} \\ \boldsymbol{m}_{t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{y,t} \\ \boldsymbol{\varepsilon}_{m,t} \end{bmatrix} \tag{C.19}$$

and the transition equation according to C.15.

Drawing the parameters of the USV latent DNS and the USV-MF-DNS is done by using the tailored Metropolis-Hastings (MH) algorithm proposed by Chib and Ergaslev [2009] and Chib and Ramamurthy [2010]. Drawing $\{l_t, s_t\}_{t=1,2,\ldots,T}$ is done by the forward-backward Kalman filtering and drawing procedure by Carter and Kohn [1994]. For drawing the volatility factors a similar forward-backward scheme is used. Due to the non-linear influence of the volatility factors drawing of $\{\boldsymbol{h}_t\}_{t=1,2,\ldots T}$ is also done according to a forward-backward scheme by using the bootstrap particle filter with conditional resampling proposed by Andrieu, Doucet and Holenstein [2010] for the forward filtering and the algorithm proposed by Whiteley [2010] for the backward drawing of $\{\boldsymbol{h}_t\}_{t=1,2,\ldots T}$.

C.3 Generalized Autoregressive Score Model (GAS)

As outlined in Creal, Koopman and Lucas [2011] the mean adjusted observed yield data y_t where y_t is a $n \times 1$ vector with the $n = 4$ maturities $\tau = 12, 36, 60, 120$ month is distributed by a multivariate Student's t distribution, with PDF:

$$f\left(\boldsymbol{y}_t|\boldsymbol{\Sigma}_t, v\right) = \frac{\Gamma\left(\frac{(v+n)}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\left((v-2)\pi\right)^{n/2}|\boldsymbol{\Sigma}_t|^{1/2}} \tag{C.20}$$

$\boldsymbol{\Sigma}_t$ is the $n \times n$ time varying covariance matrix and $v > 2$ are the degrees of freedom of the multivariate distribution. For the generalized autoregressive score (GAS) implemented here, the covariance matrix $\boldsymbol{\Sigma}_t$ and the correlation matrix $\mathbf{R}_t$ are decomposed according to the standard case described by Creal, Koopman and Lucas [2011]:

$$\boldsymbol{\Sigma}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t \quad , \quad \mathbf{R}_t = \Delta_t^{-1}\mathbf{Q}_t\boldsymbol{\Delta}_t^{-1} \quad , \quad \Delta_t = diag\left(\mathbf{Q}_t\right)^{1/2} \tag{C.21}$$

where the $n \times n$ matrix $\mathbf{D}_t$ is diagonal with the standard deviations of the mean adjusted yield $\boldsymbol{y}_t$ on its diagonal. $\mathbf{Q}_t$ is a symmetric (at least) positive semidefinite $n \times n$ matrix.

The dynamics of $\boldsymbol{\Sigma}_t$ and $\mathbf{D}_t$ are driven by a $n(n+1)/2 + n \times 1$ vector of factors $\boldsymbol{f}_t$, where $\boldsymbol{f}_t$ is specified here with $\boldsymbol{f}_t = [\boldsymbol{d}_t, q_t]^T$, where $\boldsymbol{d}_t = diag(\mathbf{D}_t)^2$ is the $n \times 1$ vector of time-varying variances from the diagonal of $\mathbf{D}_t$ and $\boldsymbol{q}_t = vech(\mathbf{Q}_t)$ is the $n(n+1)/2 \times 1$ vector of the lower triangular elements of the symmetric matrix $\mathbf{Q}_t$ with $vech(*)$ as the operator which transforms the lower triangular elements into the column vector $\boldsymbol{q}_t$. $\boldsymbol{f}_t$ is modeled by the following autoregressive process:

$$\boldsymbol{f}_{t+1} = \boldsymbol{\omega} + \sum_{i=1}^{p} \mathbf{A}_i\boldsymbol{s}_{t-i+1} + \sum_{j=1}^{q} \mathbf{B}_j\boldsymbol{f}_{t-j+1} \tag{C.22}$$

where $\boldsymbol{\omega}$ is a $n(n+1)/2 \times 1$ vector of constants and $\mathbf{A}_i$ and $\mathbf{B}_j$ with $i = 1, 2, ..., p$ and $j = 1, 2, ..., q$ are $n(n+1)/2 + n \times n(n+1)/2 + n$ diagonal matrices. $\boldsymbol{s}_t$ is the scale function which is defined by:

$$\boldsymbol{s}_t = \mathbf{S}_t\nabla_t \quad , \quad \nabla_t = \frac{\partial ln(f(\boldsymbol{y}_t|\boldsymbol{f}_t, \boldsymbol{\theta})}{\partial \boldsymbol{f}_t} \tag{C.23}$$

with PDF f of the multivariate Student distribution from C.20. $\boldsymbol{\theta}$ collects the parameters of $v, \boldsymbol{\omega}, \mathbf{A}_i$ and $\mathbf{B}_j$ with $i = 1, 2, ..., p$ and $j = 1, 2, ..., q$ of the factor dynamics defined in C.22. The scaling matrix $\mathbf{S}_t$ in C.23 is chosen with inverse Fisher information matrix, which expresses information about the curvature of the logged PDF f in C.23:

$$\mathbf{S}_t = \tilde{\mathbf{I}}_{t|t-1}^{+} \quad \tilde{\mathbf{I}}_{t|t-1} = \mathbb{E}_{t-1}\left[\nabla_t\nabla_t^T\right] \tag{C.24}$$

Due to the singularity of the information matrix, with $\tilde{\mathbf{I}}_{t|t-1}^{+}$ as the Moore-Penrose pseudo inverse is used as the scaling matrix $\mathbf{S}_t$ in C.24. ∇_t and $\tilde{\mathbf{I}}_{t|t-1}$ are defined as:

$$\nabla_t = \frac{1}{2}\boldsymbol{\Psi}_t^T\mathbf{D}_n^T\mathbf{J}_{t\otimes}^T\left(w_t\bar{\mathbf{y}}_{t\otimes} - vec(\mathbf{I}_{n\times n})\right) \tag{C.25}$$

$$\tilde{\mathbf{I}}_{t|t-1} = \frac{1}{4}\mathbf{\Psi}_t^T \mathbf{D}_n^T \mathbf{J}_{t\otimes}^T \left(g\mathbf{G} - vec(\mathbf{I}_{n\times n})vec(\mathbf{I}_{n\times n})^T\right)\mathbf{J}_{t\otimes}\mathbf{D}_n\mathbf{\Psi}_t \tag{C.26}$$

with the weights $w_t = (v+n)/(v-2+\boldsymbol{y}_t^T\mathbf{\Sigma}_t^{-1}\boldsymbol{y}_t)$ and $g = (v+n)/(v+2+n)$, $\mathbf{I}_{n\times n}$ as the $n\times n$ identity matrix, the lower triangular $\mathbf{J}_t$ implicitely defined by the Cholesky decomposition of the inverted covariance matrix $\mathbf{\Sigma}_t^{-1} = \mathbf{J}_t^T\mathbf{J}_t$, $\bar{\boldsymbol{y}}_t = \mathbf{J}_t\boldsymbol{y}_t$ and the $n^2\times n^2$ matrix $\mathbf{G}$ which is defined as:

$$\mathbf{G}[(i-1)n+l,(j-1)n+m] = \delta_{i,m}\delta_{l,m} + \delta_{i,l}\delta_{j,m} + \delta_{i,m}\delta_{j,l} \quad \forall i,j,l,m = 1,2,...,n \tag{C.27}$$

with:

$$\delta_{i,j} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$$

$\mathbf{D}_n$ is the $n^2\times n(n+1)/2$ duplicaton matrix implied by $\mathbf{D}_n vech(\mathbf{C}) = vec(\mathbf{C})$ for some symmetric $n\times n$ matrix $\mathbf{C}$. $\mathbf{C}_\otimes = \mathbf{C}\otimes\mathbf{C}$ denotes the $n^2\times n^2$ Kronecker product of the $n\times n$ matrix $\mathbf{C}$ with itself. The matrix $\mathbf{\Psi}_t = [\mathbf{\Psi}_{t,d}, \mathbf{\Psi}_{t,q}] = \partial vech(\mathbf{\Sigma}_t)/\partial \boldsymbol{f}_t^T$ is defined by:

$$\mathbf{\Psi}_{t,q} = \mathbf{L}_n\left(\mathbf{D}_t\mathbf{\Delta}_t^{-1}\otimes\mathbf{D}_t\mathbf{\Delta}_t^{-1}\right)\left(\mathbf{D}_n - \left[\mathbf{I}_{n\times n}\otimes\mathbf{Q}_t\mathbf{\Delta}_t^{-1} + \mathbf{Q}_t\mathbf{\Delta}_t^{-1}\otimes\mathbf{I}_{n\times n}\right]\mathbf{K}_\Delta\mathbf{W}_q\right) \tag{C.28}$$

$$\mathbf{\Psi}_{t,d} = \mathbf{L}_n\left[\mathbf{I}_{n\times n}\otimes\mathbf{D}_t\mathbf{R}_t + \mathbf{D}_t\mathbf{R}_t\otimes\mathbf{I}_{n\times n}\right]\mathbf{K}_d\mathbf{V} \tag{C.29}$$

where $\mathbf{L}_n = \left(\mathbf{D}_n^T\mathbf{D}_n\right)^{-1}\mathbf{D}_n^T$ defines the elimination matrix from the equation $\mathbf{L}_n vec(\mathbf{C}) = vech(\mathbf{C})$ for some $n\times n$ matrix $\mathbf{C}$, $\mathbf{K}_\Delta$ and $\mathbf{K}_d$ are selection matrices from $vec(\mathbf{\Delta}_t^2) = \mathbf{K}_\Delta\boldsymbol{q}_t$ and $vec(\mathbf{D}_t) = \mathbf{K}_d\boldsymbol{d}_t$ and $\mathbf{W}_q$ and $\mathbf{V}$ are diagonal matrices, where the $i = 1,2,...,n$ diagonal elements of $\mathbf{W}_q$ are $1/2\sqrt{|q_{t,i}|}$ if $q_{t,i}\neq 0$ and zero for $q_{t,i} = 0$. The diagonal elements of $\mathbf{V}$ are given by $1/2\sqrt{d_{t,i}}$ for $i = 1,2,...,n$. Estimation of the parameters $\boldsymbol{\theta}$ is done by MLE:

$$\begin{aligned} \hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \mathscr{L}(\boldsymbol{\theta}) = \sum_{t=1}^T \left(ln\left(\Gamma\left(\frac{v+n}{2}\right)\right) - ln\left(\Gamma\left(\frac{v}{2}\right)\right) - \frac{1}{2}ln\left(|\mathbf{\Sigma}_t|\right)\right) \\ -\sum_{t=1}^T\left(\frac{1}{2}ln\left((v-2)\pi\right) + \frac{(v+n)}{2}ln\left(1+\frac{\boldsymbol{y}_t^T\mathbf{\Sigma}_t^{-1}\boldsymbol{y}_t}{(v-2)}\right)\right) \end{aligned} \tag{C.30}$$

C.4 Decomposition of the term structure of volatilities

We focus here on the uncertainty factors related to $\boldsymbol{h}_t$. The decomposition of the term structure of volatilities is based on the empirical uncertainty shocks collected in the $T\times 1$ vector $\hat{\boldsymbol{\epsilon}}_h$, where the time t element $\hat{\epsilon}_t$ of $\hat{\boldsymbol{\epsilon}}_h$ with $t = 1,2,...,T$ is calculated as $\hat{\epsilon}_t = (h_t - \mu_h - \psi_h h_{t-1})/\sigma_h$.

For the elements of $\hat{\boldsymbol{\epsilon}}_h$ we calculate their diffusion through time, where we first define the $T\times 1$ vector $\tilde{\boldsymbol{\psi}}_h$ with:

$$\tilde{\boldsymbol{\psi}}_h = \sigma_h\sum_{t=1}^T \psi_h^t\boldsymbol{\delta}_t^T \tag{C.31}$$

$\boldsymbol{\delta}_t^T$ is a $T \times 1$ vector of zeros with the t-th element equal to 1. With $\tilde{\boldsymbol{\psi}}_h$ we next generate the $T \times T$ lower triangular matrix $\tilde{\boldsymbol{\Psi}}_h$ with:

$$\tilde{\boldsymbol{\Psi}}_h = \sum_{t=1}^{T} \sum_{i=1}^{t} \tilde{\boldsymbol{\psi}}_h \left(\boldsymbol{\delta}_i^T \otimes \boldsymbol{\delta}_i \right) \otimes \boldsymbol{\delta}_t \tag{C.32}$$

getting:

$$\tilde{\boldsymbol{\Psi}}_h = \begin{bmatrix} \sigma_h & 0 & 0 & ... & 0 \\ \sigma_h & \sigma_h \psi_h & 0 & ... & 0 \\ \sigma_h & \sigma_h \psi_h & \sigma_h \psi_h^2 & ... & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_h & \sigma_h \psi_h & \sigma_h \psi_h^2 & ... & \sigma_h \psi_h^{(T-1)} \end{bmatrix}$$

with $\tilde{\boldsymbol{\Psi}}_h$ we generate the $T \times 1$ diffusion vector $\tilde{\boldsymbol{\epsilon}}_h$ of the empirical uncertainty shocks of $\hat{\boldsymbol{\epsilon}}_h$ with:

$$\tilde{\boldsymbol{\epsilon}}_h = \tilde{\boldsymbol{\Psi}}_h \boldsymbol{\epsilon}_h \tag{C.33}$$

so that we can calculate for each t:

$$\tilde{\boldsymbol{\Sigma}}_{g,t} = \boldsymbol{\Sigma}_g exp \left(\tilde{\epsilon}_{h,t} \right) \tag{C.34}$$

with $\tilde{\boldsymbol{\Sigma}}_{g,t}$ containing the contributions of uncertainty shocks from the two latent yield factors $g_{1,t}$ and $g_{2,t}$ to the term structure of yield volatilities are summarized in the $N_\tau \times 2$ matrix:

$$\mathbf{C}_{g,t} = \sum_{m=1}^{N_\tau} \boldsymbol{\delta}_{N_\tau \times 1,m} \otimes \left(\sum_{i=1}^{2} \boldsymbol{\delta}_{2\times 1,i} \tilde{\boldsymbol{b}}_m \boldsymbol{\delta}_{2\times 1,i}^T \tilde{\boldsymbol{d}}_{g,t,i}^T \right) \tag{C.35}$$

where $\boldsymbol{\delta}_{N_\tau \times 1,i}$ and $\boldsymbol{\delta}_{2\times 1,i}$ are $N_\tau \times 1$ and 2×1 indicator vectors with 1 at the i-th position and zeros else. $N_\tau = 6$ indicates the number of used maturities $\boldsymbol{\tau} = [6, 12, 24, ..., 60]$. The two elements of the $m = 1, 2, ..., N_\tau$ th row of $\mathbf{C}_{g,t}$ are the time t contributions of uncertainty shocks from the two term structure factors $g_{1,t}$ and $g_{2,t}$ to the volatility of the zero rate $y(t, \tau_m)$ with maturity τ_m. The two 1×2 vectors $\tilde{\boldsymbol{b}}_m$ and $\tilde{\boldsymbol{d}}_{g,t,i}^T$ are determined by:

$$\tilde{\mathbf{b}}_m = \tilde{\mathbf{B}}^T \boldsymbol{\delta}_{N_c \times 1,m} \otimes \boldsymbol{e}_{2\times 1}^T \tag{C.36}$$

with $\tilde{B} = [\mathbf{B}\tilde{\mathbf{L}}^T]$ where $\tilde{\mathbf{L}} = [\mathbf{0}_{2\times 1}, \mathbf{I}_{2\times 2}]$ is a selection matrix used for composing the $N_\tau \times 2$ matrix $\tilde{\mathbf{B}}$ from the last two colums of $\mathbf{B}$. $\mathbf{B}^T = [\boldsymbol{b}_6^T, \boldsymbol{b}_{12}^T, \boldsymbol{b}_{24}^T, \boldsymbol{b}_{36}^T, \boldsymbol{b}_{48}^T, \boldsymbol{b}_{60}^T]$ is the $N_\tau \times 3$ matrix of maturity dependent bond loadings from the recursive bond pricing scheme outlined in section 4.2.3.2. $\boldsymbol{e}_{2\times 1}$ is a 2×1 unit vector.

$$\tilde{\boldsymbol{d}}_{g,t,i} = \tilde{\mathbf{D}}_{g,t} \boldsymbol{\delta}_{N_\tau \times 1,m} \otimes \boldsymbol{e}_{2\times 1} \tag{C.37}$$

where the $2N_\tau \times 2$ matrix $\tilde{D}_{g,t}$ is defined as:

$$\tilde{\mathbf{D}}_{g,t} = \tilde{\boldsymbol{\Sigma}}_{g,t} \tilde{\boldsymbol{\Sigma}}_{g,t}^T \otimes \tilde{\mathbf{B}}^T \tag{C.38}$$

C.5 Macroeconomic response on uncertainty shocks

For calculating the macroeconomic responses on uncertainty shocks originating in the macroeconomic volatility's 7×1 process expressed in 4.30 we first define the transition of a one standard deviation shock for each of the 7 macroeconomic volatilities $\boldsymbol{\sigma}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t}]$. For a given number of steps n_q (in quarters) we define the $n_q \times 7$ matrix $\mathbf{R}_v$ with its (i,j) element:

$$r_{v,i,j} = \sum_{k=1}^{i} \rho_{\sigma,j}^{k-1} \mu_{\sigma,j} + \rho_{\sigma,j}^{i-1} \sigma_{\sigma,j} \tag{C.39}$$

indicating the transmitted standard deviation shock from the $j = 1, 2, ..., 7$ volatility after $i = 1, 2, ..., n_q$ steps diffused through the volatility process 4.30. For getting the macroeconomic responses we transfer the shock transitions in $\mathbf{R}_v$ into the transition process of the macroeconomic DSGE state variables expressed in its canonical linear form in 4.32. Therefore we define the $49n_q \times 7$ block matrix $\mathbf{R}_s$:

$$\mathbf{R}_v = \begin{bmatrix} \boldsymbol{r}_{v,1,1} & \dots & \boldsymbol{r}_{v,1,7} \\ \vdots & & \vdots \\ \boldsymbol{r}_{v,n_q,1} & \dots & \boldsymbol{r}_{v,n_q,7} \end{bmatrix} \tag{C.40}$$

with its (i,j) 49×1 vector element $\boldsymbol{r}_{v,i,j}$ defined as:

$$\boldsymbol{r}_{v,i,j} = \sum_{k=1}^{i} \boldsymbol{\Theta}_0^{i-k} \boldsymbol{\Delta}_{k,j} \tag{C.41}$$

where $\boldsymbol{\Delta}_{k,j}$ for $k = 1, 2, ..., n_q$ is:

$$\boldsymbol{\Delta}_k = diag(\Theta_1(\tilde{\boldsymbol{r}}_v^{(k,j)})\Theta_1(\tilde{\boldsymbol{r}}_v^{(k,j)})^T) \tag{C.42}$$

$\Theta_1(\tilde{\boldsymbol{r}}_v^{(k,j)})\Theta_1(\tilde{\boldsymbol{r}}_v^{(k,j)})^T$ is the rational equilibrium covariance matrix of our used DSGE model after running Sim's QZ algorithm. $\tilde{\boldsymbol{r}}_v^{(k,j)}$ is a 7×1 column vector with $r_{v,k,j}$ from $\mathbf{R}_v$ in the k th row and all other elements $-\infty$. Last one ensures that only the k th volatility shock in step $i = 1, 2, ..., n_q$ is transferred into the transition process of the DSGE. In using the diagonal elements in $\boldsymbol{\Delta}_k$ we focus on the macroeconomic shock responses directly coming from the macroeconomic volatilities.

C.6 Bootstrap particle filter with conditional resampling

In the Gibbs sampling procedure applied in our model estimation the particle filter with the conditional resampling proposed by Andrieu, Doucet and Holenstein [2010] is used. For filtering the volatility factors $\boldsymbol{h}_t^{(i)}$ in the current step i of the Gibbs sampler the particle resampling of the filter uses the draw of the path $\{\tilde{\boldsymbol{h}}_t^{(i-1)}\}_{t=0,1,2,...,T}$ of filtered volatility factors in the previous step $i-1$ of the Gibbs sampler. The implementation of the particle filter is as follows:

Initialize the particle filter at time $t = 0$:

- Set $\boldsymbol{h}_0^{(i,1)} = \tilde{\boldsymbol{h}}_0^{(i-1)}$ and draw $\boldsymbol{h}_0^{(i,j)}$ from the initial distribution $h_0^{(i,j)} \sim P(\boldsymbol{h}_0|\boldsymbol{\theta})$ for the remaining draws $j = 2, 3, ..., J$ of the particle filter

- Set the normalized weights $\hat{\omega}_0^{(i,j)} = 1/J$ for all $j = 1, 2, ..., J$

- Store the draws and their normalized weights $\{\boldsymbol{h}_0^{(i,j)}, \hat{\omega}_0^{(i,j)}\}_{j=1,2,...,J}$

Continue at times $t = 1, 2, ..., T$ with:

- Set $\boldsymbol{h}_t^{(i,1)} = \tilde{\boldsymbol{h}}_t^{(i-1)}$ and draw $\boldsymbol{h}_t^{(i,j)}$ with $j = 2, 3, ..., J$ from the proposal distribution $\boldsymbol{h}_t^{(i,j)} \sim P(\boldsymbol{h}_t^{(i,j)}|\boldsymbol{h}_{t-1}^{(i,j)}, \boldsymbol{\theta})$ where $P(\boldsymbol{h}_t^{(i,j)}|\boldsymbol{h}_{t-1}^{(i,j)}, \boldsymbol{\theta})$ is the transition probability. With this definition of the proposal the implemented particle filter becomes a bootstrap filter.

- For each draw $j = 1, 2, ..., J$ of the particle filter calculate the importance weights:
$$\omega_t^{(i,j)} \propto \omega_{t-1}^{(i,j)} P(\boldsymbol{g}_t^{(i)}, \boldsymbol{m}_t^{(i)}|\boldsymbol{h}_t^{(i,j)}, \boldsymbol{g}_{t-1}^{(i)}, \boldsymbol{m}_{t-1}^{(i)}, \boldsymbol{\theta}) \tag{C.43}$$
where $P(\boldsymbol{g}_t^{(i)}, \boldsymbol{m}_t^{(i)}|\boldsymbol{h}_t^{(i,j)}, \boldsymbol{g}_{t-1}^{(i)}, \boldsymbol{m}_{t-1}^{(i)}, \boldsymbol{\theta})$ is the probability distribution defined by the measurement equation.

- For each draw $j = 1, 2, ..., J$ calculate the normalized weights:
$$\bar{\omega}_t^{(i,j)} = \frac{\omega_t^{(i,j)}}{\sum_{j=1}^{J} \omega_t^{(i,j)}} \tag{C.44}$$

- Resample the draws $\{\boldsymbol{h}_t^{(i,j)}\}_{j=1,2,...,J}$ with the probabilities $\{\bar{\omega}_t^{(i,j)}\}_{j=1,2,...,J}$ and set all weights to the constant value $\hat{\omega}_t^{(i,j)} = \omega_t^{(i,j)} = 1/J$. Following the conditional resampling suggested by Andrieu, Doucet and Holenstein [2010] the draw $\tilde{\boldsymbol{h}}_t^{(i-1)}$ of the Gibbs sampler in the previous step is always resampled.

- Store the draws and the normalized weights $\{\boldsymbol{h}_t^{(i,j), \hat{\omega}_t^{(i,j)}}\}_{j=1,2,...,J}$

C.7 Backward drawing from the particle filter output

Drawing a path $\{\tilde{\boldsymbol{h}}_t^{(i)}\}_{t=0,1,2,...,T}$ of the volatility factors in step i of the block Gibbs sampler is done by the algorithm proposed by Whiteley [2010]. The algorithm works as follows:

Starting at time $t = T$ one volatility factor $\tilde{\boldsymbol{h}}_T^{(i)} = \boldsymbol{h}_T^{(i,j)}$ is drawn with probability $\hat{\omega}_t^{(i,j)}$.

Iterating backwards for $t = T-1, T-2, ..., 0$ the algorithm proceeds with the steps:

- For each draw of the particle filter $j = 1, 2, ..., J$ calculate the backward weights:

$$\omega_{t|T}^{(i,j)} \propto \omega_t^{(i,j)} P(\tilde{\boldsymbol{h}}_{t+1}^{(i)} | \boldsymbol{h}_t^{(i,j)}, \boldsymbol{\theta}) \tag{C.45}$$

- For each draw $j = 1, 2, ..., J$ normalize the backward weights:

$$\hat{\omega}_{t|T}^{(i,j)} = \frac{\omega_{t|T}^{(i,j)}}{\sum_{j=1}^{J} \omega_{t|T}^{(i,j)}} \tag{C.46}$$

- Draw $\tilde{\boldsymbol{h}}_t^{(i)} = \boldsymbol{h}_t^{(i,j)}$ with the normalized backward probability $\hat{\omega}_{t|T}^{(i,j)}$

With the last draw in $t = 0$ the backward algorithm generates a draw $\{\tilde{\boldsymbol{h}}_t^{(i)}\}_{t=0,1,2,...,T}$ of the path of filtered volatility factors $\boldsymbol{h}_t$ in the i-th step of the Gibbs sampler.

C.8 Priors of the DSGE-USV-ATSM

SW DSGE Prior Distributions		
Structural Parameters	$r_{\Delta y} \sim N(0.12, 0.05)$	$\rho_\sigma^r \sim beta(0.90, 0.20)$
$\bar{\gamma} \sim N(0.40, 0.10)$	**Shock AR Coefficients**	$\rho_\sigma^p \sim beta(0.90, 0.20)$
$\bar{\pi} \sim Gamma(0.62, 0.10)$	$\rho_a \sim beta(0.50, 0.20)$	$\rho_\sigma^w \sim beta(0.90, 0.20)$
$L \sim N(0.00, 2.00)$	$\rho_b \sim beta(0.50, 0.20)$	**Stoch. Volatility Sigmas**
Preferences	$\rho_g \sim beta(0.50, 0.20)$	$\sigma_\sigma^a \sim InvGamma(0.01, 2.00)$
$(\beta^{-1} - 1)100 \sim Gamma(0.25, 0.10)$	$\rho_i \sim beta(0.50, 0.20)$	$\sigma_\sigma^b \sim InvGamma(0.01, 2.00)$
$\lambda \sim \beta(0.70, 0.10)$	$\rho_r \sim beta(0.50, 0.20)$	$\sigma_\sigma^g \sim InvGamma(0.01, 2.00)$
$\sigma_c \sim N(1.50, 0.37)$	$\rho_p \sim beta(0.50, 0.20)$	$\sigma_\sigma^i \sim InvGamma(0.01, 2.00)$
Price and Wage Setting	$\rho_w \sim beta(0.50, 0.20)$	$\sigma_\sigma^r \sim InvGamma(0.01, 2.00)$
$\xi_p \sim beta(0.50, 0.10)$	**Stoch. Volatility Constants**	$\sigma_\sigma^p \sim InvGamma(0.01, 2.00)$
$\iota_p \sim beta(0.50, 0.15)$	$\mu_\sigma^a \sim N(0.00, 0.10)$	$\sigma_\sigma^w \sim InvGamma(0.01, 2.00)$
$\xi_w \sim beta(0.50, 0.10)$	$\mu_\sigma^b \sim N(0.00, 0.10)$	**Measurement Errors**
$\iota_w \sim beta(0.50, 0.15)$	$\mu_\sigma^g \sim N(0.00, 0.10)$	$y \sim InvGamma(0.10, 2.00)$
Production	$\mu_\sigma^i \sim N(0.00, 0.10)$	$c \sim InvGamma(0.10, 2.00)$
$\varphi \sim N(4.00, 1.50)$	$\mu_\sigma^r \sim N(0.00, 0.10)$	$i \sim InvGamma(0.10, 2.00)$
$\alpha \sim N(0.30, 0.05)$	$\mu_\sigma^p \sim N(0.00, 0.10)$	$l \sim InvGamma(0.10, 2.00)$
$\Phi \sim N(1.25, 0.12)$	$\mu_\sigma^w \sim N(0.00, 0.10)$	$\pi \sim InvGamma(0.10, 2.00)$
$\psi \sim beta(0.50, 0.15)$	**Stoch. Volatility Coefficients**	$w \sim InvGamma(0.10, 2.00)$
Monetary Policy	$\rho_\sigma^a \sim beta(0.90, 0.20)$	$r \sim InvGamma(0.10, 2.00)$
$\rho \sim beta(0.75, 0.10)$	$\rho_\sigma^b \sim beta(0.90, 0.20)$	
$r_\pi \sim N(1.50, 0.25)$	$\rho_\sigma^g \sim beta(0.90, 0.20)$	
$r_y \sim N(0.12, 0.05)$	$\rho_\sigma^i \sim beta(0.90, 0.20)$	
Prior Distributions of USV ATSM Parameters		
Real World Dynamics	$\mu_{\sigma,2}^Q \sim N(0.10, 0.10)$	$\mu_h \sim N(0.01, 0.05)$
$\mu_{\sigma,1} \sim N(0.10, 0.10)$	$\mu_{\sigma,3}^Q \sim N(0.10, 0.10)$	$\psi_h \sim beta(0.65, 0.10)$
$\mu_{\sigma,2} \sim N(0.10, 0.10)$	$\psi_{1,1}^Q \sim beta(0.90, 0.10)$	$\sigma_h \sim N(0.30, 0.10)$
$\psi_{\sigma,1,1} \sim beta(0.00, 0.10)$	$\psi_{\sigma,1,2}^Q \sim beta(0.92, 0.10)$	**Measurement Errors**
$\psi_{\sigma,1,2} \sim beta(0.00, 0.10)$	$\psi_{\sigma,2,2}^Q \sim beta(0.09, 0.05)$	$\sigma_{6M} \sim InvGamma(0.10, 2.00)$
$\psi_{\sigma,2,1} \sim beta(0.00, 0.10)$	$\psi_{\sigma,2,1}^Q \sim beta(0.09, 0.05)$	$\sigma_{12M} \sim InvGamma(0.10, 2.00)$
$\psi_{\sigma,2,2} \sim beta(0.00, 0.10)$	$\psi_{\sigma,2,2}^Q \sim beta(0.92, 0.10)$	$\sigma_{24M} \sim InvGamma(0.10, 2.00)$
$\sigma_{\sigma,1,1} \sim InvGamma(0.00, 0.10)$	**Short term rate**	$\sigma_{36M} \sim InvGamma(0.10, 2.00)$
$\sigma_{\sigma,2,1} \sim InvGamma(0.10, 2.00)$	$\delta_1 \sim N(0.05, 0.05)$	$\sigma_{48M} \sim InvGamma(0.10, 2.00)$
$\sigma_{\sigma,2,2} \sim InvGamma(0.10, 2.00)$	$\delta_2 \sim N(0.10, 0.10)$	$\sigma_{60M} \sim InvGamma(0.10, 2.00)$
Risk neutral Dynamics	$\delta_3 \sim N(0.10, 0.10)$	
$\mu_{\sigma,1}^Q \sim N(0.10, 0.10)$	**Stoch. Volatility Factor**	

Table C.1: Prior distributions of the DSGE-USV-ATSM (We report the distribution's mean and standard deviation respectively. For the inverse gamma distribution we report the scale and shape parameters).

C.9 DSGE-USV-ATSM and alternative USV-ATSM parameter estimates

C.9.1 DSGE-USV-ATSM parameter estimates Germany

DSGE Structural Parameters						**DSGE Disturbance Parameters**						
$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c	ρ_a	ρ_b	ρ_i	ρ_g	ρ_p	ρ_w	ρ_r
0.389	0.643	3.426	0.271	0.854	1.708	0.992	0.289	0.716	0.987	0.631	0.797	0.186
(0.103)	(0.041)	(0.091)	(0.095)	(0.064)	(0.055)	(0.037)	(0.085)	(0.094)	(0.018)	(0.095)	(0.100)	(0.061)
β	φ	ψ	ι_p	Φ	ξ_p	**Macroeconomic Uncertainty Parameters**						
0.999	6.704	0.628	0.631	1.313	0.326	μ_σ^a	μ_σ^b	μ_σ^g	μ_σ^i	μ_σ^r	μ_σ^p	μ_σ^w
(0.005)	(0.032)	(0.052)	(0.031)	(0.056)	(0.074)	0.063	0.008	-0.012	0.110	0.046	0.049	-0.020
ι_w	ξ_w	σ_l	ρ	r	r	(0.046)	(0.048)	(0.070)	(0.052)	(0.036)	(0.048)	(0.036)
0.175	0.363	2.071	0.763	2.279	0.115	ρ_σ^a	ρ_σ^b	ρ_σ^g	ρ_σ^i	ρ_σ^r	ρ_σ^p	ρ_σ^w
(0.053)	(0.107)	(0.046)	(0.093)	(0.018)	(0.039)	0.787	0.880	0.935	0.949	0.905	0.864	0.872
$r_{\Delta y}$						(0.048)	(0.035)	(0.077)	(0.065)	(0.090)	(0.040)	(0.059)
0.023						σ_σ^a	σ_σ^b	σ_σ^g	σ_σ^i	σ_σ^r	σ_σ^p	σ_σ^w
(0.054)						0.197	0.183	0.133	0.083	0.091	0.077	0.042
						(0.062)	(0.067)	(0.090)	(0.089)	(0.050)	(0.044)	(0.034)
DSGE Measurement Errors												
v	c	i	w	l	π	r						
0.145	0.417	0.172	0.266	0.041	0.029	0.105						
(0.104)	(0.038)	(0.042)	(0.059)	(0.042)	(0.040)	(0.028)						
USV ATSM Structural Parameters												
$\boldsymbol{\mu}_g^T$		Ψ_g		$\boldsymbol{\Sigma}_g$		μ_1^Q		$\boldsymbol{\mu}_g^{QT}$		$\psi_{1,1}^Q$	$\boldsymbol{\Psi}_g^Q$	
0.091	0.086	0.547	0.590	0.178		0.165	0.121	0.095	0.683	0.948	0.095	
(0.094)	(0.025)	(0.042)	(0.111)	(0.043)		(0.049)	(0.029)	(0.177)	(0.110)	(0.059)	(0.048)	
		0.527	0.590	0.066	0.113					0.038	0.937	
		(0.093)	(0.081)	(0.031)	(0.050)					(0.013)	(0.063)	
μ_h	ψ_h	σ_h	$\boldsymbol{\delta}_1^T$			σ_{6M}	σ_{12M}	σ_{24M}	σ_{36M}	σ_{48M}	σ_{60M}	
0.001	0.945	0.414	0.057	0.119	0.126	1.030	0.001	0.001	0.001	0.079	0.377	
(0.004)	(0.030)	(0.028)	(0.016)	(0.060)	(0.057)	(0.103)	(0.005)	(0.057)	(0.002)	(0.053)	(0.072)	

C.9.2 Structural DSGE posterior parameter distributions from MCMC for Germany

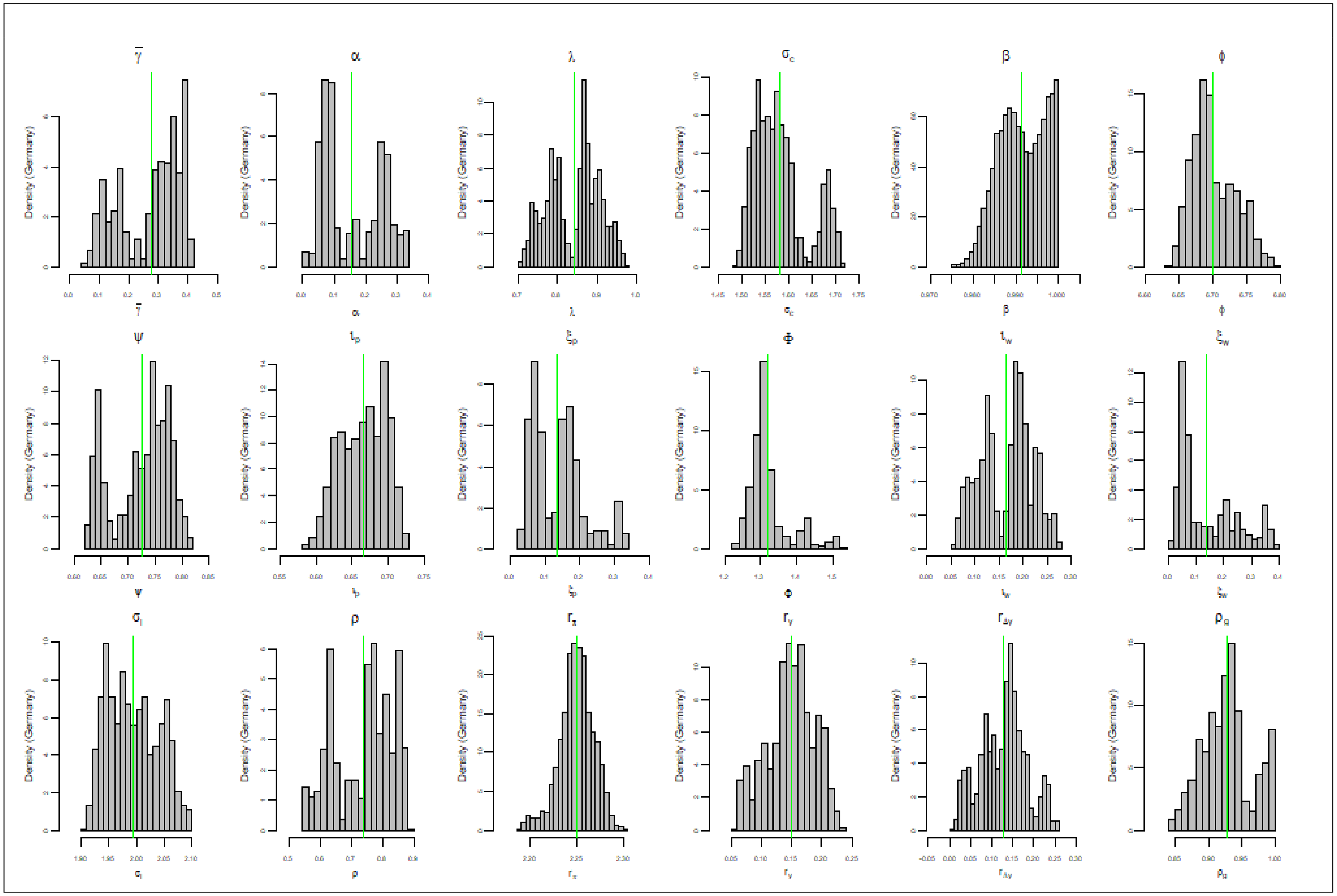


Table C.2: German MCMC diagnostic structural macroeconomic DSGE parameter distributions from MH drawing (1/3).

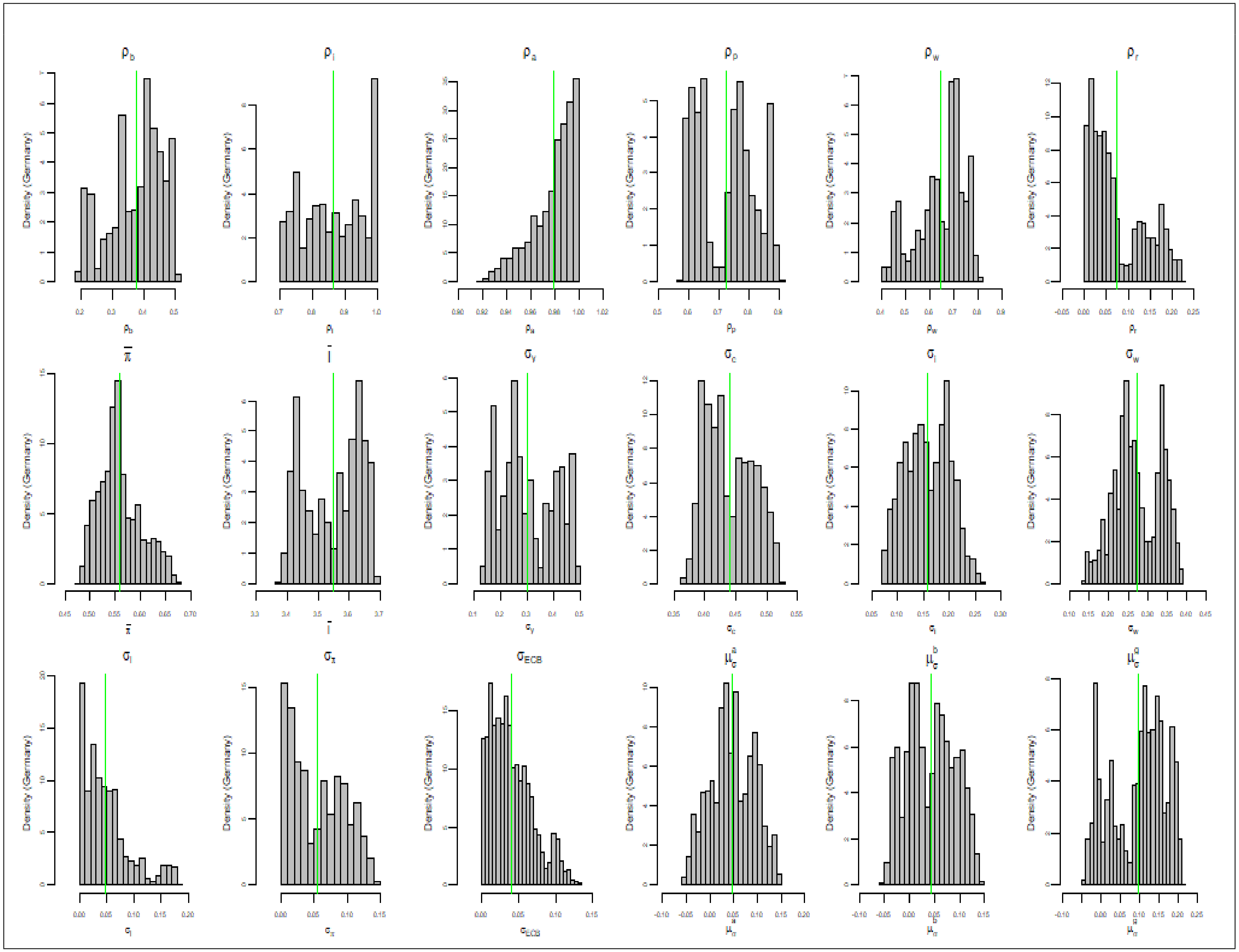


Table C.3: German MCMC diagnostic structural macroeconomic DSGE parameter distributions from MH drawing (2/3).

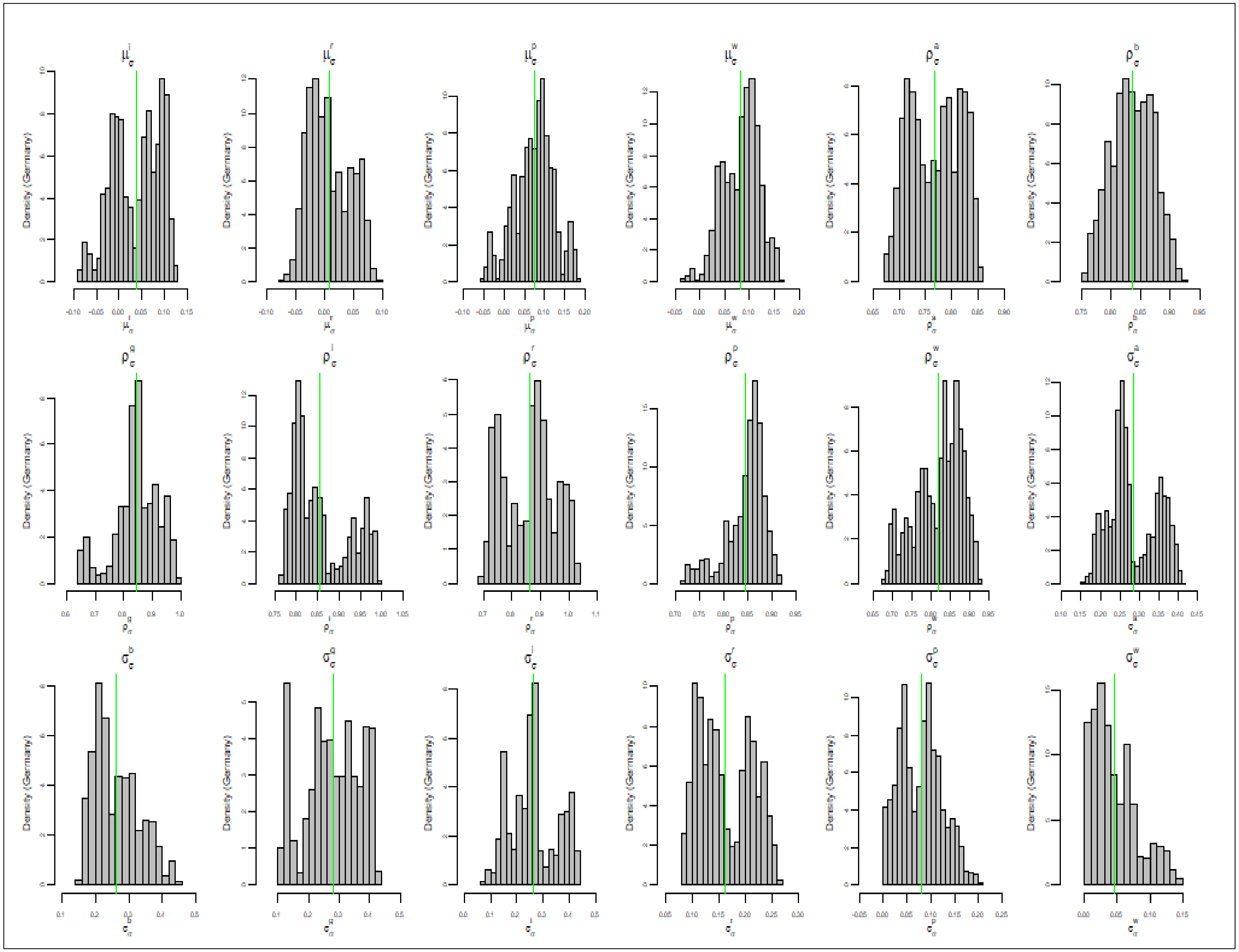


Table C.4: German MCMC diagnostic structural macroeconomic DSGE parameter distributions from MH drawing (3/3).

C.9.3 DSGE-USV-ATSM parameter estimates Italy

DSGE Structural Parameters

$\bar{\gamma}$	$\bar{\pi}$	$\bar{l}$	α	λ	σ_c
0.377	0.623	3.383	0.219	0.769	1.509
(0.028)	(0.063)	(0.023)	(0.048)	(0.031)	(0.049)
β	φ	ψ	ι_p	Φ	ξ_p
0.991	6.634	0.643	0.409	1.643	0.346
(0.005)	(0.029)	(0.026)	(0.086)	(0.056)	(0.111)
ι_w	ξ_w	σ_l	ρ	r	r
0.173	0.401	1.972	0.735	2.113	0.078
(0.082)	(0.139)	(0.049)	(0.029)	(0.045)	(0.030)
$r_{\Delta y}$					
0.229					
(0.018)					

DSGE Disturbance Parameters

ρ_a	ρ_b	ρ_i	ρ_g	ρ_p	ρ_w	ρ_r
0.848	0.382	0.798	0.997	0.757	0.764	0.068
(0.026)	(0.054)	(0.063)	(0.007)	(0.023)	(0.024)	(0.032)

Macroeconomic Uncertainty Parameters

μ_σ^a	μ_σ^b	μ_σ^g	μ_σ^i	μ_σ^r	μ_σ^p	μ_σ^w
0.030	0.037	0.082	0.011	-0.132	-0.059	-0.009
(0.021)	(0.051)	(0.030)	(0.109)	(0.066)	(0.115)	(0.053)
ρ_σ^a	ρ_σ^b	ρ_σ^g	ρ_σ^i	ρ_σ^r	ρ_σ^p	ρ_σ^w
0.833	0.964	0.928	0.662	0.675	0.793	0.857
(0.036)	(0.088)	(0.057)	(0.045)	(0.066)	(0.064)	(0.072)
σ_σ^a	σ_σ^b	σ_σ^g	σ_σ^i	σ_σ^r	σ_σ^p	σ_σ^w
0.672	0.668	0.598	0.563	0.569	0.270	0.028
(0.129)	(0.101)	(0.085)	(0.064)	(0.053)	(0.073)	(0.041)

DSGE Measurement Errors

v	c	i	w	l	π	r
0.074	0.331	0.041	0.057	0.002	0.041	0.070
(0.051)	(0.045)	(0.034)	(0.027)	(0.019)	(0.057)	(0.025)

USV ATSM Structural Parameters

$\boldsymbol{\mu}_g^T$		Ψ_g		$\boldsymbol{\Sigma}_g$		μ_1^Q		$\boldsymbol{\mu}_g^{QT}$		$\psi_{1,1}^Q$	$\boldsymbol{\Psi}_g^Q$
0.115	0.071	0.656	0.652	0.082		0.025	0.346	0.169	0.907	0.973	0.030
(0.039)	(0.285)	(0.181)	(0.170)	(0.065)		(0.010)	(0.133)	(0.034)	(0.226)	(0.162)	(0.078)
		0.525	0.433	0.259	0.232					0.076	0.962
		(0.179)	(0.211)	(0.079)	(0.057)					(0.063)	(0.145)
μ_h	ψ_h	σ_h	$\boldsymbol{\delta}_1^T$			σ_{6M}	σ_{12M}	σ_{24M}	σ_{36M}	σ_{48M}	σ_{60M}
0.000	0.977	0.277	0.124	0.064	0.152	0.000	0.001	0.008	0.005	0.002	0.003
(0.005)	(0.318)	(0.111)	(0.047)	(0.064)	(0.075)	(0.000)	(0.000)	(0.003)	(0.002)	(0.001)	(0.001)

C.9.4 Structural DSGE posterior parameter distributions from MCMC for Italy

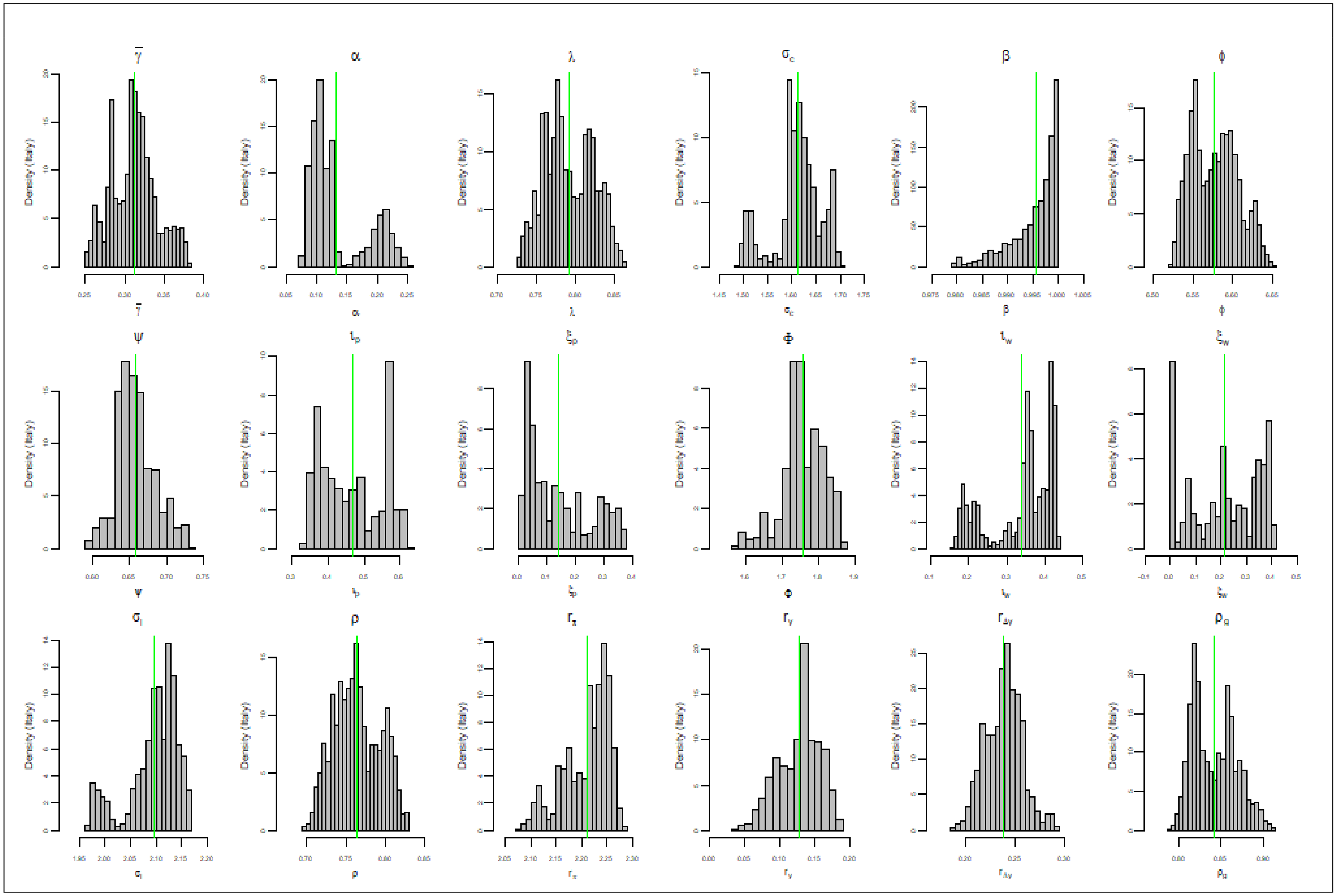


Table C.5: Italian MCMC diagnostic structural macroeconomic DSGE parameter distributions from MH drawing (1/3).

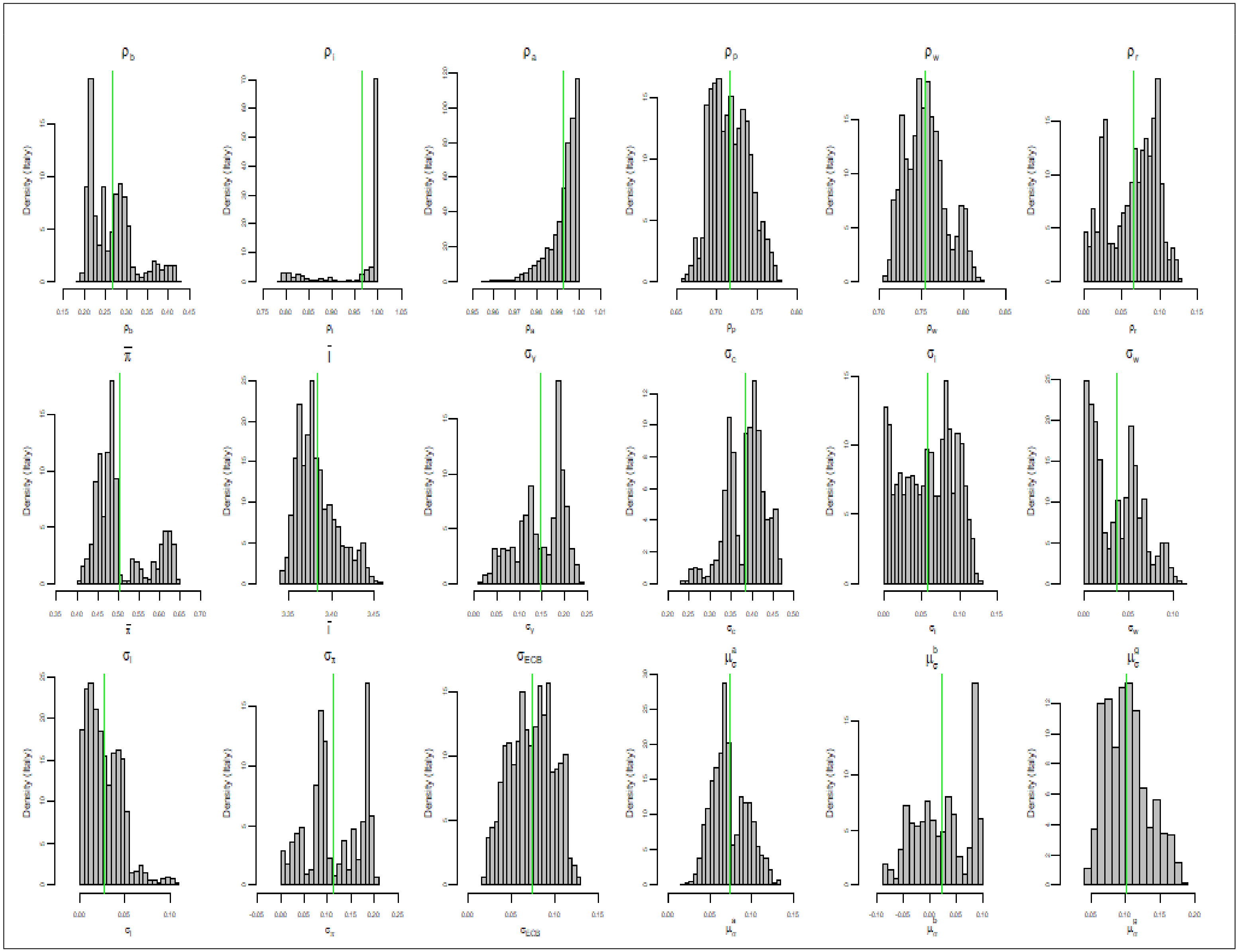


Table C.6: Italian MCMC diagnostic structural macroeconomic DSGE parameter distributions from MH drawing (2/3).

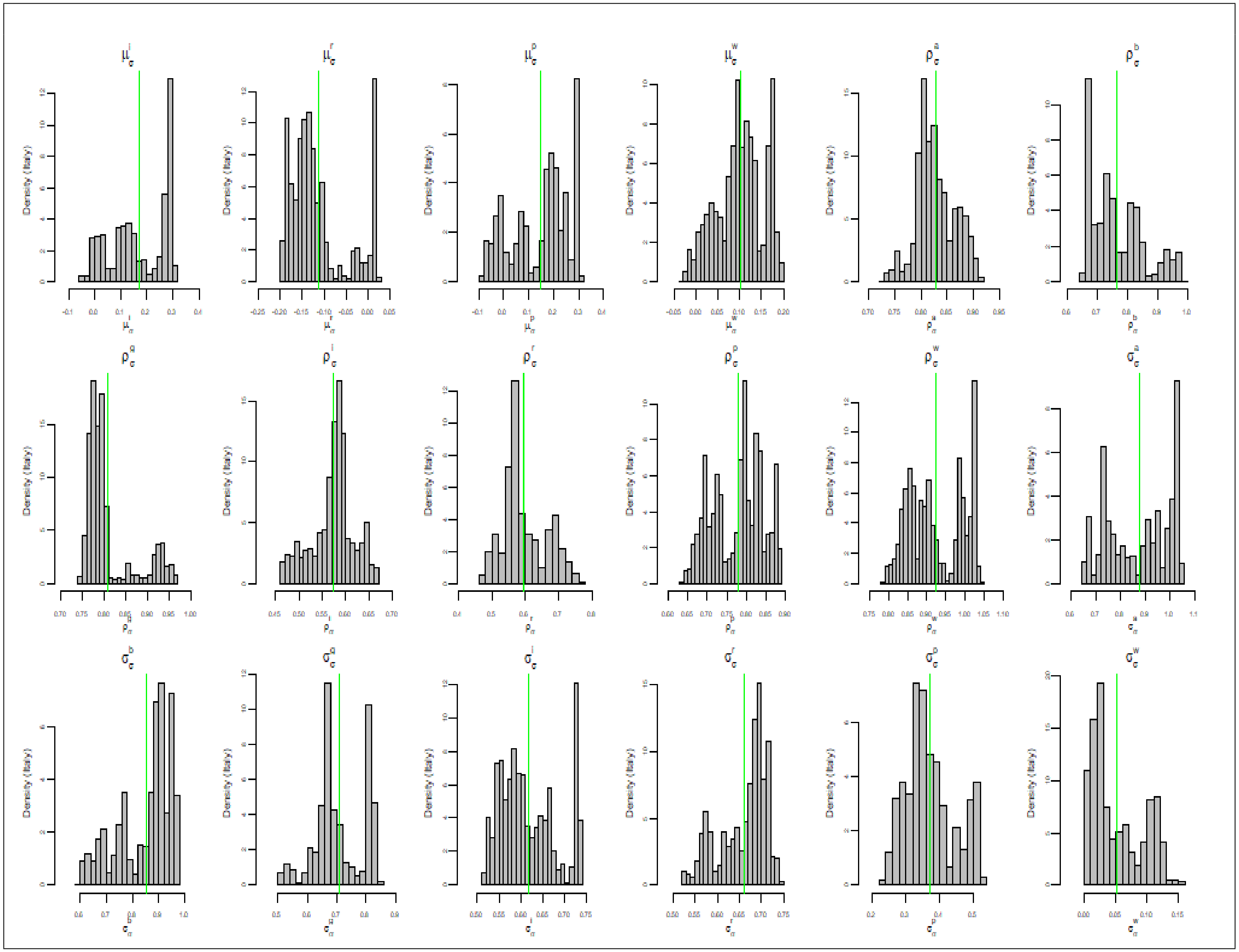


Table C.7: Italian MCMC diagnostic structural macroeconomic DSGE parameter distributions from MH drawing (3/3).

C.9.5 Alternative USV macro-finance ATSM parameter estimates Germany

		$\bar{\boldsymbol{\mu}}^{Qu}$							$\boldsymbol{\Gamma}_0$			$\boldsymbol{\Gamma}_1$					
		0.568	0.655						0			1	0	0	0		
		(0.350)	(0.331)						–			–	–	–	–		
		$\boldsymbol{\Psi}^{Q}$							0			0	1	0	0		
		-0.304	1.876	1.872					–			–	–	–	–		
		(0.012)	(0.091)	(0.090)					0			0	0	1	0		
		-53.615	78.918	77.754					–			–	–	–	–		
		(0.410)	(3.756)	(3.748)					1.808			0	0	0.000	0.997		
		53.599	-77.909	-76.745					(0.256)			–	–	(0.081)	(0.122)		
		(0.410)	(3.757)	(3.749)					0			0	0	0	1		
									–			–	–	–	–		
		$\boldsymbol{\lambda}_g^{Q}$							$\boldsymbol{\lambda}_f$								
		0.987	0.977	0.094					0.997	0.992	0.991	0.989	0.988	0.940	0.896		
		(0.002)	(0.002)	(0.000)					(0.002)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)		
$\bar{\boldsymbol{\mu}}_m$		$\bar{\boldsymbol{\mu}}_g$			$\bar{\boldsymbol{\mu}}_h$				diag($\boldsymbol{\Omega}$)	σ_{6M}	σ_{12M}	σ_{24M}	σ_{36M}	σ_{48M}	σ_{60M}		
0.375	-0.049	0.568	0.568	0.655	2.837	-0.579	-1.617	-4.705		0.025	0.006	0.000	0.000	0.000	0.001		
(0.631)	(0.624)	(0.350)	(0.350)	(0.331)	(2.105)	(0.661)	(0.903)	(2.095)		(0.025)	(0.005)	(0.004)	(0.001)	(0.001)	(0.003)		
$\bar{\boldsymbol{\Psi}}_m$		$\bar{\boldsymbol{\Psi}}_{mg}$			$\bar{\boldsymbol{\Psi}}_{mh}$				$\boldsymbol{\Sigma}_m$								
0.993	-0.001	-0.005	0.012	-0.005	0.135	0.043	-0.031	0.116	1.000								
(0.001)	(0.000)	(0.003)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)								
0.001	0.991	0.087	-0.095	-0.005	-0.021	-0.035	0.017	0.011	0.000	1.000							
(0.001)	(0.001)	(0.016)	(0.019)	(0.002)	(0.003)	(0.003)	(0.001)	(0.003)	(0.557)	(0.000)							
$\bar{\boldsymbol{\Psi}}_{gm}$		$\bar{\boldsymbol{\Psi}}_g$			$\bar{\boldsymbol{\Psi}}_{gh}$				$\boldsymbol{\Sigma}_{gm}$		$\boldsymbol{\Sigma}_g$						
0.000	0.000	0.962	0.038	0.000	-0.023	-0.017	0.009	-0.015	0.000	-0.591	1.000						
(0.000)	(0.000)	(0.001)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.278)	(1.846)	(0.000)						
0.000	0.000	-0.032	1.032	0.000	0.004	0.003	-0.002	0.002	0.000	-2.594	-3.697	1.000					
(0.000)	(0.000)	(0.001)	(0.020)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.308)	(8.977)	(0.348)	(0.000)					
0.000	0.002	-0.019	0.018	0.998	-0.1000	-0.031	0.023	-0.094	0.000	2.357	4.171	0.000	1.000				
(0.000)	(0.001)	(0.007)	(0.008)	(0.001)	(0.003)	(0.003)	(0.002)	(0.004)	(1.062)	(7.676)	(0.250)	(0.005)	(0.000)				
					$\bar{\boldsymbol{\Psi}}_{hh}$				$\boldsymbol{\Sigma}_{hm}$		$\boldsymbol{\Sigma}_{hg}$			$\boldsymbol{\Sigma}_h$			
					0.849	-0.051	0.033	-0.107	0.006	-0.022	0.022	-0.014	-0.012	0.112			
					(0.001)	(0.001)	(0.001)	(0.002)	(0.026)	(0.025)	(0.025)	(0.023)	(0.023)	(0.018)			
					0.115	0.989	-0.009	0.098	0.003	0.038	0.027	0.028	0.003	0.034	0.140		
					(0.002)	(0.001)	(0.001)	(0.001)	(0.026)	(0.026)	(0.025)	(0.024)	(0.024)	(0.024)	(0.017)		
					0.090	0.036	0.967	0.069	-0.012	0.045	0.004	-0.020	0.008	-0.031	-0.022	0.156	
					(0.001)	(0.001)	(0.001)	(0.001)	(0.026)	(0.024)	(0.024)	(0.024)	(0.023)	(0.023)	(0.022)	(0.017)	
					0.031	0.030	-0.015	1.008	-0.025	-0.021	-0.027	-0.030	-0.014	-0.033	-0.023	-0.013	0.132
					(0.001)	(0.001)	(0.001)	(0.002)	(0.028)	(0.026)	(0.026)	(0.024)	(0.025)	(0.023)	(0.023)	(0.022)	(0.017)

C.9.6 Alternative USV macro-finance ATSM parameter estimates Italy

		$\bar{\boldsymbol{\mu}}^{Qu}$							$\boldsymbol{\Gamma}_0$			$\boldsymbol{\Gamma}_1$					
		0.292	0.599						0			1	0	0	0		
		(0.194)	(0.058)						–			–	–	–	–		
		$\boldsymbol{\Psi}^Q$							0			0	1	0	0		
		-0.316	2.128	2.123					–			–	–	–	–		
		(0.457)	(0.767)	(0.757)					0			0	0	1	0		
		-52.125	86.394	85.218					–			–	–	–	–		
		(14.340)	(24.097)	(23.717)					1.114			0	0	0.817	0.852		
		52.11	-85.386	-84.211					(0.471)			–	–	(0.263)	(0.085)		
		(14.343)	(24.102)	(23.722)					0			0	0	0	1		
									–			–	–	–	–		
		$\boldsymbol{\lambda}_g^Q$							$\boldsymbol{\lambda}_f$								
		0.982	0.970	0.085					0.989	0.989	0.954	0.927	0.918	0.913	0.865		
		(0.005)	(0.017)	(0.204)					(0.002)	(0.002)	(0.022)	(0.028)	(0.012)	(0.022)	(0.012)		
$\bar{\boldsymbol{\mu}}_m$		$\bar{\boldsymbol{\mu}}_g$			$\bar{\boldsymbol{\mu}}_h$				diag($\boldsymbol{\Omega}$)	σ_{6M}	σ_{12M}	σ_{24M}	σ_{36M}	σ_{48M}	σ_{60M}		
-98.123	145.732	0.292	0.292	0.599	-0.051	-0.784	-2.883	-4.650		0.140	0.012	0.001	0.002	0.001	0.001		
(34.394)	(71.140)	(0.194)	(0.194)	(0.058)	(11.100)	(11.076)	(18.426)	(16.435)		(0.020)	(0.003)	(0.001)	(0.001)	(0.001)	(0.000)		
$\bar{\boldsymbol{\Psi}}_m$		$\bar{\boldsymbol{\Psi}}_{mg}$			$\bar{\boldsymbol{\Psi}}_{mh}$				$\boldsymbol{\Sigma}_m$								
0.925	-0.008	-0.204	0.213	-0.005	0.155	0.072	-0.026	0.035	1.000								
(0.024)	(0.020)	(0.078)	(0.340)	(0.047)	(0.133)	(0.104)	(0.014)	(0.207)	(0.000)								
-0.011	0.924	0.016	-0.022	-0.004	-0.012	-0.030	0.043	-0.056	-0.211	1.000							
(0.016)	(0.048)	(0.110)	(0.519)	(0.063)	(0.203)	(0.159)	(0.022)	(0.327)	(0.895)	(0.000)							
$\bar{\boldsymbol{\Psi}}_{gm}$		$\bar{\boldsymbol{\Psi}}_g$			$\bar{\boldsymbol{\Psi}}_{gh}$				$\boldsymbol{\Sigma}_{gm}$		$\boldsymbol{\Sigma}_g$						
-0.022	-0.019	0.794	0.252	-0.040	0.057	0.051	-0.028	0.050	2.616	0.213	1.000						
(0.009)	(0.008)	(0.096)	(0.477)	(0.016)	(0.268)	(0.204)	(0.022)	(0.402)	(0.549)	(0.156)	(0.000)						
0.001	0.001	-0.018	1.016	0.002	-0.001	-0.001	0.001	0.000	1.678	0.243	-3.157	1.000					
(0.000)	(0.000)	(0.001)	(0.007)	(0.001)	(0.004)	(0.003)	(0.000)	(0.006)	(0.401)	(0.917)	(0.978)	(0.000)					
0.029	0.022	0.131	-0.094	0.967	-0.088	-0.027	0.003	0.040	-1.765	1.331	4.251	0.001	1.000				
(0.011)	(0.016)	(0.136)	(0.686)	(0.028)	(0.355)	(0.274)	(0.026)	(0.537)	(0.651)	(0.275)	(1.517)	(0.080)	(0.000)				
					$\bar{\boldsymbol{\Psi}}_{hh}$				$\boldsymbol{\Sigma}_{hm}$		$\boldsymbol{\Sigma}_{hg}$			$\boldsymbol{\Sigma}_h$			
					0.803	-0.068	0.044	-0.14	-0.012	-0.039	-0.007	0.001	0.010	0.126			
					(0.020)	(0.008)	(0.005)	(0.017)	(0.025)	(0.025)	(0.024)	(0.024)	(0.023)	(0.051)			
					0.138	0.974	-0.004	0.118	-0.006	-0.013	-0.001	0.045	-0.003	-0.008	0.164		
					(0.033)	(0.031)	(0.016)	(0.022)	(0.025)	(0.027)	(0.024)	(0.024)	(0.026)	(0.022)	(0.025)		
					0.121	0.048	0.959	0.091	-0.003	-0.061	0.010	0.032	-0.005	0.015	0.001	0.145	
					(0.014)	(0.006)	(0.003)	(0.012)	(0.026)	(0.025)	(0.025)	(0.024)	(0.023)	(0.023)	(0.022)	(0.024)	
					0.049	0.049	-0.025	1.019	0.030	0.032	0.017	-0.002	-0.012	-0.022	-0.011	-0.015	0.148
					(0.015)	(0.015)	(0.007)	(0.011)	(0.025)	(0.026)	(0.024)	(0.024)	(0.023)	(0.023)	(0.024)	(0.022)	(0.024)

C.9.7 Italian uncertainty responses of GDP, investment and ECB's short rate

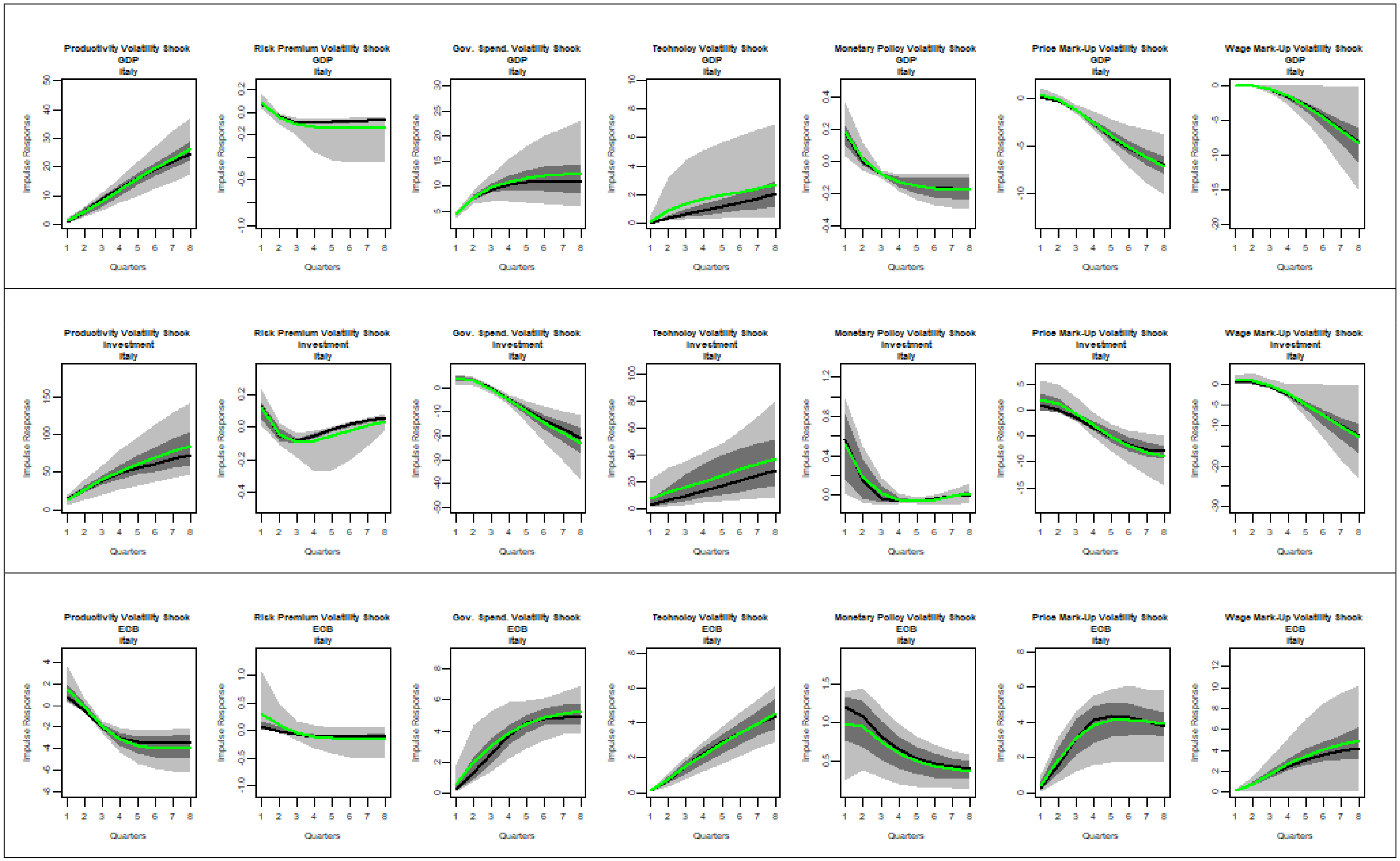


Table C.8: Macroeconomic responses to one standard deviation uncertainty shocks to $\boldsymbol{\sigma}_t^T = [\sigma_{a,t}, \sigma_{b,t}, \sigma_{g,t}, \sigma_{i,t}, \sigma_{r,t}, \sigma_{p,t}, \sigma_{w,t}]$ of y_t, i_t and r_t for Italy. Based on the models posterior distribution we calculate 1000 responses and show the median (black), the mean (green) and the 40% and 80% confidence intervals.

C.9.8 Uncertainty responses of current and expected inflation for Germany and Italy

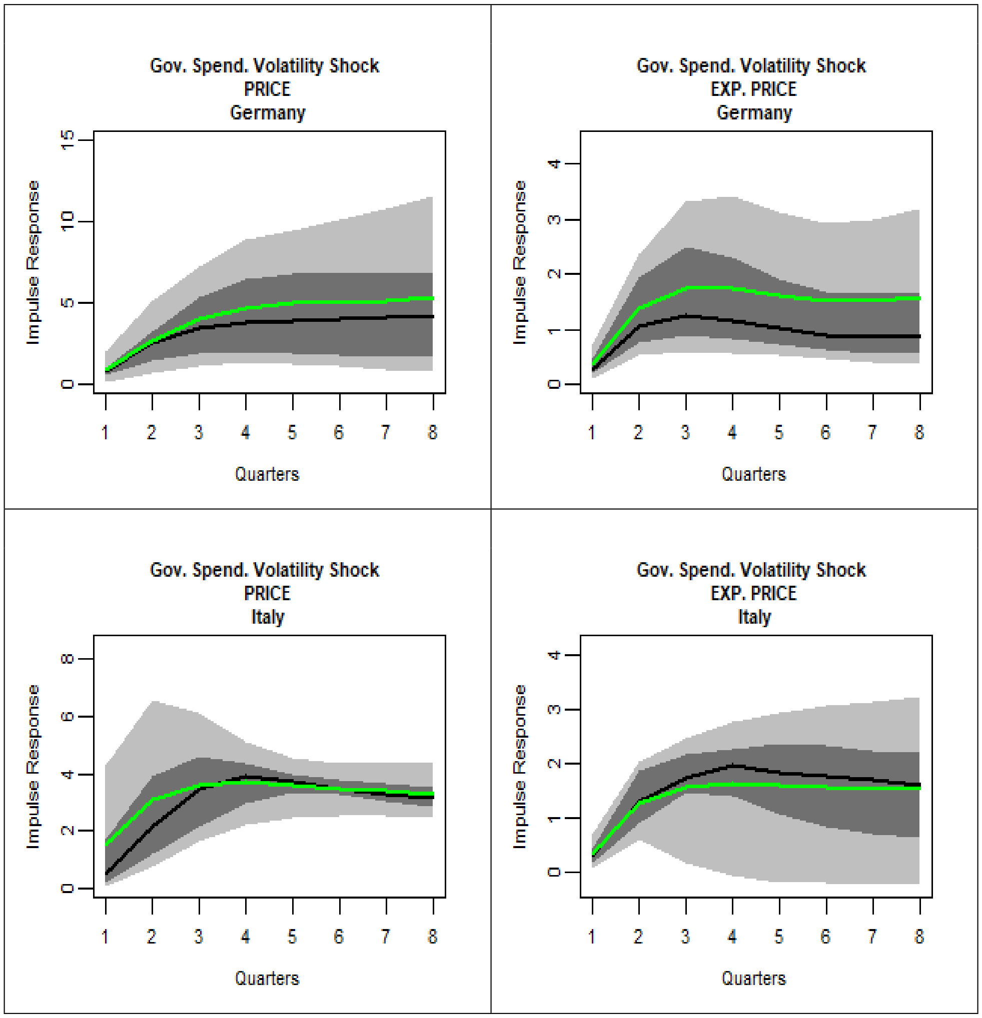


Table C.9: Responses of current and expected future inflation π_t and $\mathbb{E}_t[\pi_{t+1}]$ to a one standard deviation uncertainty shock in government spending activities. We show the median (black), the mean (green) and the 40% and 80% confidence intervals for Germany and Italy respectively.

D. Appendix Chapter 5

D.1 Financial market stock data

Descriptive statistics of company stock's excess returns for companies listed in the DAX 30, CAC 40, AEX 25, FTSE MIB and IBEX 35. Excess returns are calculated from the companies stocks opening prices between 03/2005 and 02/2014.

Germany - DAX 30 -								
Company	*Mean*	*Std.*	$Q(5\%)$	$Q(95\%)$	*Skewness*	*Kurtosis*	$ACF[1]$	$ACF[12]$
Adidas	-1.198	10.971	-14.538	11.163	-3.330	21.136	0.114	0.061
Allianz	-1.121	9.274	-15.310	12.091	-1.086	3.478	0.101	0.166
BASF	-0.937	9.971	-15.732	11.085	-1.898	10.003	0.194	0.095
Bayer	-0.327	7.292	-12.332	9.444	0.092	1.779	0.066	-0.016
Beiersdorf	-0.777	5.922	-11.166	8.063	-0.224	-0.173	0.114	0.090
BMW	-0.569	9.109	-16.067	14.447	-0.115	1.098	0.107	0.086
Commerzbank	4.605	77.994	-23.554	25.379	9.520	93.203	-0.042	-0.008
Continental	0.261	13.983	-24.912	18.687	-0.404	4.046	0.249	0.154
Daimler	-0.705	10.673	-18.877	16.209	0.345	2.041	0.139	0.114
Deutsche Bank	-1.655	12.044	-16.719	15.432	0.556	4.466	0.252	0.120
Deutsche Boerse	-1.148	10.630	-18.606	12.306	-1.452	6.093	0.229	0.201
Deutsche Lufthansa	-0.964	8.669	-14.841	11.795	-0.247	-0.055	0.211	0.051
Deutsche Post	-1.058	9.050	-15.789	11.225	-1.196	4.531	0.056	0.214
Deutsche Telekom	-1.866	6.422	-12.735	7.501	-0.443	-0.188	0.096	0.136
Eon	-2.583	10.220	-17.445	7.151	-2.981	17.796	0.176	-0.080
Fresenius	-0.377	6.599	-11.674	8.903	-0.426	0.691	0.272	-0.092
Fresenius Medical Care	-0.956	5.144	-10.047	7.843	-0.176	-0.242	0.001	-0.072
Heidelberg Cement	-0.562	12.697	-20.513	15.523	0.546	4.134	0.256	0.040
Henkel	-0.425	6.953	-12.590	10.511	-0.589	1.001	0.182	0.111
Infineon	-0.152	20.473	-29.591	20.783	2.748	16.292	0.423	0.153
Linde	-0.653	6.523	-12.438	8.141	-0.312	0.748	0.147	0.059
Merck	-0.852	7.374	-12.211	10.282	-0.162	0.309	0.010	0.095
MunichRE	-1.178	5.834	-10.292	7.862	-0.092	-0.569	0.097	0.191
ProSiebenSat1	0.488	19.519	-28.487	24.792	1.890	9.260	0.290	-0.104
RWE	-1.969	7.451	-11.824	8.897	-0.268	1.241	0.151	0.063
SAP	-1.016	6.861	-12.237	8.950	-0.641	1.973	0.107	0.236
Siemens	-1.019	8.225	-17.206	9.900	-0.845	1.717	0.144	0.079
Thyssen-Krupp	-1.069	11.300	-20.078	16.017	-0.563	0.918	0.175	-0.035
VW	0.662	13.779	-19.849	17.881	0.385	6.175	-0.006	0.257

France - CAC 40 -								
Company	*Mean*	*Std.*	*Q*(5%)	*Q*(95%)	*Skewness*	*Kurtosis*	*ACF*[1]	*ACF*[12]
Total	-2.266	9.392	-9.925	7.044	-5.145	40.183	0.043	-0.010
BNP Paribas	-1.216	10.413	-17.984	13.890	-0.029	0.898	0.305	-0.048
LVMH	-0.758	7.482	-15.569	10.920	-0.169	1.141	0.097	0.104
Axa	-1.236	11.340	-18.584	14.316	-0.310	2.912	0.130	0.126
L'Oreal	-0.978	5.956	-11.944	8.590	-0.747	1.549	0.216	0.133
Danone	-1.788	7.560	-9.678	6.016	-3.705	22.332	0.040	0.109
Airbus	-0.574	10.307	-19.875	15.544	-0.347	0.126	-0.036	0.094
Air Liquide	-0.970	16.256	-14.435	6.404	2.713	20.322	-0.504	-0.015
Vinci	-1.344	16.119	-17.310	10.553	2.713	25.853	-0.253	0.045
Schneider Electric	-1.210	9.533	-14.341	10.340	-2.232	11.408	0.100	0.019
Orange	-2.468	6.155	-11.587	8.145	0.490	0.644	0.134	-0.048
Societe Generale	-1.439	13.097	-22.726	18.503	0.047	0.131	0.220	0.138
Engie	-1.657	7.162	-12.137	9.807	0.222	-0.192	-0.045	0.008
Essilor	1.196	32.867	-14.636	10.095	7.018	60.579	-0.297	-0.009
Vivendi	-1.749	6.898	-12.873	8.559	-0.277	-0.392	0.057	0.197
Pernod Ricard	-0.966	6.715	-12.110	9.791	-0.077	0.749	0.069	0.064
Safran	-0.402	9.672	-18.703	12.546	-0.271	0.884	0.072	-0.068
Saint Goban	-1.418	9.920	-20.010	13.298	-0.257	1.401	0.127	0.090
Michelin	-0.921	9.527	-16.856	12.033	0.374	2.051	0.206	-0.020
Renault	-0.905	13.594	-24.711	17.094	0.113	3.272	0.292	0.059
Capgemini	-0.777	9.136	-17.465	14.090	-0.347	0.254	0.125	0.203
Carrefour	-1.876	7.919	-14.207	9.940	-0.335	0.361	0.209	-0.043
Kering	-0.884	9.459	-19.056	14.375	-0.053	1.740	0.045	0.078
Publicis Groupe	-0.590	7.035	-9.669	10.011	-0.401	1.739	0.132	0.238
Veolia Environment	-1.852	10.724	-22.099	13.685	-0.199	1.912	0.169	0.043
Credit Agricole	-1.651	12.458	-22.757	17.289	0.266	0.263	0.209	0.033
Klepierre	0.577	29.998	-20.085	13.743	4.415	28.414	-0.262	-0.022
Sodexo	-0.580	5.892	-11.072	7.827	-0.453	0.235	0.031	0.153
Peugeot	-2.184	12.770	-21.013	17.740	0.571	2.366	0.174	-0.123
Bouygues	-1.605	8.236	-14.028	12.663	0.041	0.658	0.172	0.131
Accor	-1.093	8.855	-15.927	11.768	-0.026	0.276	0.124	0.070
Solvay	-1.226	8.853	-15.521	14.182	0.386	1.803	0.185	0.045
Technip	-1.387	12.173	-14.496	14.644	-2.402	11.750	0.085	0.082

Netherlands - AEX 25 -								
Company	*Mean*	*Std.*	*Q*(5%)	*Q*(95%)	*Skewness*	*Kurtosis*	*ACF*[1]	*ACF*[12]
Unilever	-1.112	5.453	-9.600	7.290	-0.307	1.244	0.088	-0.050
ING Group	-1.313	14.236	-22.298	16.707	0.242	6.068	0.229	0.151
ASML	-0.017	9.297	-15.966	15.450	-0.262	0.041	0.023	0.060
Unibail Rodamco	-0.883	7.664	-14.084	10.062	-0.266	0.292	0.092	0.079
Heineken	-0.944	7.014	-14.607	9.015	-0.356	0.763	0.062	0.041
Ahold	-1.010	6.263	-11.269	8.994	-0.082	-0.455	0.041	0.088
Relx	-1.342	5.784	-11.180	7.727	-0.138	0.347	0.072	0.090
Akzo Nobel	-0.933	8.749	-14.141	12.175	-0.222	0.302	0.007	0.085
KPN	-1.980	7.819	-14.993	7.681	-0.596	3.908	-0.008	0.031
Aegon	3.544	40.288	-19.172	17.094	6.495	46.405	-0.001	-0.019
DSM	-1.434	9.711	-16.495	11.583	-1.722	6.399	0.289	0.020

Company	*Mean*	*Std.*	*Q(5%)*	*Q(95%)*	*Skewness*	*Kurtosis*	*ACF[1]*	*ACF[12]*
Randstad	-0.943	11.445	-21.315	16.767	0.058	0.432	0.217	0.183
Gemalto	-0.407	8.932	-17.663	13.209	-0.469	0.495	0.022	0.019
Vopak	-0.535	9.145	-14.148	11.442	-1.784	7.773	0.131	0.027
Aalberts Industries	-1.158	13.174	-19.794	15.705	-1.500	9.247	0.215	0.113
Boskalis Westminster	-0.734	12.085	-18.010	15.777	-1.659	9.007	0.111	0.159
SBM Offshore	-1.341	10.742	-18.193	15.873	-0.278	1.726	0.047	0.039

Italy
- FTSE MIB -

Company	*Mean*	*Std.*	*Q(5%)*	*Q(95%)*	*Skewness*	*Kurtosis*	*ACF[1]*	*ACF[12]*
A2A	-1.936	9.719	-18.542	12.970	0.118	0.251	0.039	-0.160
Atlantia	-1.731	8.102	-14.918	11.526	-0.086	0.280	0.159	0.136
Banca Monte Dei Paschi	-3.456	11.286	-21.870	13.863	0.225	0.778	0.127	-0.004
Banca Pop. E. Romagna	-1.649	10.729	-18.937	19.043	0.212	0.987	-0.057	-0.003
Banca Pop. Di Milano	-2.235	12.866	-21.532	21.264	0.154	0.012	0.111	0.140
Banco Popolare	-1.958	15.688	-22.739	21.170	1.163	4.282	0.019	-0.002
Buzzi Unicem	-1.123	10.845	-19.098	16.570	0.034	-0.104	0.114	0.202
Campari	-2.084	12.003	-13.406	8.247	-4.442	28.360	-0.021	0.046
Enel	-2.073	7.128	-13.253	9.977	0.102	-0.025	0.242	0.006
Eni	-1.763	6.026	-10.252	7.134	-0.075	0.968	0.044	0.112
Exor	1.414	20.001	-18.603	14.312	5.968	49.598	0.283	-0.035
Fiat Chrysler	0.128	13.774	-20.040	24.264	0.515	1.942	0.248	0.122
Finmeccanica	-0.492	13.321	-16.292	19.559	2.712	13.369	-0.127	-0.058
Intesa Sanpoalo	-1.682	10.583	-18.638	15.957	-0.183	0.141	0.037	0.063
Italcementi	-1.869	12.752	-20.130	18.823	1.180	3.168	-0.123	-0.057
Luxottica Group	-0.606	8.989	-14.144	10.706	0.534	4.625	-0.016	0.027
Mediaset	-2.157	10.796	-17.282	18.390	0.409	0.699	0.160	0.060
Mediobanca	-1.811	10.926	-16.767	15.700	0.768	0.973	0.022	-0.017
Saipem	-0.725	10.954	-19.669	15.150	-0.676	1.682	0.101	-0.055
Snam	-1.538	5.085	-9.075	4.987	-1.161	3.214	0.112	0.031
ST Microelectronics	-1.904	11.113	-19.504	13.727	0.001	-0.005	0.189	0.082
Telecom Italia	-2.656	8.418	-16.921	10.980	-0.055	0.360	0.020	0.037
Tenaris	-0.026	12.485	-22.170	19.794	-0.351	1.287	0.160	-0.087
UniCredit	4.591	71.455	-25.417	23.534	9.485	92.733	-0.026	0.012
Unipolsai	82.207	883.209	-24.735	19.903	10.100	100.981	-0.015	-0.012

Spain
- IBEX 35 -

Company	*Mean*	*Std.*	*Q(5%)*	*Q(95%)*	*Skewness*	*Kurtosis*	*ACF[1]*	*ACF[12]*
Banco Santander	-1.771	9.359	-18.558	11.852	0.456	3.912	0.249	-0.033
Telefonica	-1.800	6.709	-12.593	8.427	0.021	0.572	-0.029	0.016
Inditex	-0.236	6.748	-11.208	9.574	-0.133	1.353	0.229	-0.031
Iberdrola	-3.196	12.759	-18.524	11.529	-2.553	11.498	0.140	0.098
Repsol	-1.644	8.063	-13.388	10.480	-0.618	1.429	0.176	-0.046
Gas Natural	-2.180	10.962	-16.792	12.904	-2.832	16.578	0.135	0.047
International Airlines	-0.673	11.854	-22.235	14.757	-0.625	0.670	0.043	0.006
Ferrovial	-0.797	10.367	-15.488	16.644	0.300	2.091	0.096	0.156
Banco Pop. Espanol	0.007	28.769	-20.564	21.222	7.802	70.834	0.171	0.038
Grupo ACS	-1.179	8.339	-13.523	11.289	-0.109	1.592	-0.018	-0.017
Banco Sabadell	-1.982	10.381	-17.183	16.049	0.709	2.09	0.080	0.140
Endesa	-1.119	8.953	-14.211	13.236	-0.158	1.154	-0.019	0.082
Enagas	-1.082	7.125	-11.022	8.204	-0.415	1.761	0.016	0.035
Bankinter	-2.338	14.870	-20.576	17.131	-1.661	8.618	0.155	0.042

Company	*Mean*	*Std.*	*Q(5%)*	*Q(95%)*	*Skewness*	*Kurtosis*	*ACF[1]*	*ACF[12]*
Mediaset Espana	-1.736	11.212	-21.010	17.587	0.050	0.378	0.107	0.345
Acciona	-1.456	10.625	-20.401	16.865	-0.198	0.206	0.209	-0.100
Fomento de Const.	-2.066	11.105	-19.485	15.651	0.763	1.885	0.230	0.066
Gamesa	-1.121	13.591	-21.468	21.011	0.147	2.313	0.223	-0.083
Sacyr	-1.541	16.876	-22.379	26.565	0.438	0.448	0.235	-0.101
Abengoa	-0.907	15.430	-21.626	23.926	-0.915	5.513	0.132	-0.105
Indra Sistemas	-1.629	6.163	-11.404	8.542	0.065	-0.119	0.086	0.017

D.2 Market clearing and aggregate constraint

In this Appendix we list the market clearing conditions in the markets for labor, capital, domestic and imported intermediate goods, final goods as well as in the domestic market for government bonds and the market for foreign bonds. The market clearing conditions are analogue to Christoffel, Coenen and Warne [2008].

Market clearing in the labor market

In equilibrium aggregating over the differentiated labor-services leads to:

$$N_{h,t} = \int_0^1 N_{f,t}^h df = N_t^h \tag{D.1}$$

Aggregating over the households h leads to:

$$\begin{aligned} \int_0^1 N_{h,t} dh &= \int_0^1 N_t^h dh \\ &= \int_0^1 \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} dh \end{aligned} \tag{D.2}$$

where for the LHS the following equation holds true:

$$\begin{aligned} &\int_0^1 \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} dh = \\ &(1 - \xi_w) \left(\frac{\tilde{W}_t}{W_t} \right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} + \xi_w \left(\frac{\tilde{W}_t}{W_t} \frac{\Pi_{C,t}}{\Pi_{C,t-1}^{\chi_w} \Pi_t^{1-\chi_w}} \right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} \int_0^1 \left(\frac{W_{h,t-1}}{W_{t-1}} \right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} dh \end{aligned} \tag{D.3}$$

$\tilde{W}_t$ denotes the optimal wage chosen by the wage setting households in t. Total wage sum paid by the firms to the households in equilibrium leads to:

$$\int_0^1 W_{h,t} N_{h,t} dh = N_t \int_0^1 W_{h,t} \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{\varphi_t^W}{(\varphi_t^W - 1)}} dh = W_t N_t \tag{D.4}$$

Market clearing in the capital market

In equilibrium the effective utilization of capital by the households equals the firms used capital services rented from the households:

$$u_t K_t = u_t \int_0^1 K_{h,t} dh = \int_0^1 K_{f,t}^s df = K_t^s \tag{D.5}$$

Market clearing in the domestic intermediate goods market

In equilibrium each intermediate goods producing firm f generates a supply $Y_{f,t}$ equal to the domestic and foreign demand:

$$Y_{f,t} = H_{f,t} + X_{f,t} \tag{D.6}$$

Aggregating over the firms f in equilibrium leads to:

$$\begin{aligned} Y_t &= \int_0^1 Y_{f,t} df = \int_0^1 H_{f,t} df + \int_0^1 X_{f,t} df \\ &= \int_0^1 \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{(\varphi_t^H - 1)}} H_t df + \left(\frac{P_{X,f,t}}{P_{X,t}}\right)^{-\frac{\varphi_t^X}{(\varphi_t^X - 1)}} X_t df \\ &= \bar{s}_{H,t} H_t + \bar{s}_{X,t} X_t \end{aligned} \tag{D.7}$$

where $\bar{s}_{H,t}$ and $\bar{s}_{X,t}$ on the LHS hold:

$$\bar{s}_{H,t} = (1 - \xi_H) \left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{(\varphi_t^H - 1)}} + \xi_H \left(\frac{\Pi_{H,t}}{\Pi_{H,t-1}^{\chi_H} \Pi_t^{1-\chi_H}}\right)^{-\frac{\varphi_t^H}{(\varphi_t^H - 1)}} \bar{s}_{H,t-1} \tag{D.8}$$

and

$$\bar{s}_{X,t} = (1 - \xi_X) \left(\frac{\tilde{P}_{X,t}}{P_{X,t}}\right)^{-\frac{\varphi_t^X}{(\varphi_t^X - 1)}} + \xi_X \left(\frac{\Pi_{X,t}}{\Pi_{X,t-1}^{\chi_X} \Pi_t^{1-\chi_X}}\right)^{-\frac{\varphi_t^X}{(\varphi_t^X - 1)}} \bar{s}_{X,t-1} \tag{D.9}$$

$\tilde{P}_{H;t}$ and $\tilde{P}_{X,t}$ denote the optimal prices the domestic and foreign price setting firms chose in$t$. In nominal terms it holds:

$$\begin{aligned} P_{Y,t} Y_t &= \int_0^1 P_{H,f,t} H_{f,t} df + \int_0^1 P_{X,f,t} X_{f,t} df \\ &= H_t \int_0^1 P_{H,f,t} \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{(\varphi_t^H - 1)}} df + X_t \int_0^1 P_{X,f,t} \left(\frac{P_{X,f,t}}{P_{X,t}}\right)^{-\frac{\varphi_t^X}{(\varphi_t^X - 1)}} df \\ &= P_{H,t} H_t + P_{X,t} X_t \end{aligned} \tag{D.10}$$

For the intermediate firm's profits in equilibrium it holds:

$$\begin{aligned} D_t &= \int_0^1 D_{H,f,t} df + \int_0^1 D_{X,f,t} df \\ &= P_{H,t} H_t + P_{X,t} X_t - MC_t \left(\bar{s}_{H,t} H_t + \bar{s}_{X,t} X_t + \psi z_t\right) \end{aligned} \tag{D.11}$$

and in profit shares of nominal overall income:

$$s_{D,t} = \frac{D_t}{P_{Y,t}Y_t} = 1 - \frac{MC_t\left(\bar{s}_{H,t}H_t + \bar{s}_{X,t}X_t + \psi z_t\right)}{P_{Y,t}Y_t} \tag{D.12}$$

Market clearing in the market for imported intermediate goods

In equilibrium the supply of each foreign exporter f^* equals the demand $IM_{f^*,t}$. Aggregating over the exporters f^* leads to:

$$\begin{aligned}\int_0^1 IM_{f^*,t}df^* &= int_0^1 \left(\frac{P_{IM,f^*,t}}{P_{IM,t}}\right)^{-\frac{\varphi_t^*}{(\varphi_t^*-1)}} IM_t df^* \\ &= \bar{s}_{IM,t} IM_t\end{aligned} \tag{D.13}$$

$$\bar{s}_{IM,t} = (1-\xi^*)\left(\frac{\tilde{P}_{IM,t}}{P_{IM,t}}\right)^{-\frac{\varphi_t^*}{(\varphi_t^*-1)}} + \xi^*\left(\frac{\Pi_{IM,t}}{\Pi_{IM,t-1}^{\chi^*}\Pi_t^{1-\chi^*}}\right)^{-\frac{\varphi_t^H}{\left(\varphi_t^H-1\right)}} \bar{s}_{IM,t-1} \tag{D.14}$$

$\tilde{P}_{IM,t}$ is the optimal price chosen by the price setting exporters.

Market clearing in the final goods market

In equilibrium in the final goods market the following conditions hold:

$$\begin{aligned}Q_t^C &= C_t \\ Q_t^I &= I_t + \Gamma_u(u_t)K_t \\ Q_t^G &= G_t\end{aligned} \tag{D.15}$$

Combining the market clearing conditions for the domestic intermediate goods and final goods market leads to the nominal aggregate resource constraint:

$$\begin{aligned}P_{Y,t}Y_t = {} & P_{H,t}H_t + P_{X,t}X_t \\ & - P_{C,t}C_t + P_{I,t}\left(I_t + \Gamma_u(u_t)K_t\right) + P_{G,t}G_t + P_{X,t}X_t \\ & - P_{IM,t}\left(\frac{1-\Gamma_{IM^C}\left(IM_t^C/Q_t^C, \epsilon_t^{IM}\right)IM_t^C}{\tilde{\Gamma}_{IM^C}\left(IM_t^C/Q_t^C, \epsilon_t^{IM}\right)} + \frac{1-\Gamma_{IM^I}\left(IM_t^I/Q_t^I, \epsilon_t^{IM}\right)IM_t^I}{\tilde{\Gamma}_{IM^I}\left(IM_t^I/Q_t^I, \epsilon_t^{IM}\right)}\right)\end{aligned} \tag{D.16}$$

Market clearing in the domestic government bond market

In equilibrium it is assumed that the outstanding debt is zero. Budget deficits are financed by lump-sum taxes, such that:

$$B_t = \int_0^1 B_{h,t} dh = 0 \tag{D.17}$$

Market clearing in the market for foreign bonds

At every time t the supply of internationally traded foreign bonds matches the demand of domestic households for foreign bonds expressed in their (net) holdings:

$$B_t^* = \int_0^1 B_{h,t}^* dh = 0 \tag{D.18}$$

D.3 Stochastic volatility NAWM priors and estimation results

D.3.1 Prior and posterior distributions

Marginal prior distributions used for estimating the stochastic volatility EMU wide DSGE model's structural parameters. (Note: The inverse Gamma distribution is parameterized by the shape and the scale parameter).

		Prior Distributions			**Posterior Distributions**				
		Distribution	Mean	Std	Mode	Mean	Median	Q(5%)	Q(95%)
Preferences									
Habit formation	κ	Beta	0.700	0.050	0.820	0.821	0.821	0.820	0.822
Intertemp. pref.	β^*	Normal	0.700	0.050	1.197	1.197	1.197	1.194	1.198
Wage and Price Setting									
Calvo: wages	ξ_w	Beta	0.750	0.050	0.636	0.637	0.637	0.635	0.638
Index: wages	χ_w	Beta	0.750	0.100	0.984	0.985	0.985	0.984	0.986
Calvo: dom. prices	ξ_h	Beta	0.750	0.050	0.933	0.934	0.934	0.932	0.935
Index: dom. prices	χ_h	Beta	0.750	0.100	0.325	0.326	0.326	0.325	0.327
Calvo: exp. prices	ξ_x	Beta	0.750	0.050	0.896	0.897	0.897	0.895	0.898
Index: exp. prices	χ_x	Beta	0.750	0.100	0.434	0.434	0.434	0.432	0.435
Calvo: imp. prices	ξ^*	Beta	0.750	0.050	0.784	0.783	0.782	0.781	0.785
Index: imp. prices	χ^*	Beta	0.750	0.100	0.576	0.575	0.576	0.574	0.577
Oil import share	ω^*	Beta	0.750	0.100	0.086	0.088	0.088	0.085	0.091
Adjustment Costs									
Investment	γ_i	Gamma	4.000	0.500	4.974	4.977	4.977	4.974	4.978
Import: Cons.	γ_{I^c}	Gamma	2.500	1.000	5.686	5.683	5.682	5.681	5.686
Import: Inv	γ_{I^i}	Gamma	2.500	1.000	0.455	0.457	0.457	0.455	0.461
Export Share	γ^*	Gamma	2.500	1.000	2.814	2.816	2.816	2.815	2.818
Capital utilizing	$\gamma_{u,2}$	Gamma	0.100	0.500	0.152	0.154	0.154	0.149	0.154
Employment									
Sensitivity: Hours	ξ_E	Beta	0.500	0.150	0.470	0.469	0.469	0.468	0.471
Final-goods Prod.									
Subs. Elast.:Cons	μ_c	Gamma	1.500	0.250	1.910	1.914	1.913	1.910	1.919
Subs. Elast.:Inv	μ_i	Gamma	1.500	0.250	1.599	1.599	1.599	1.598	1.601
Price Elast.:Exp	μ^*	Gamma	1.500	0.250	0.933	0.934	0.934	0.932	0.936
Monetary Policy									
Int. Smoothing	ϕ_r	Beta	0.900	0.050	0.996	0.998	0.998	0.996	1.000
Inflation	ϕ_π	Normal	1.700	0.100	1.877	1.876	1.875	1.874	1.877
Output	π_y	Normal	0.000	0.400	0.051	0.052	0.053	0.051	0.054
Target Inflation	$\phi_{\bar{\pi}}$	Normal	1.000	0.100	0.759	0.759	0.759	0.758	0.761
Change Inflation	$\phi_{\Delta\pi}$	Normal	0.300	0.100	0.151	0.153	0.153	0.150	0.155
Change Output	$\phi_{\Delta y}$	Normal	0.200	0.050	0.219	0.219	0.219	0.217	0.220

		Prior Distributions			Posterior Distributions				
		Distribution	Mean	Std	Mode	Mean	Median	Q(5%)	Q(95%)
Autoregressive Coefficients									
Cons. Shock	ρ_c	Beta	0.750	0.100	0.910	0.912	0.911	0.910	0.915
Perm. Tech. Shock	ρ_{g_z}	Beta	0.750	0.100	0.942	0.940	0.940	0.939	0.942
Inv. Tech. Shock	ρ_i	Beta	0.750	0.100	0.928	0.928	0.928	0.926	0.929
Dom. RP Shock	ρ_{RP}	Beta	0.750	0.100	0.999	0.999	1.000	0.998	1.000
Foreign RP Shock	ρ_{RP^*}	Beta	0.750	0.100	0.979	0.977	0.977	0.975	0.979
Wage markup	ρ_{φ^w}	Beta	0.750	0.100	0.690	0.693	0.693	0.690	0.696
Emp. Shock	ρ_N	Beta	0.750	0.100	0.670	0.670	0.670	0.668	0.671
Dom. Price markup	ρ_{φ^h}	Beta	0.750	0.100	0.439	0.441	0.441	0.439	0.442
Exp. Price markup	ρ_{φ^x}	Beta	0.750	0.100	0.311	0.312	0.311	0.310	0.313
Imp. Price markup	ρ_{φ^*}	Beta	0.750	0.100	0.981	0.979	0.979	0.977	0.982
Imp. Demand	ρ_I	Beta	0.750	0.10	0 0.911	0.912	0.912	0.910	0.913
Interest Rate	ρ_r	Beta	0.750	0.100	0.684	0.683	0.683	0.682	0.686
Inflation Target	ρ_π	Beta	0.750	0.100	0.916	0.917	0.917	0.915	0.919
Export Deman	ρ_x	Beta	0.750	0.100	0.851	0.852	0.851	0.849	0.857
Trans. Tech. Shock	ρ	Beta	0.750	0.100	0.972	0.972	0.972	0.971	0.973
Export Prefernce	ρ_{v^x}	Beta	0.750	0.100	0.782	0.783	0.783	0.781	0.784
Global Output	ρ_{y^*}	Beta	0.750	0.100	0.802	0.800	0.800	0.798	0.802
Global Int. Rate	ρ_{r^*}	Beta	0.750	0.100	0.796	0.797	0.797	0.796	0.798
Oil Shock	ρ_{p_o}	Beta	0.750	0.100	0.804	0.803	0.803	0.801	0.805
Global Inflation	$\rho_{p_{y^*}}$	Beta	0.750	0.100	0.798	0.798	0.798	0.797	0.800
Gov. Spending	ρ_g	Beta	0.750	0.100	0.797	0.799	0.800	0.798	0.800
Foreign Interrelations									
Export Inflation	$a_{p_{y^*}}$	Normal	0.900	0.010	0.904	0.906	0.905	0.900	0.907
World GDP	$a_{y^*p_{y^*}}$	Normal	0.000	0.010	0.001	0.003	0.001	0.000	0.003
World GDP	a_{y^*}	Normal	0.900	0.010	0.901	0.900	0.901	0.900	0.901
World Short Rate	$a_{r^*p_{y^*}}$	Normal	0.000	0.010	0.001	0.004	0.001	0.000	0.004
World Short Rate	$a_{r^*y^*}$	Normal	0.000	0.010	-0.001	-0.003	-0.001	-0.004	0.001
World Short Rate	a_{r^*}	Normal	0.900	0.010	0.900	0.900	0.900	0.899	0.901
Oil Price	$a_{p_op_{y^*}}$	Normal	0.000	0.010	0.001	0.000	-0.003	-0.005	0.001
Oil Price	$a_{p_oy^*}$	Normal	0.000	0.010	0.000	0.003	0.000	-0.002	0.004
Oil Price	$a_{p_or^*}$	Normal	0.000	0.010	-0.001	-0.003	-0.002	-0.004	0.001
Oil Price	a_{p_o}	Normal	0.900	0.010	0.903	0.905	0.902	0.901	0.905
Gov. Spending	a	Normal	0.900	0.010	0.898	0.897	0.898	0.895	0.901
Stochastic Volatility Constants									
Cons. Shock	μ_σ^c	Normal	0.000	0.010	0.001	0.003	0.001	0.000	0.003
Perm. Tech. Shock	$\mu_\sigma^{g_z}$	Normal	0.000	0.010	0.004	0.003	0.003	0.002	0.005
Inv. Tech. Shock	μ_σ^i	Normal	0.000	0.010	0.000	0.000	0.000	-0.001	0.001
Dom. RP Shock	μ_σ^{RP}	Normal	0.000	0.010	0.000	0.001	-0.001	-0.002	0.001
Foreign RP Shock	$\mu_\sigma^{RP^*}$	Normal	0.000	0.010	0.000	-0.001	0.000	-0.001	0.001
Wage markup	$\mu_\sigma^{\varphi^*}$	Normal	0.000	0.010	0.007	0.008	0.007	0.003	0.009
Emp. Shock	μ_σ^N	Normal	0.000	0.010	-0.001	0.002	-0.001	-0.003	0.002
Dom. Price markup	$\mu_\sigma^{\varphi^h}$	Normal	0.000	0.010	0.000	0.001	0.000	-0.001	0.001
Exp. Price markup	$\mu_\sigma^{\varphi^x}$	Normal	0.000	0.010	0.002	0.003	0.002	0.001	0.004
Imp. Price markup	$\mu_\sigma^{\varphi^*}$	Normal	0.000	0.010	0.000	-0.002	0.000	-0.002	0.001
Imp. Demand	μ_σ^{im}	Normal	0.000	0.010	-0.001	0.000	-0.001	-0.002	0.001
Interest Rate	μ_σ^r	Normal	0.000	0.010	-0.001	-0.001	-0.001	-0.002	0.000

		Prior Distributions			Posterior Distributions				
		Distribution	Mean	Std	Mode	Mean	Median	Q(5%)	Q(95%)
Inflation Target	μ_σ^π	Normal	0.000	0.010	0.001	0.002	0.001	-0.001	0.002
Export Deman	μ_σ^x	Normal	0.000	0.010	0.001	-0.001	0.002	-0.002	0.003
Trans. Tech. Shock	μ_σ	Normal	0.000	0.010	-0.001	-0.002	-0.001	-0.002	0.000
Export Prefernce	$\mu_\sigma^{v^*}$	Normal	0.000	0.010	0.003	0.002	0.003	0.001	0.005
Global Output	$\mu_\sigma^{y^*}$	Normal	0.000	0.010	0.000	0.002	0.000	-0.002	0.002
Global Int. Rate	$\mu_\sigma^{r^*}$	Normal	0.000	0.010	-0.001	-0.001	-0.001	-0.002	0.000
Oil Shock	$\mu_\sigma^{p_o}$	Normal	0.000	0.010	-0.001	-0.002	-0.001	-0.002	-0.001
Global Inflation	$\mu_\sigma^{p_{y^*}}$	Normal	0.000	0.010	-0.001	-0.002	-0.001	-0.003	0.002
Gov. Spending	μ_σ^g	Normal	0.000	0.010	0.001	0.001	0.001	0.000	0.002
Stochastic Volatility Autoregressive Coefficients									
Cons. Shock	ρ_σ^c	Normal	0.900	0.010	0.899	0.897	0.898	0.897	0.901
Perm. Tech. Shock	$\rho_\sigma^{g_z}$	Normal	0.900	0.010	0.906	0.908	0.906	0.903	0.908
Inv. Tech. Shock	ρ_σ^i	Normal	0.900	0.010	0.902	0.903	0.902	0.900	0.903
Dom. RP Shock	ρ_σ^{RP}	Normal	0.900	0.010	0.898	0.899	0.898	0.897	0.901
Foreign RP Shock	$\rho_\sigma^{RP^*}$	Normal	0.900	0.010	0.896	0.893	0.896	0.892	0.899
Wage markup	$\rho_\sigma^{\varphi^*}$	Normal	0.900	0.010	0.900	0.900	0.900	0.899	0.901
Emp. Shock	ρ_σ^N	Normal	0.900	0.010	0.901	0.901	0.900	0.899	0.902
Dom. Price markup	$\rho_\sigma^{\varphi^h}$	Normal	0.900	0.010	0.900	0.902	0.900	0.898	0.902
Exp. Price markup	$\rho_\sigma^{\varphi^x}$	Normal	0.900	0.010	0.897	0.896	0.897	0.896	0.898
Imp. Price markup	$\rho_\sigma^{\varphi^*}$	Normal	0.900	0.010	0.900	0.899	0.900	0.898	0.901
Imp. Demand	ρ_σ^{im}	Normal	0.900	0.010	0.900	0.899	0.900	0.898	0.902
Interest Rate	ρ_σ^r	Normal	0.900	0.010	0.901	0.902	0.901	0.898	0.902
Inflation Target	ρ_σ^π	Normal	0.900	0.010	0.900	0.902	0.900	0.898	0.903
Export Deman	ρ_σ^x	Normal	0.900	0.010	0.900	0.902	0.900	0.898	0.902
Trans. Tech. Shock	ρ_σ	Normal	0.900	0.010	0.904	0.907	0.904	0.902	0.908
Export Prefernce	$\rho_\sigma^{v^*}$	Normal	0.900	0.010	0.898	0.898	0.898	0.896	0.899
Global Output	$\rho_\sigma^{y^*}$	Normal	0.900	0.010	0.899	0.900	0.899	0.898	0.900
Global Int. Rate	$\rho_\sigma^{r^*}$	Normal	0.900	0.010	0.898	0.899	0.898	0.897	0.900
Oil Shock	$\rho_\sigma^{p_o}$	Normal	0.900	0.010	0.902	0.903	0.902	0.901	0.903
Global Inflation	$\rho_\sigma^{p_{y^*}}$	Normal	0.900	0.010	0.900	0.900	0.900	0.898	0.901
Gov. Spending	ρ_σ^g	Normal	0.900	0.010	0.902	0.900	0.902	0.897	0.904
Stochastic Volatility Standard Deviation									
Cons. Shock	σ_σ^c	Inv. Gamma	0.010	2.000	0.012	0.016	0.011	0.010	0.017
Perm. Tech. Shock	$\sigma_\sigma^{g_z}$	Inv. Gamma	0.010	2.000	0.012	0.016	0.011	0.008	0.015
Inv. Tech. Shock	σ_σ^i	Inv. Gamma	0.010	2.000	0.013	0.014	0.013	0.011	0.016
Dom. RP Shock	σ_σ^{RP}	Inv. Gamma	0.010	2.000	0.013	0.016	0.012	0.010	0.017
Foreign RP Shock	$\sigma_\sigma^{RP^*}$	Inv. Gamma	0.010	2.000	0.015	0.019	0.015	0.011	0.019
Wage markup	$\sigma_\sigma^{\varphi^*}$	Inv. Gamma	0.010	2.000	0.014	0.017	0.015	0.010	0.018
Emp. Shock	σ_σ^N	Inv. Gamma	0.010	2.000	0.014	0.017	0.014	0.011	0.018
Dom. Price markup	$\sigma_\sigma^{\varphi^h}$	Inv. Gamma	0.010	2.000	0.012	0.016	0.012	0.010	0.016
Exp. Price markup	$\sigma_\sigma^{\varphi^x}$	Inv. Gamma	0.010	2.000	0.013	0.013	0.013	0.011	0.016
Imp. Price markup	$\sigma_\sigma^{\varphi^*}$	Inv. Gamma	0.010	2.000	0.011	0.014	0.011	0.009	0.014
Imp. Demand	σ_σ^{im}	Inv. Gamma	0.010	2.000	0.013	0.017	0.013	0.009	0.017
Interest Rate	σ_σ^r	Inv. Gamma	0.010	2.000	0.011	0.013	0.011	0.009	0.015
Inflation Target	σ_σ^π	Inv. Gamma	0.010	2.000	0.014	0.017	0.014	0.013	0.017

		Prior Distributions			Posterior Distributions				
		Distribution	Mean	Std	Mode	Mean	Median	Q(5%)	Q(95%)
Export Deman	σ_{σ}^{x}	Inv. Gamma	0.010	2.000	0.012	0.015	0.012	0.010	0.016
Trans. Tech. Shock	σ_{σ}	Inv. Gamma	0.010	2.000	0.013	0.015	0.015	0.010	0.016
Export Prefernce	$\sigma_{\sigma}^{v^*}$	Inv. Gamma	0.010	2.000	0.014	0.019	0.013	0.010	0.019
Global Output	$\sigma_{\sigma}^{y^*}$	Inv. Gamma	0.010	2.000	0.013	0.016	0.012	0.011	0.017
Global Int. Rate	$\sigma_{\sigma}^{r^*}$	Inv. Gamma	0.010	2.000	0.016	0.020	0.015	0.012	0.020
Oil Shock	$\sigma_{\sigma}^{p_o}$	Inv. Gamma	0.010	2.000	0.014	0.016	0.014	0.011	0.017
Global Inflation	$\sigma_{\sigma}^{p_{y^*}}$	Inv. Gamma	0.010	2.000	0.016	0.018	0.016	0.011	0.019
Gov. Spending	σ_{σ}^{g}	Inv. Gamma	0.010	2.000	0.012	0.012	0.012	0.010	0.013
Measurement Errors									
EMU GDP	ΔGDP^{EMU}	Inv. Gamma	0.100	2.000	0.007	0.010	0.007	0.003	0.011
EMU Consumption	$\Delta CONS$	Inv. Gamma	0.100	2.000	0.003	0.005	0.003	0.002	0.006
EMU Investment	ΔINV	Inv. Gamma	0.100	2.000	0.005	0.010	0.005	0.002	0.009
EMU Gov. Spending	GOV	Inv. Gamma	0.100	2.000	0.002	0.002	0.002	0.001	0.003
EMU Exports	$\Delta EXPORT$	Inv. Gamma	0.100	2.000	0.002	0.004	0.002	0.000	0.004
EMU Imports	$\Delta IMPORT$	Inv. Gamma	0.100	2.000	0.008	0.012	0.009	0.003	0.012
EMU GDP Inflation	ΔINF_{Y}^{EMU}	Inv. Gamma	0.100	2.000	0.002	0.003	0.002	0.001	0.003
EMU Cons. Inflation	$\Delta INFC$	Inv. Gamma	0.100	2.000	0.004	0.009	0.003	0.001	0.009
EMU Imp. Inflation	ΔINF_{I}	Inv. Gamma	0.100	2.000	0.009	0.013	0.010	0.003	0.013
EMU Employment	$LABOR$	Inv. Gamma	0.100	2.000	0.002	0.003	0.003	0.000	0.004
EMU Wages	$\Delta WAGE$	Inv. Gamma	0.100	2.000	0.005	0.009	0.004	0.002	0.009
ECB's Short Rate	ECB	Inv. Gamma	0.100	2.000	0.003	0.004	0.003	0.002	0.005
EMU Exchange Rate	FX	Inv. Gamma	0.100	2.000	0.003	0.005	0.003	0.002	0.005
World GDP	ΔGDP_{t}^{WORLD}	Inv. Gamma	0.100	2.000	0.004	0.006	0.004	0.000	0.007
World Inflation	$INF_{Y,t}^{WORLD}$	Inv. Gamma	0.100	2.000	0.003	0.003	0.003	0.001	0.005
World Short Rate	$LIBOR_{t}^{\$}$	Inv. Gamma	0.100	2.000	0.006	0.007	0.006	0.003	0.008
Export Inflation	$INF_{EXPORT,t}$	Inv. Gamma	0.100	2.000	0.002	0.002	0.002	0.000	0.003
Oil Price (UK-Brent)	$PRICE_{t}^{OIL}$	Inv. Gamma	0.100	2.000	0.017	0.028	0.016	0.007	0.028

D.3.2 Non-stochastic constant volatility NAWM (pre-) estimation results

Houshold's Preferences		**Final Goods**		**Adjustment Cost**					**Employment**
κ	β^*	μ_c	μ_i	γ_i	γ_{ι^c}	γ_{ι^i}	γ^*	$\gamma_{u,2}$	ξ_E
0.802	1.203	1.925	1.597	4.966	5.695	0.454	2.816	0.135	0.493
(0.012)	(0.006)	(0.004)	(0.003)	(0.004)	(0.005)	(0.007)	(0.003)	(0.011)	(0.019)

Wage and Price setting			
ξ_w	χ_w	μ^*	
0.635	0.985	0.926	
(0.004)	(0.003)	(0.007)	
		Monetary Policy	
ξ_h	χ_h	ϕ_r	ϕ_y
0.635	0.318	0.996	0.063
(0.004)	(0.008)	(0.003)	(0.010)
ξ_x	χ_x	ϕ_π	$\phi_{\Delta\pi}$
0.897	0.428	1.872	0.161
(0.010)	(0.007)	(0.011)	(0.003)
ξ^*	χ^*	$\phi_{\Delta y}$	$\phi_{\bar{\pi}}$
0.782	0.586	0.229	0.725
(0.002)	(0.003)	(0.009)	(0.004)
ω^*			
0.079			
(0.004)			

NAWM Structural Shock Processes

Autoregressive Coefficients							
ρ_c	ρ_{g_z}	ρ_i	ρ_{RP}	ρ_{RP^*}	ρ_{φ^w}	ρ_N	ρ_{φ^h}
0.996	0.940	0.914	0.997	0.971	0.706	0.665	0.441
(0.002)	(0.002)	(0.004)	(0.003)	(0.003)	(0.005)	(0.005)	(0.008)
ρ_{φ_x}	ρ_{φ^*}	ρ_ι	ρ_r	$\rho_{\bar{\pi}}$	ρ_x	ρ	ρ_{v^x}
0.313	0.978	0.910	0.686	0.908	0.858	0.966	0.783
(0.003)	(0.007)	(0.003)	(0.003)	(0.010)	(0.008)	(0.009)	(0.004)

Standard Deviations							
σ_c	σ_{g_z}	σ_i	σ_{RP}	σ_{RP^*}	σ_{φ^w}	σ_N	σ_{φ^h}
0.066	0.074	0.112	0.460	0.443	0.223	0.161	0.121
(0.003)	(0.005)	(0.006)	(0.003)	(0.004)	(0.003)	(0.002)	(0.004)
σ_{φ_x}	σ_{φ^*}	σ_ι	σ_r	$\sigma_{\bar{\pi}}$	σ_x	σ	σ_{v^x}
1.030	1.335	4.847	0.145	0.119	0.149	1.135	8.977
(0.006)	(0.005)	(0.021)	(0.006)	(0.006)	(0.004)	(0.008)	(0.011)

(Squared) Measurement Errors diag($\mathbf{\Sigma}$)

ΔGDP^{EMU}	$\Delta CONS$	ΔINV	GOV	$\Delta EXPORT$	$\Delta IMPORT$	INF_Y^{EMU}	INF_C	INF_ι	$LABOR$	$\Delta WAGE$	ECB	FX
0.290	0.212	0.587	0.725	0.123	0.123	0.122	0.094	0.499	0.002	0.675	0.205	0.046
(0.005)	(0.003)	(0.003)	(0.003)	(0.008)	(0.006)	(0.006)	(0.003)	(0.005)	(0.001)	(0.003)	(0.004)	(0.005)

D.3.3 Q1/1987 - Q1/2014 stochastic NAWM volatilities

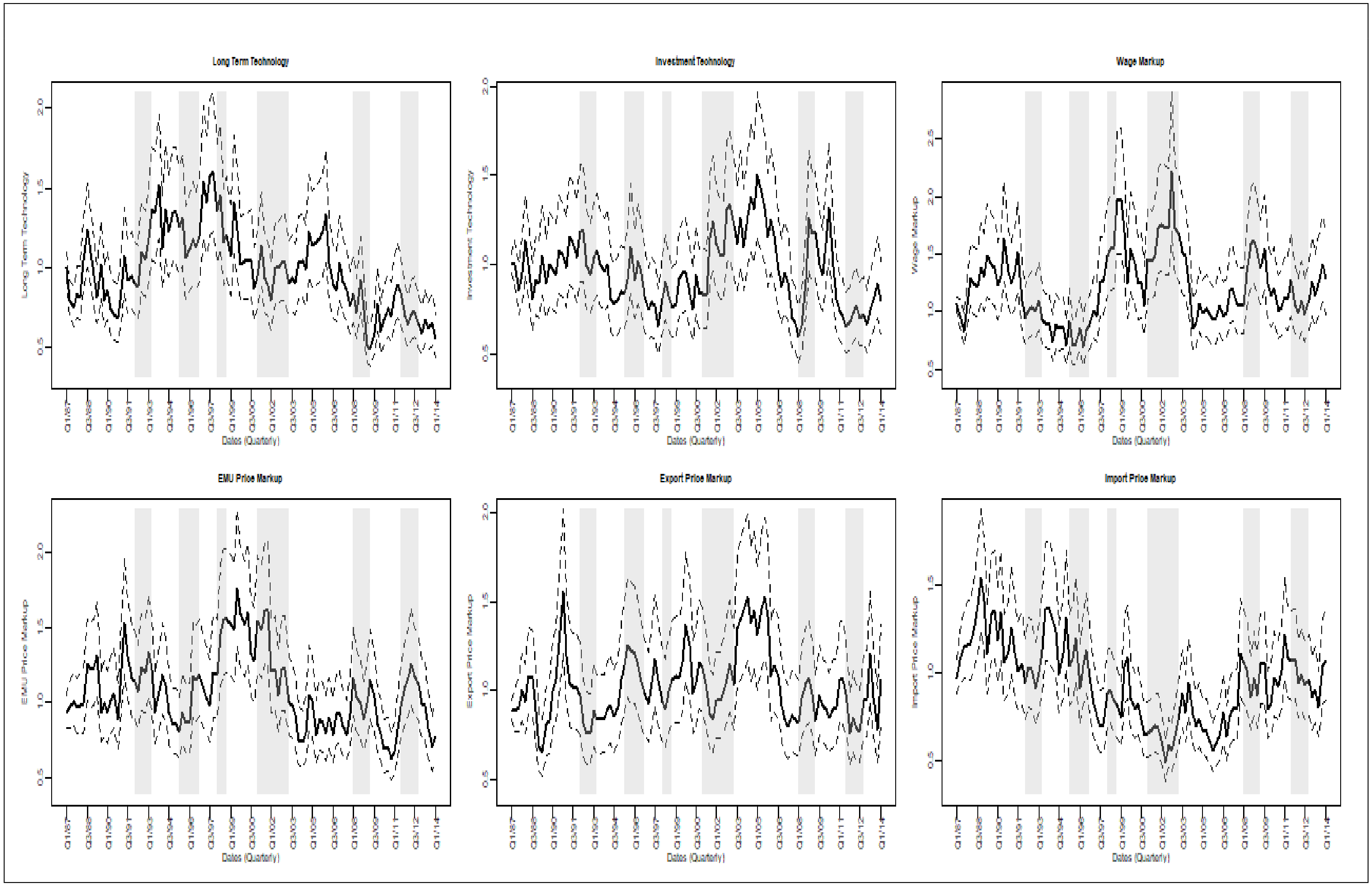


Table D.1: DSGE model implied stochastic volatilities of structural shock components ranging from long term and investment technology to import price markups evaluated at the mode of the models posteriors (dashed lines show the 2σ band)

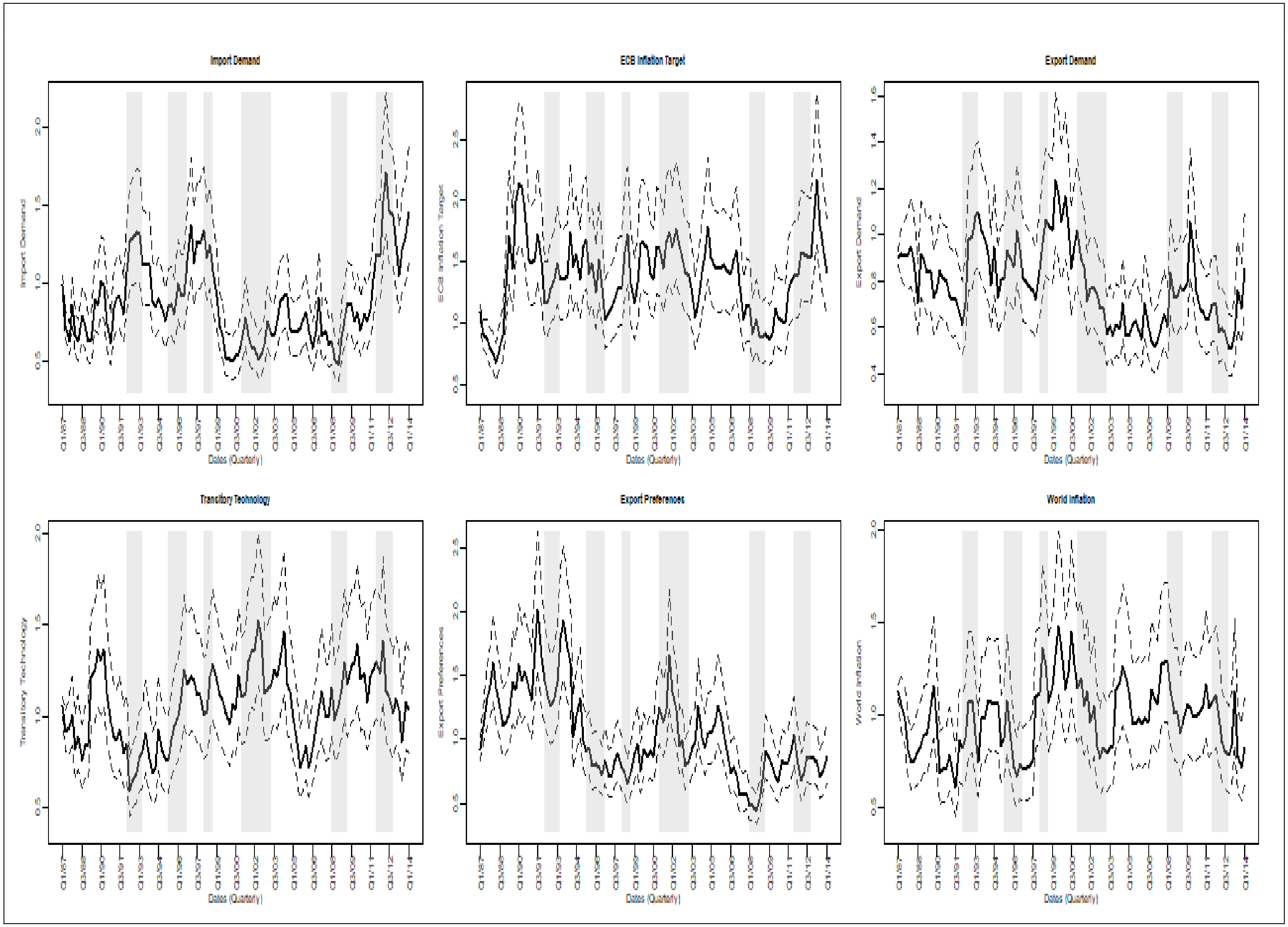
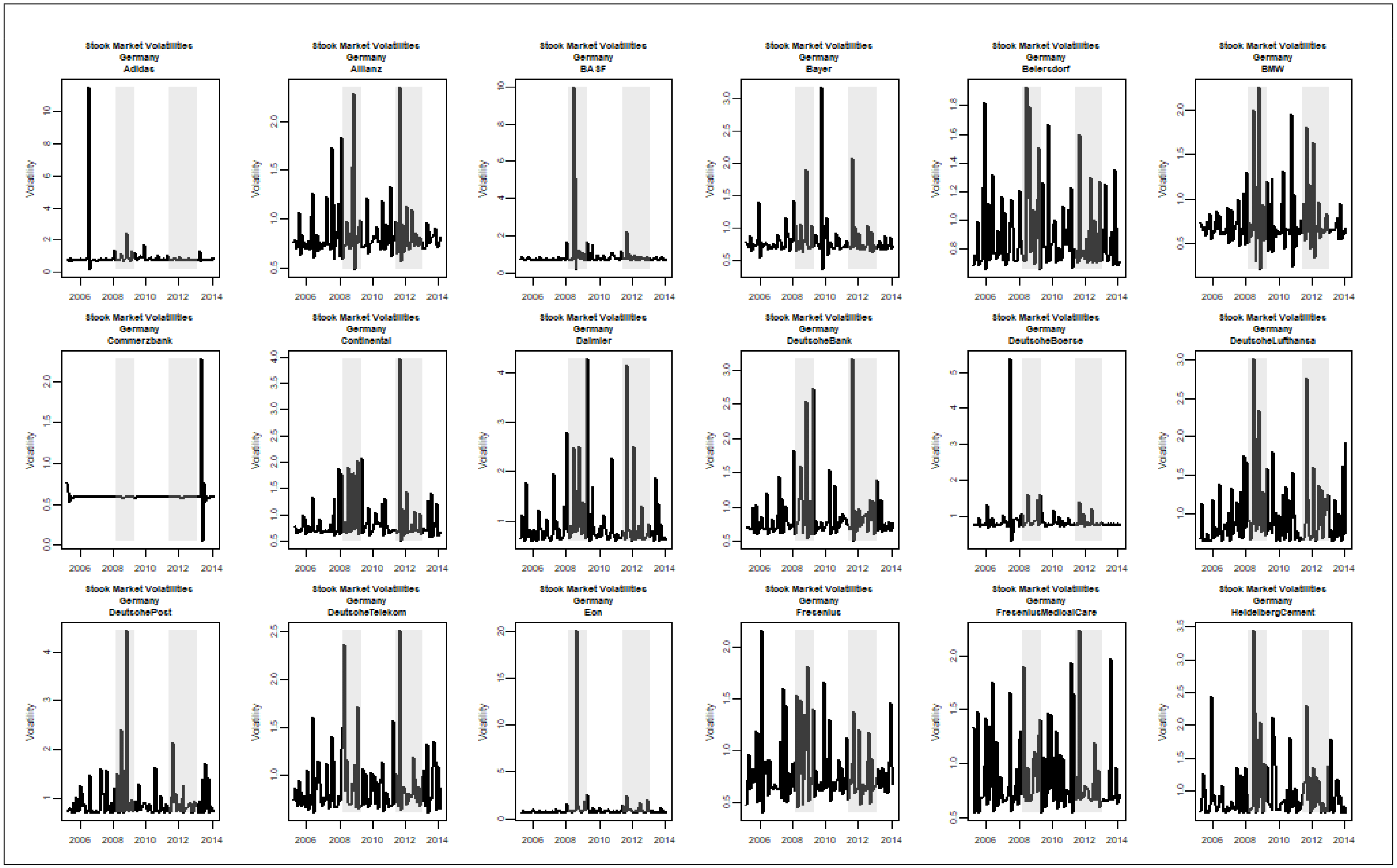
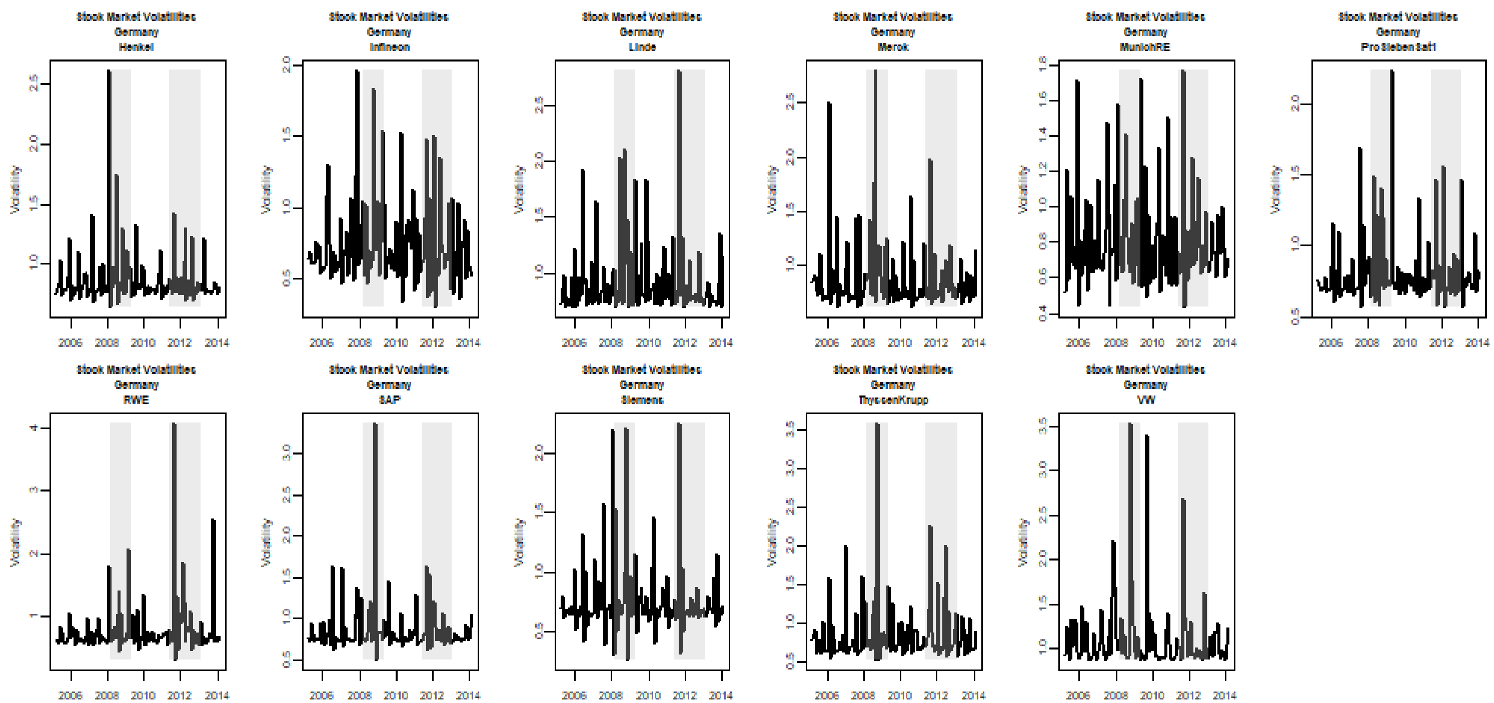


Table D.2: DSGE model implied stochastic volatilities of structural shock components ranging from export and import demand to international inlfation evaluated at the mode of the models posteriors (dashed lines show the 2σ band)

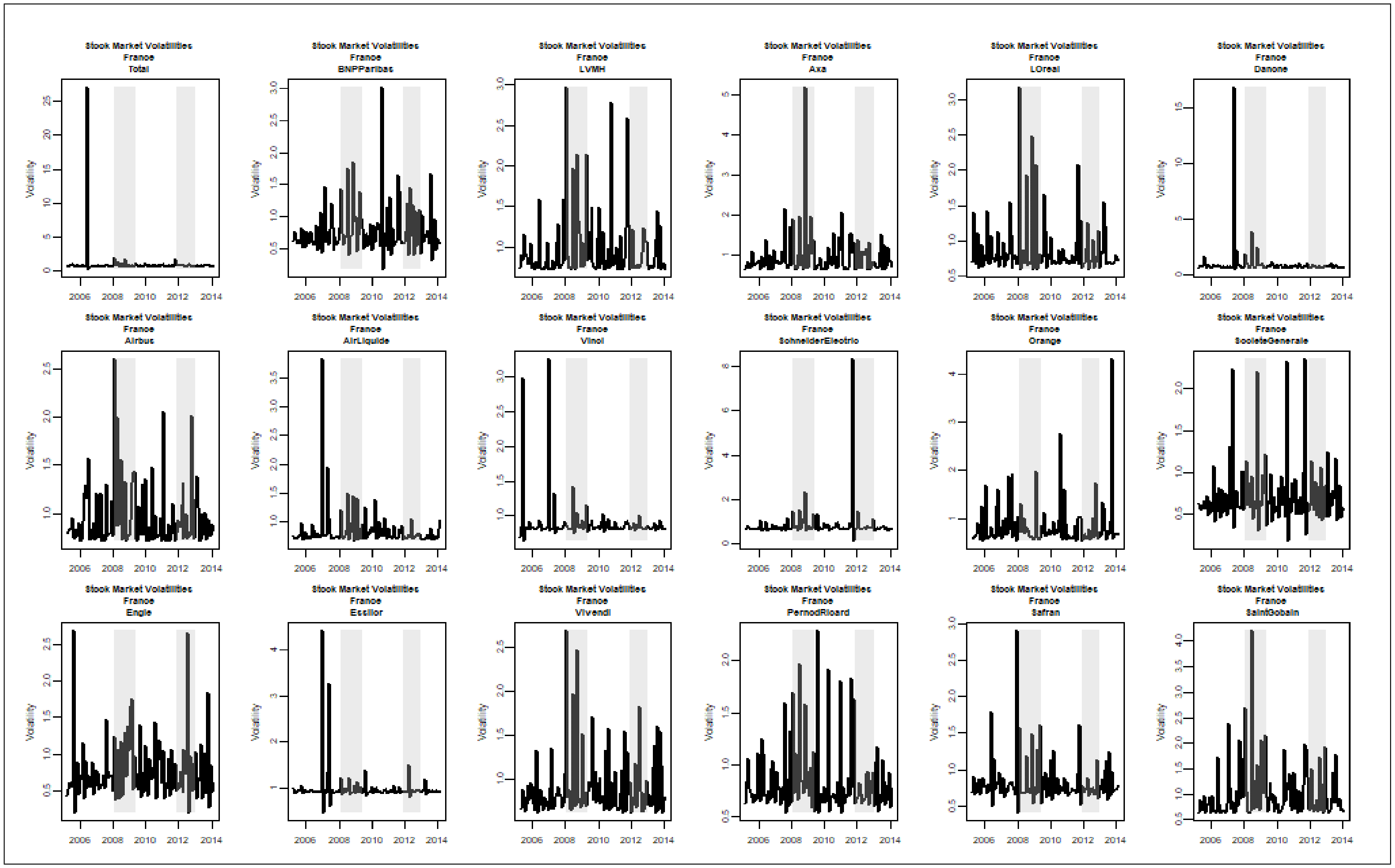
D.4 Time varying GAS stock volatility estimation results

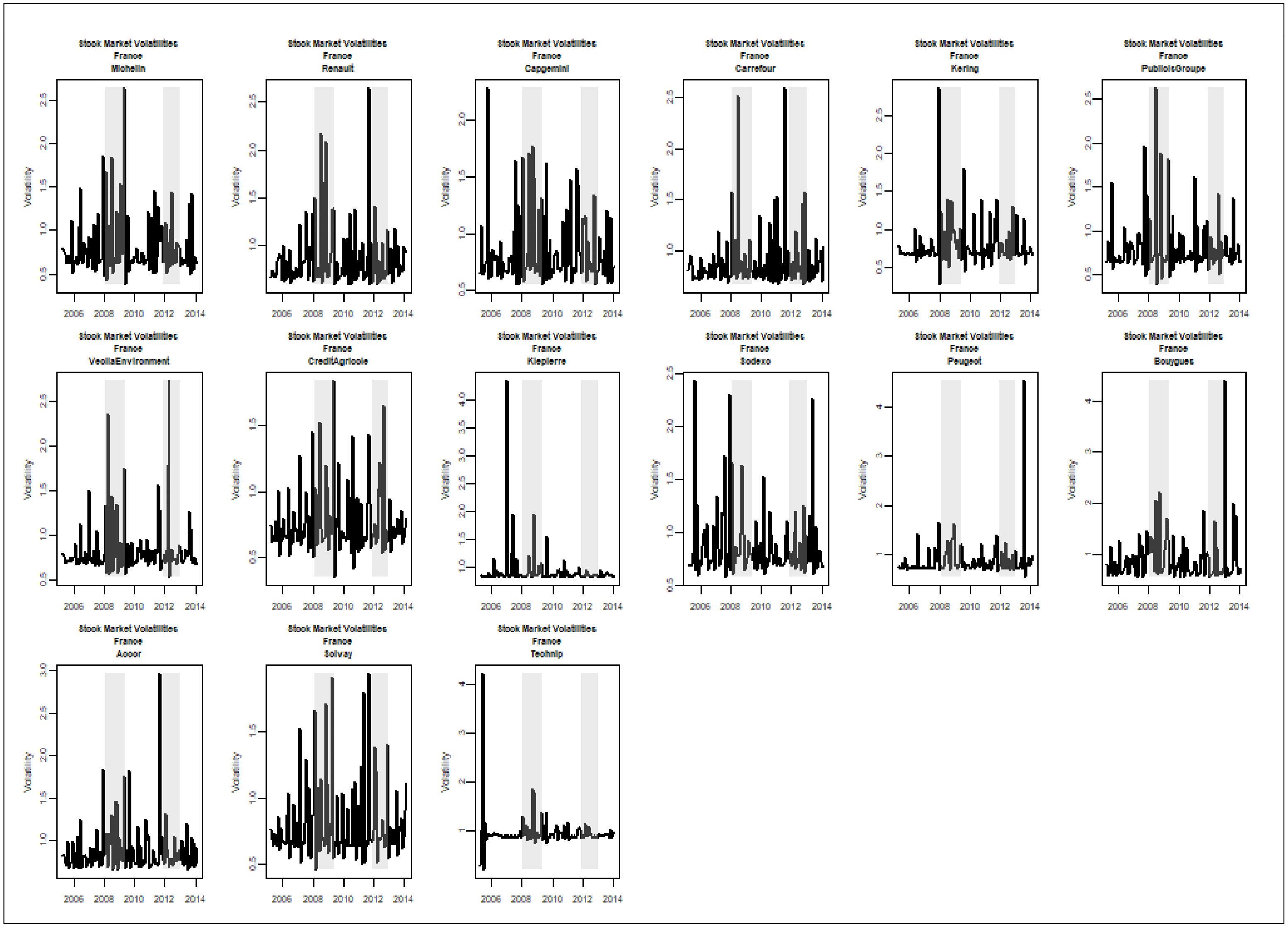
D.4.1 DAX 30 time varying GAS stock volatilities Germany



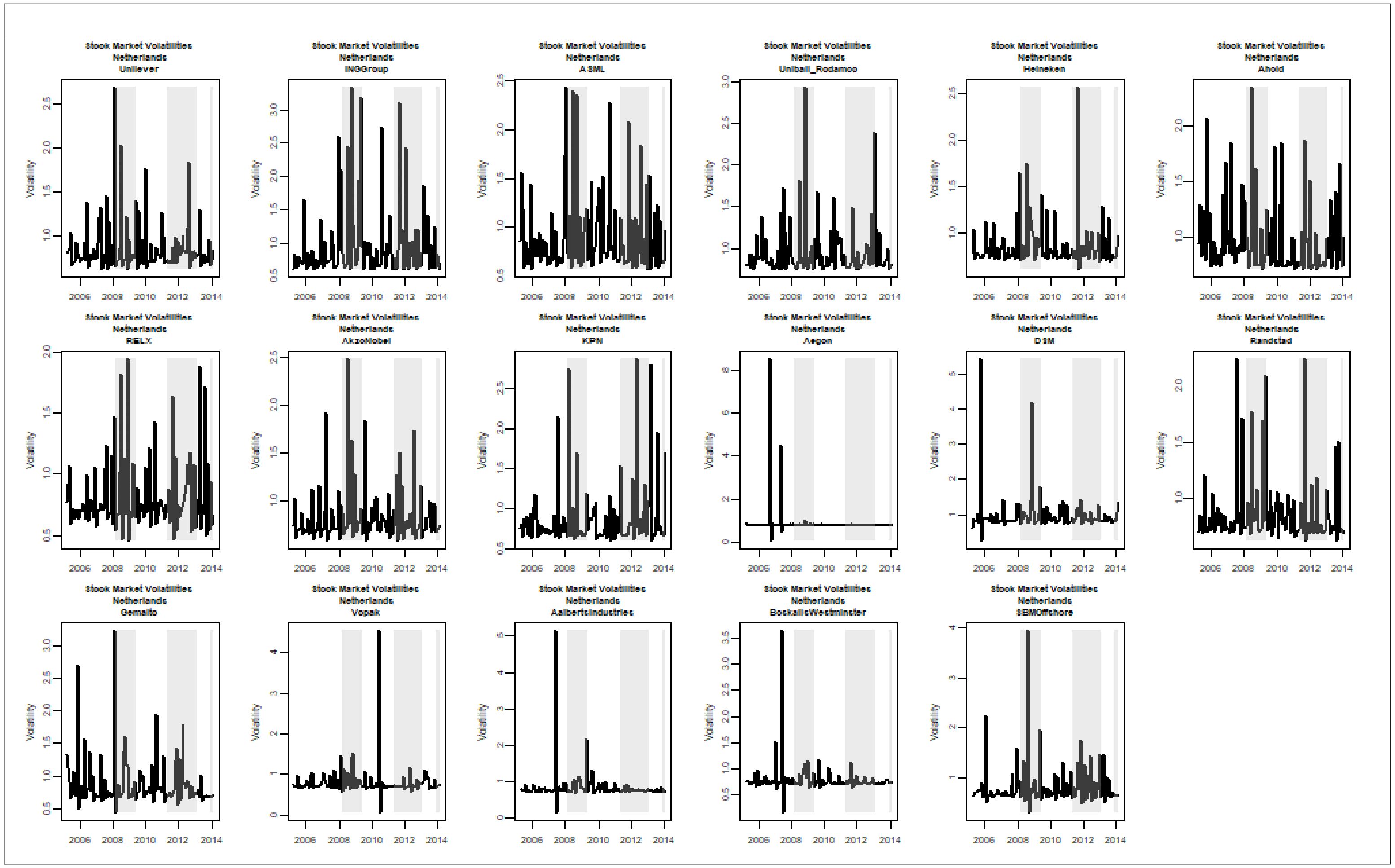

Stock Market Volatilities
Germany
Henkel
Stock Market Volatilities
Germany
Infineon
Stock Market Volatilities
Germany
Linde
Stock Market Volatilities
Germany
Merck
Stock Market Volatilities
Germany
MunichRE
Stock Market Volatilities
Germany
ProSiebenSat1
Stock Market Volatilities
Germany
RWE
Stock Market Volatilities
Germany
SAP
Stock Market Volatilities
Germany
Siemens
Stock Market Volatilities
Germany
ThyssenKrupp
Stock Market Volatilities
Germany
VW
Volatility
2006 2008 2010 2012 2014

D.4.2 CAC 40 time varying GAS stock volatilities France

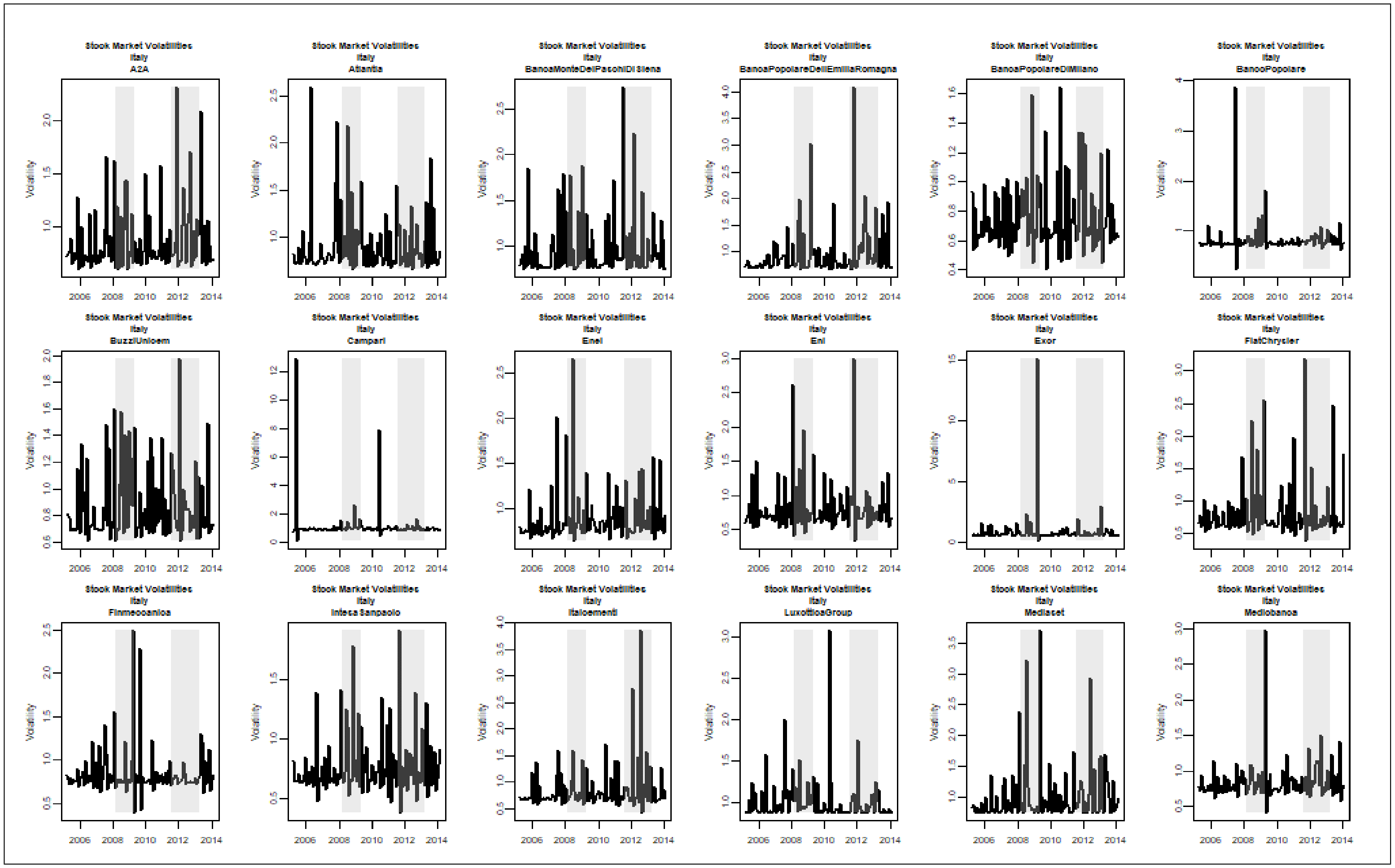


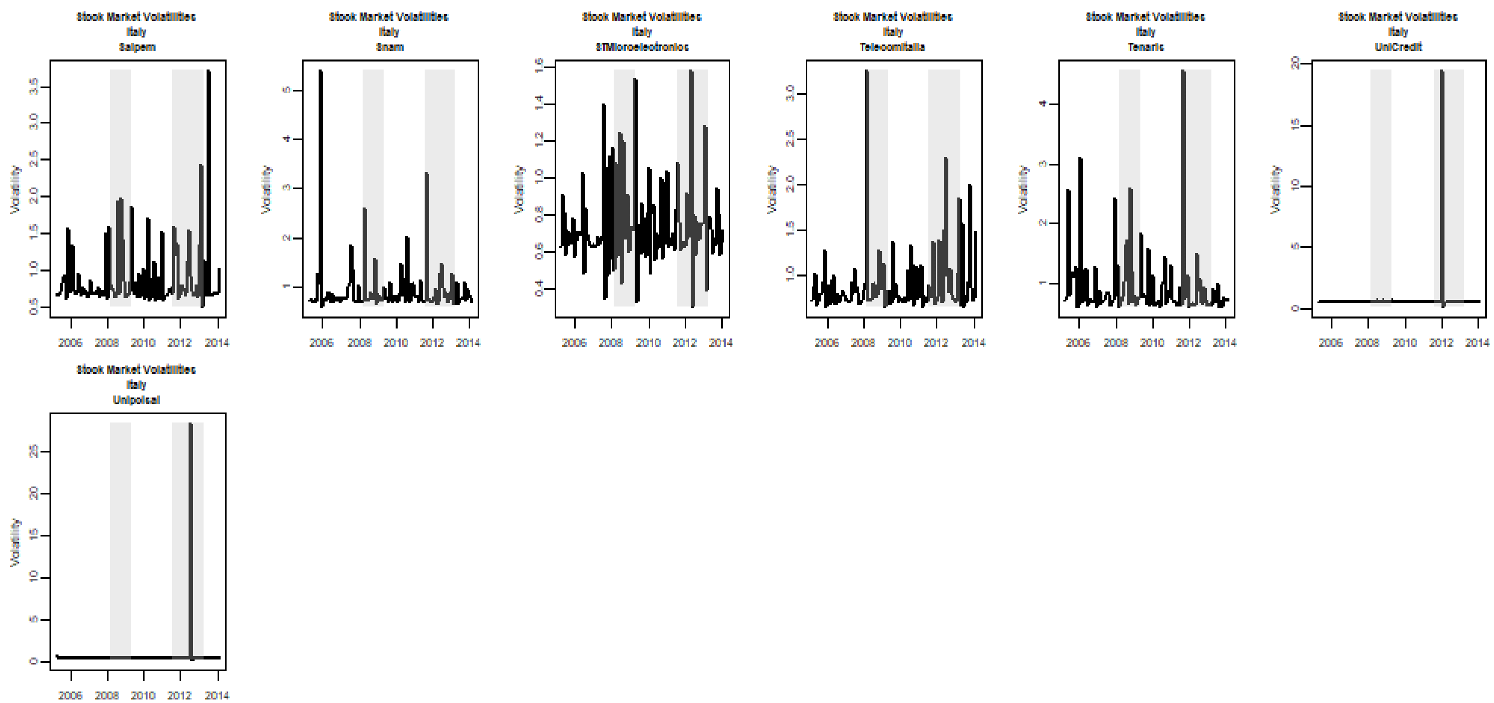

Stock Market Volatilities France Michelin
Stock Market Volatilities France Renault
Stock Market Volatilities France Capgemini
Stock Market Volatilities France Carrefour
Stock Market Volatilities France Kering
Stock Market Volatilities France PublicisGroupe
Stock Market Volatilities France VeoliaEnvironment
Stock Market Volatilities France CreditAgricole
Stock Market Volatilities France Klepierre
Stock Market Volatilities France Sodexo
Stock Market Volatilities France Peugeot
Stock Market Volatilities France Bouygues
Stock Market Volatilities France Accor
Stock Market Volatilities France Solvay
Stock Market Volatilities France Technip
Volatility
2006 2008 2010 2012 2014

D.4.3 AEX 25 time varying GAS stock volatilities Netherlands

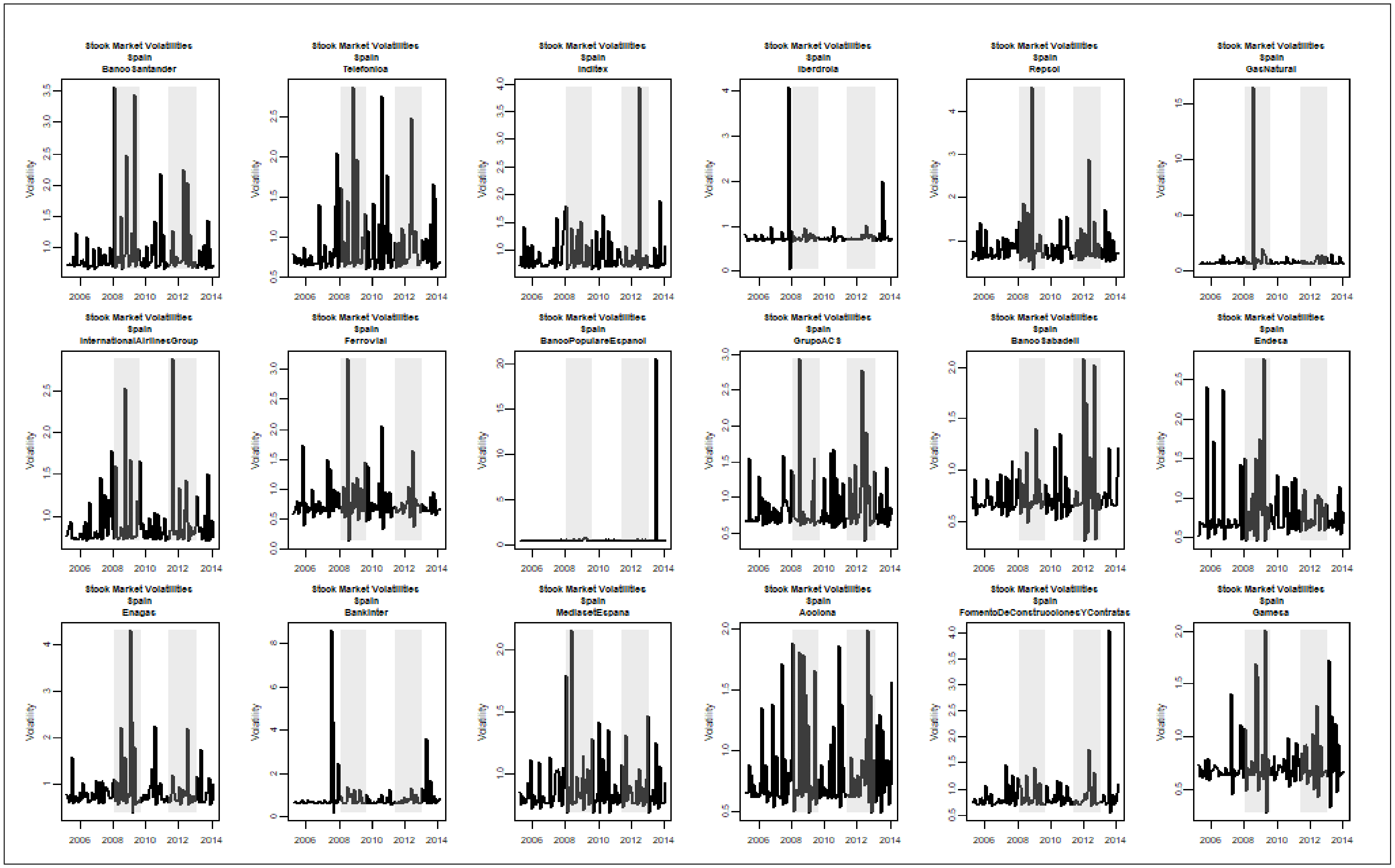


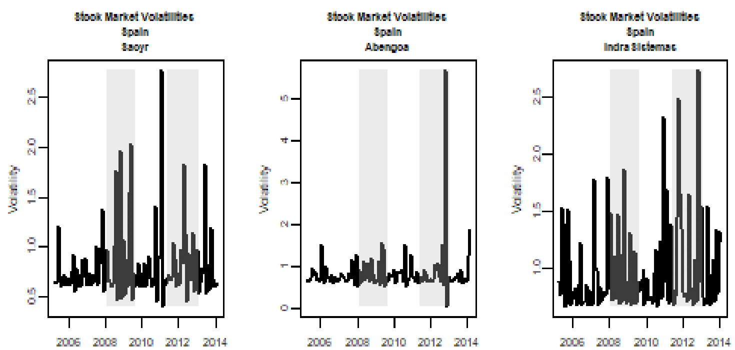
D.4.4 FTSE MIB time varying GAS stock volatilities Italy



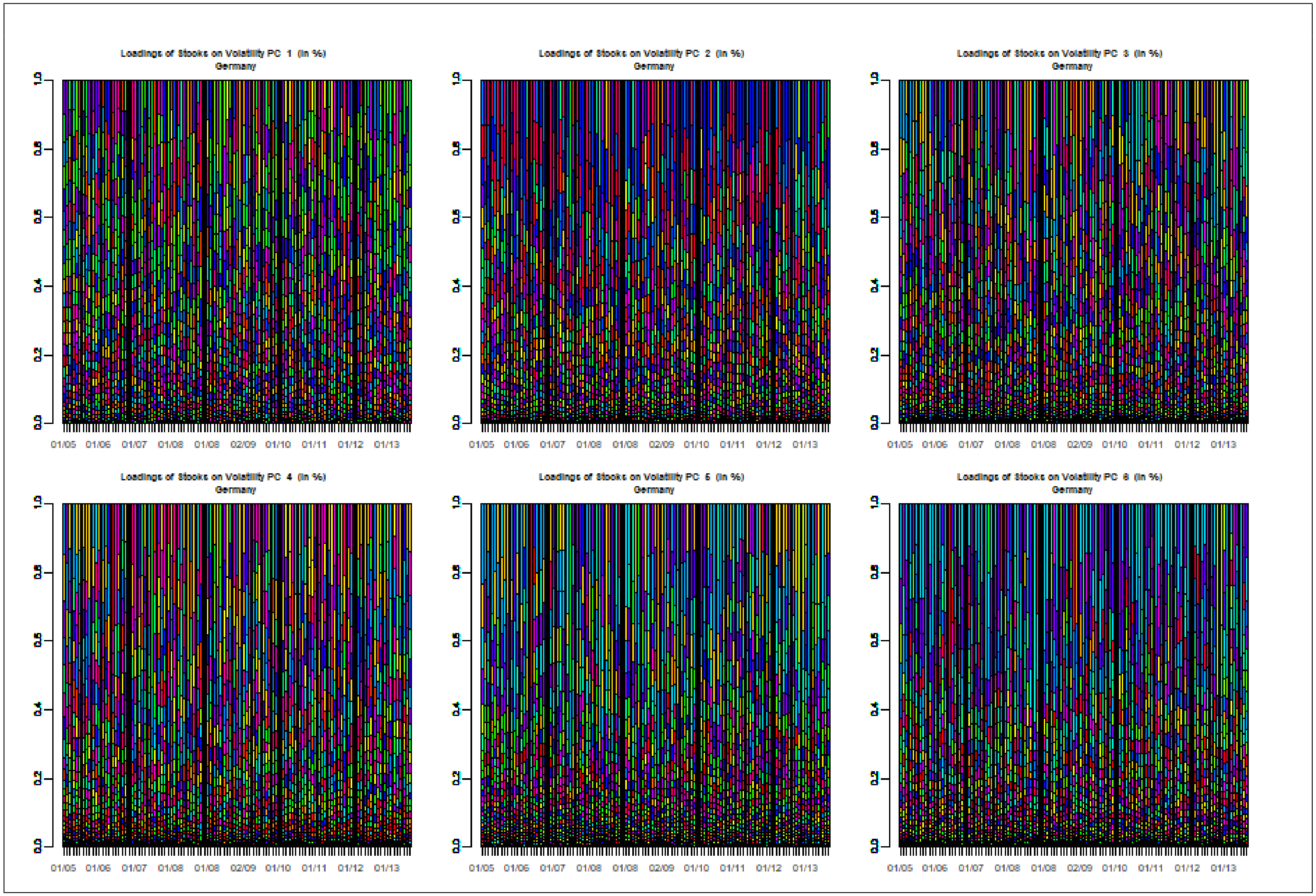

Stock Market Volatilities
Italy
Saipem
Stock Market Volatilities
Italy
Snam
Stock Market Volatilities
Italy
STMicroelectronics
Stock Market Volatilities
Italy
Telecomitalia
Stock Market Volatilities
Italy
Tenaris
Stock Market Volatilities
Italy
UniCredit
Stock Market Volatilities
Italy
Unipolsai
Volatility
2006
2008
2010
2012
2014

D.4.5 IBEX 35 time varying GAS stock volatilities Spain

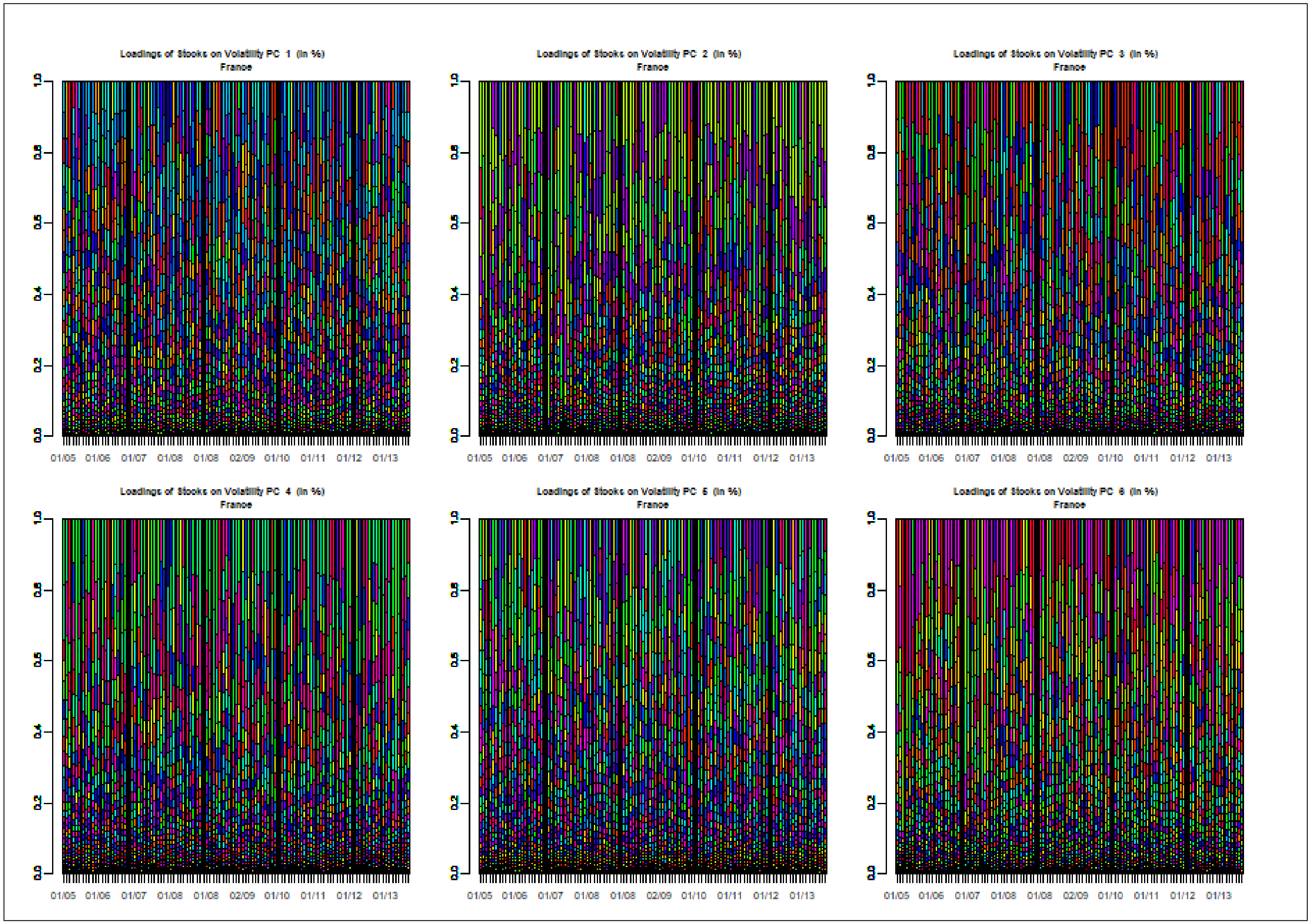



Stock Market Volatilities
Spain
Sacyr
Stock Market Volatilities
Spain
Abengoa
Stock Market Volatilities
Spain
Indra Sistemas
Volatility
2006
2008
2010
2012
2014

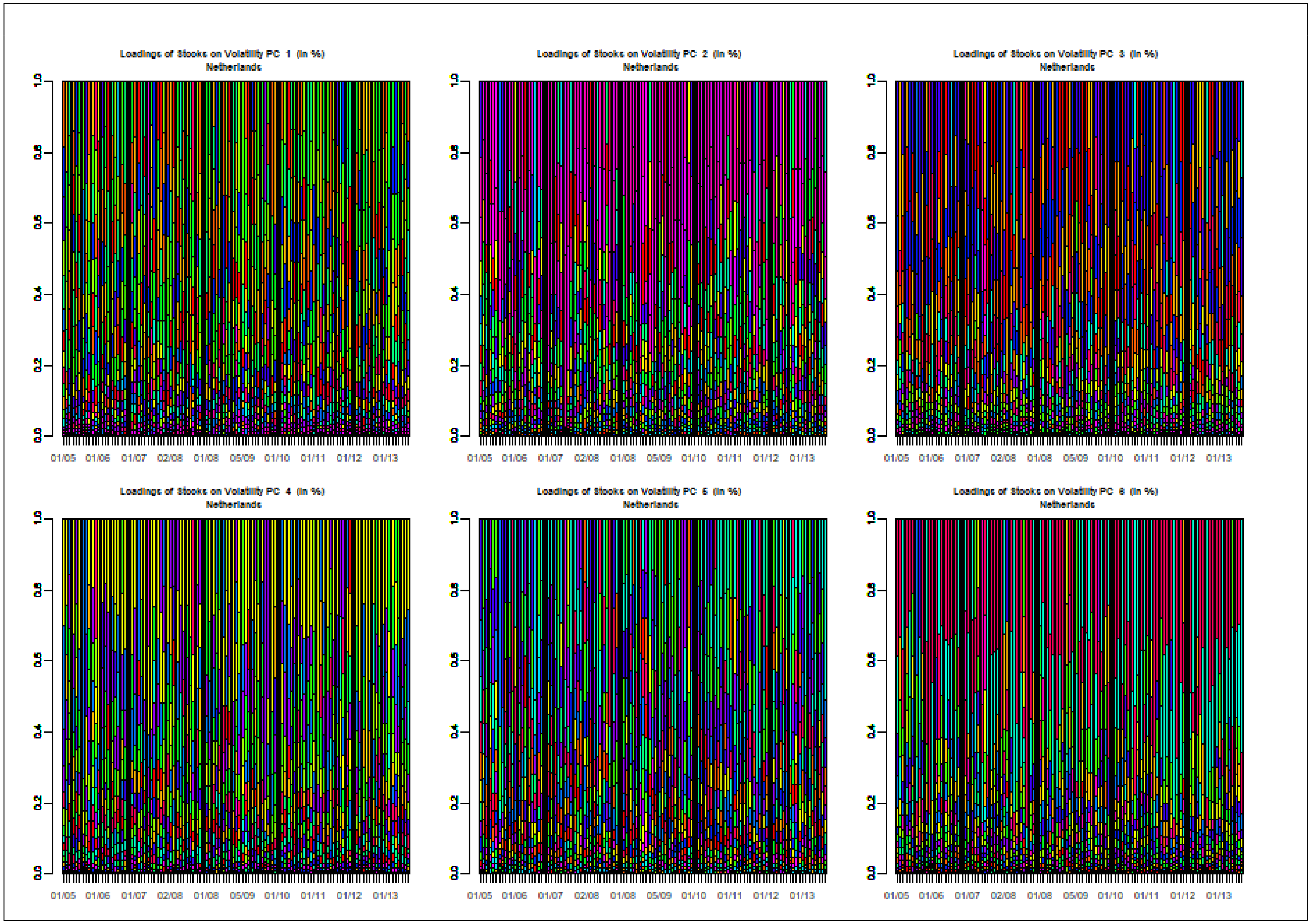
D.4.6 Composition of the DAX 30 Stock Volatility Principle Components for Germany



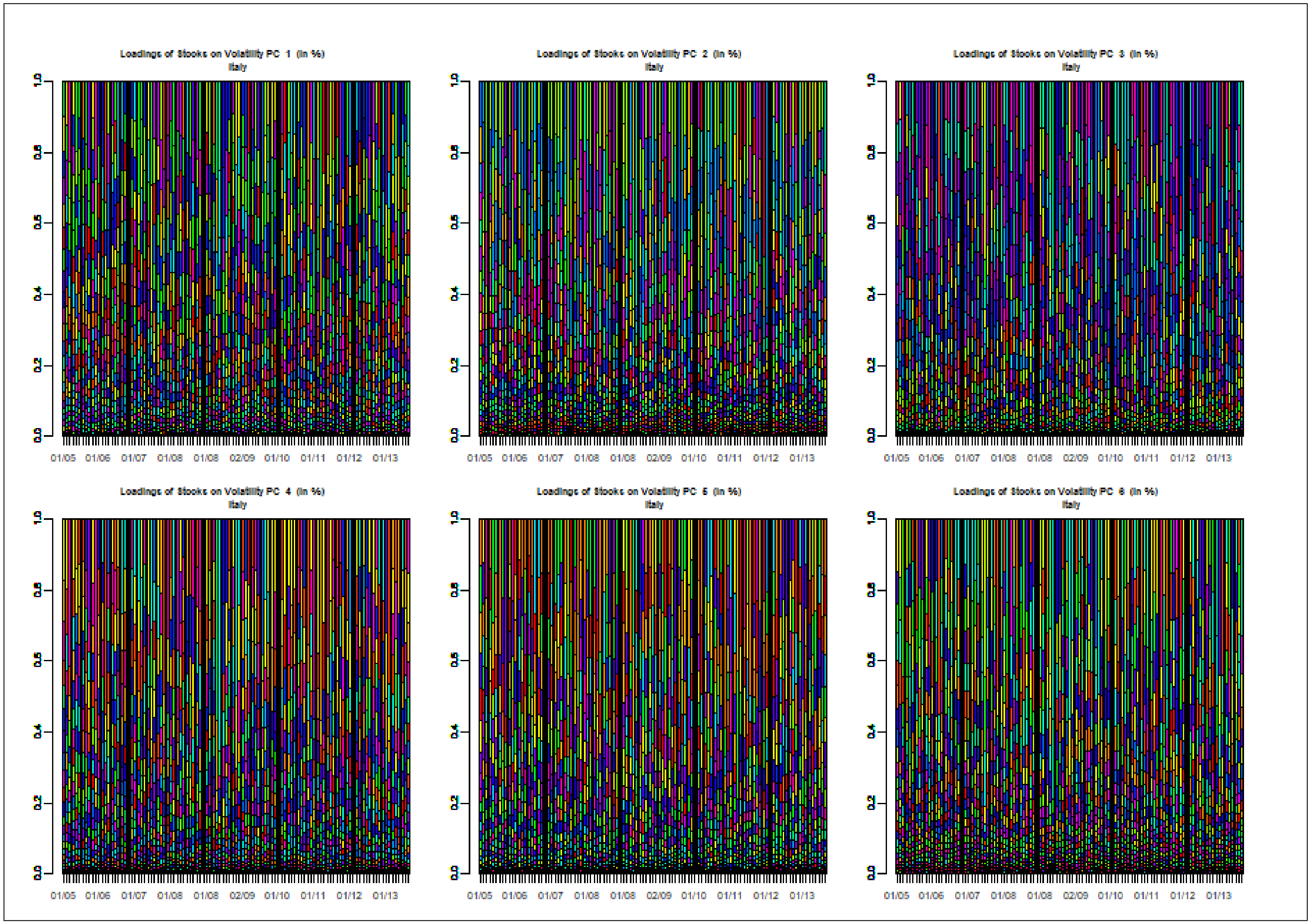
D.4.7 Composition of the CAC 40 Stock Volatility Principle Components for France



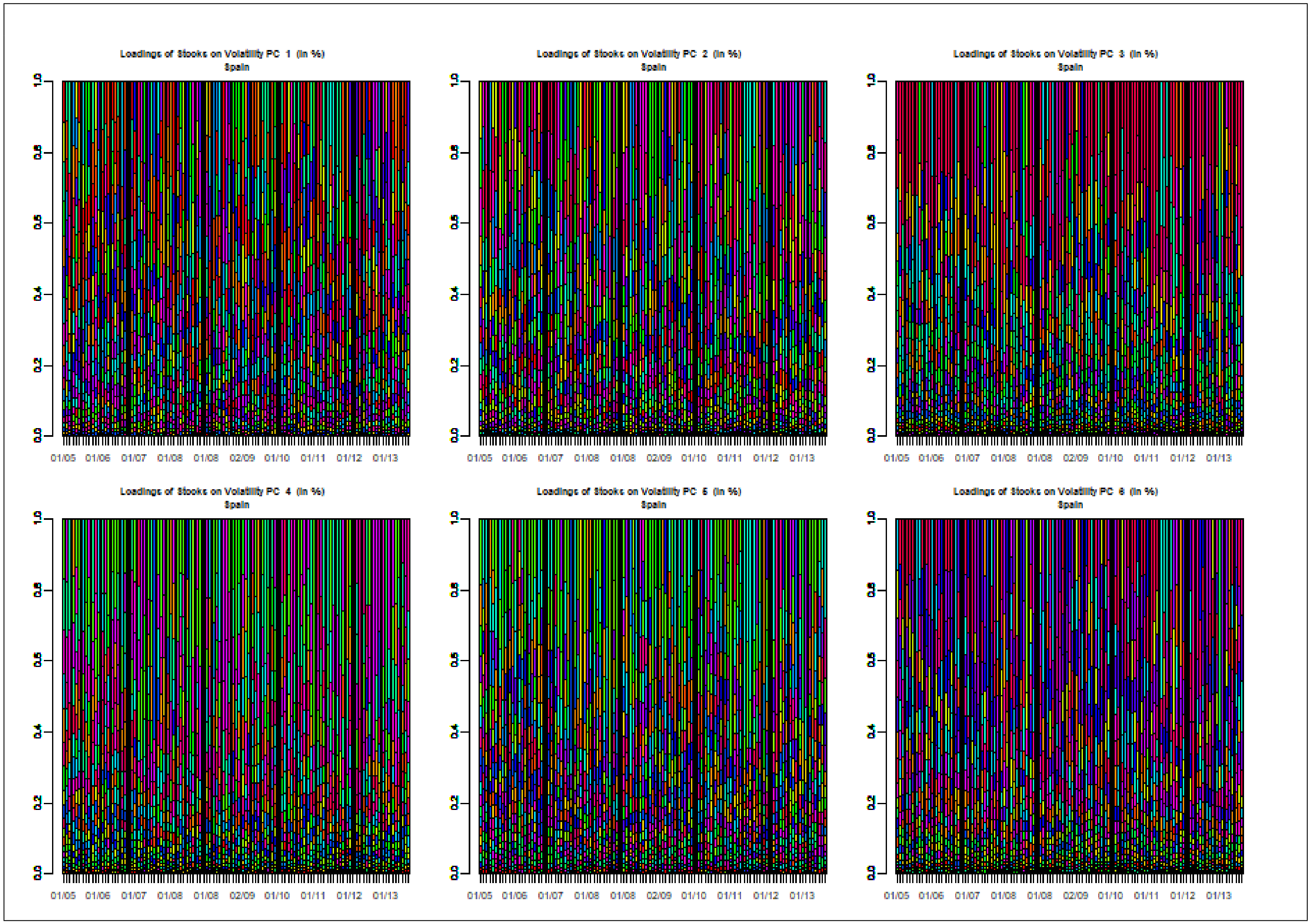
D.4.8 Composition of the AEX 25 Stock Volatility Principle Components for the Netherlands



D.4.9 Composition of the FTSE MIB Stock Volatility Principle Components for Italy



D.4.10 Composition of the IBEX 35 Stock Volatility Principle Components for Spain



E. Appendix Chapter 6

E.1 Calculation of the continuous AF-DNS conditional covariance matrix

As outlined in Christensen, Diebold and Rudebusch [2011] for calculating the covariance $\mathbf{Q} = \mathbf{V}^P\left[\mathbf{X}_t|\mathbf{X}_{t-1}\right]$ of the state variables $\mathbf{X}_t$ from the transition in the AF-DNS the diagonalization of state-variable's reversion $\mathbf{K}^P$ is used:

$$\mathbf{K}^P = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{-1} \tag{E.1}$$

where $\mathbf{V}$ is a $K \times K$ matrix of the eigenvectors of $\mathbf{K}^P$ in its columns and $\boldsymbol{\Lambda}$ is a $K \times K$ diagonal matrix containing the K eigenvalues $\lambda_1, \lambda_2, ..., \lambda_K$ of $\mathbf{K}^P$ on its diagonal. With diagonalization of $\mathbf{K}^P$ we get:

$$exp(-\mathbf{K}^P s) = \mathbf{V} exp(-\boldsymbol{\Lambda} s)\mathbf{V}^{-1} \tag{E.2}$$

and

$$exp(-\left(\mathbf{K}^P\right)^T s) = \left(\mathbf{V}^{-1}\right)^T exp(-\boldsymbol{\Lambda} s)\mathbf{V}^T \tag{E.3}$$

so that we obtain:

$$\mathbf{Q} = \mathbf{V}\left(\int_0^{\Delta t} exp(-\boldsymbol{\Lambda} s)\tilde{\boldsymbol{\Omega}} exp(-\boldsymbol{\Lambda} s) ds\right)\mathbf{V}^T \tag{E.4}$$

where $\tilde{\boldsymbol{\Omega}} = \mathbf{V}^{-1}\boldsymbol{\Omega}\boldsymbol{\Omega}^T\left(\mathbf{V}^{-1}\right)^T$ which leads to the expression for the covariance $\mathbf{Q}$:

$$\mathbf{Q} = \mathbf{V}\left(\frac{\omega_{i,j}}{(\lambda_i + \lambda_j)}\left[1 - exp(-(\lambda_i + \lambda_j)\Delta t)\right]\right)_{K\times K}\mathbf{V}^T \tag{E.5}$$

where $(*)_{K\times K}$ denotes a $K \times K$ matrix with its (i, j) element defined by the expression in the inner of the brackets. For the initialization of the Kalman filter in $t = 0$ beside $\mathbf{X}_0 = \boldsymbol{\theta}^P$ the unconditional covariance

$$\mathbf{Q}_0 = \int_0^{\infty} exp(-\mathbf{K}^P s)\boldsymbol{\Omega}\boldsymbol{\Omega}^T exp(-\left(\mathbf{K}^P\right)^T s) ds \tag{E.6}$$

is used, which is under the assumption $\lambda_i > 0$ for all $i = 1, 2, ..., K$ given by:

$$\mathbf{Q}_0 = \mathbf{V}\left(\frac{\omega_{i,j}}{(\lambda_i + \lambda_j)}\right)_{K\times K}\mathbf{V}^T \tag{E.7}$$

E.2 AF-DNS yield adjustment term

Following Christensen, Diebold and Rudebusch [2011] the analytical form of the yield adjustment term of the AF-DNS with $K = 3$ latent term structure factors is:

$$
\begin{aligned}
\frac{\Gamma(t,T)}{(T-t)} &= \frac{1}{2(T-t)} \sum_{i=1}^{3} \int_t^T \left[\mathbf{\Omega}^T \mathbf{B}(t,T)\mathbf{B}(t,T)^T \mathbf{\Omega}\right]_{i,i} ds \\
&= \bar{a}\frac{(T-t)^2}{6} + \bar{b}\left(\frac{1}{2\lambda^2} - \frac{1}{\lambda^3}\frac{(1-e^{-\lambda(T-t)})}{(T-t)} + \frac{1}{4\lambda^3}\frac{(1-e^{-2\lambda(T-t)})}{(T-t)}\right) \\
&+ \bar{c}\left(\frac{1}{2\lambda^2} + \frac{1}{\lambda^2}e^{-\lambda(T-t)} - \frac{1}{4\lambda}(T-t)e^{-2\lambda(T-t)} - \frac{3}{4\lambda^2}e^{-2\lambda(T-t)}\right) \\
&- \bar{c}\left(\frac{2}{\lambda^3}\frac{(1-e^{-\lambda(T-t)})}{(T-t)} - \frac{5}{8\lambda^3}\frac{(1-e^{-2\lambda(T-t)})}{(T-t)}\right) \\
&+ \bar{d}\left(\frac{1}{2\lambda^2}(T-t) + \frac{1}{\lambda^2}e^{-\lambda(T-t)} - \frac{1}{\lambda^3}\frac{(1-e^{-\lambda(T-t)})}{(T-t)}\right) \\
&+ \bar{e}\left(\frac{3}{\lambda^2}e^{-\lambda(T-t)} + \frac{1}{2\lambda}(T-t) + \frac{1}{\lambda}(T-t)e^{-\lambda(T-t)} - \frac{3}{\lambda^3}\frac{(1-e^{-\lambda(T-t)})}{(T-t)}\right) \\
&+ \bar{f}\left(\frac{1}{\lambda^2} + \frac{1}{\lambda^2}e^{-\lambda(T-t)} - \frac{1}{2\lambda^2}e^{-2\lambda(T-t)} - \frac{3}{\lambda^3}\frac{(1-e^{-\lambda(T-t)})}{(T-t)} + \frac{3}{4\lambda^3}\frac{(1-e^{-2\lambda(T-t)})}{(T-t)}\right)
\end{aligned}
\tag{E.8}
$$

where for the *independent-factor AF-DNS* the constants $\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e}$ and $\bar{f}$ are:

$$\bar{a} = \omega_{1,1}^2,\ \bar{b} = \omega_{2,2}^2,\ \bar{c} = \omega_{3,3}^2,\ \bar{d} = \bar{e} = \bar{f} = 0$$

and for the *correlated-factor AF-DNS* the constants are:

$$\bar{a} = \omega_{1,1}^2,\ \bar{b} = \omega_{2,1}^2 + \omega_{2,2}^2,\ \bar{c} = \omega_{3,1}^2 + \omega_{3,2}^2 + \omega_{3,3}^2,\ \bar{d} = \omega_{1,1}\omega_{2,1},\ \bar{e} = \omega_{1,1}\omega_{3,1},\ \bar{f} = \omega_{2,1}\omega_{3,1} + \omega_{2,2}\omega_{3,2}$$

E.3 Derivation of recursive ATSM bond pricing with stochastic volatility

Following Creal and Wu [2015] for the derivation of the recursive pricing scheme of the arbitrage-free ATSM with stochastic volatility the Laplace transforms for the Gaussian and multivariate non-central Gamma distributed state variables are introduced:

$$
\begin{aligned}
\boldsymbol{g}_{t+1} \sim N(&\boldsymbol{\mu}_g - \boldsymbol{\lambda}_g + (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g)\,\boldsymbol{g}_t + (\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh})\,\boldsymbol{h}_t \\
&+ \boldsymbol{\Sigma}_{gh}\boldsymbol{h}_{t+1} - \boldsymbol{\Sigma}_{gh}\left[(\mathbf{I}_{H\times H} + \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\,\boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h(\boldsymbol{v}_h - \boldsymbol{\lambda}_h) + (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\,\boldsymbol{h}_t\right], \boldsymbol{\Sigma}_{g,t}\boldsymbol{\Sigma}_{g,t}^T)
\end{aligned}
\tag{E.9}
$$

$$\boldsymbol{h}_{t+1} \sim MultNCG\left(v_{hi} - \lambda_{hi}, \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h, \boldsymbol{\Sigma}_h, \boldsymbol{\mu}_h\right) \tag{E.10}$$

where $\boldsymbol{\lambda}_g, \boldsymbol{\Lambda}_g, \boldsymbol{\lambda}_h$ and $\boldsymbol{\Lambda}_h$ are the parameters determining the time-varying market prices of risk for the risk-adjustment done by the risk-averse investor. For more details see Creal and Wu [2015]. For any real $G \times 1$ vector $\bar{\boldsymbol{u}}_g$ the Laplace transform of the Gaussian $\boldsymbol{g}_{t+1}$ conditional on $\boldsymbol{h}_{t+1}$ is expressed as:

$$\begin{aligned}\mathbb{E}_{g|h}\left[exp(\bar{\boldsymbol{u}}_g^T \boldsymbol{g}_{t+1})\right] = exp((&\boldsymbol{\mu}_g - \boldsymbol{\lambda}_g + (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g)\boldsymbol{g}_t + (\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh})\boldsymbol{h}_t \\ &+ \boldsymbol{\Sigma}_{gh}\boldsymbol{h}_{t+1} - \boldsymbol{\Sigma}_{gh}[(\mathbf{I}_{H\times H} + \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h(\boldsymbol{v}_h - \boldsymbol{\lambda}_h) \\ &+ (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{h}_t])^T \bar{\boldsymbol{u}}_g + \frac{1}{2}\bar{\boldsymbol{u}}_g^T \boldsymbol{\Sigma}_{gt}\boldsymbol{\Sigma}_{gt}^T \bar{\boldsymbol{u}}_g)\end{aligned} \tag{E.11}$$

The Laplace transform of the multivariate non-central Gamma distributed $\boldsymbol{h}_{t+1}$ for any real $H \times 1$ vector $\bar{\boldsymbol{u}}_h$ is given by:

$$\begin{aligned}\mathbb{E}\left[exp(\bar{\boldsymbol{u}}_h^T \boldsymbol{h}_{t+1})\right] = exp(&\bar{\boldsymbol{u}}_h^T \boldsymbol{\mu}_h + \sum_{i=1}^{H} \frac{\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T \bar{\boldsymbol{u}}_h}{\left(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T \bar{\boldsymbol{u}}_h\right)} \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1} \left(\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h\right)\left(\boldsymbol{h}_t - \boldsymbol{\mu}_h\right) \\ &- \sum_{i=1}^{H} \left(v_{hi} - \lambda_{hi}\right) ln\left(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T \bar{\boldsymbol{u}}_h\right))\end{aligned} \tag{E.12}$$

where the Laplace transform exists only for $\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T \bar{\boldsymbol{u}}_h < 1$ for all $i = 1, 2, ..., H$. Defining the $G + H \times 1$ vectors $\boldsymbol{u}^T = [\boldsymbol{u}_g, \boldsymbol{u}_h]$ and $\boldsymbol{x}_{t+1}^T = [\boldsymbol{g}_{t+1}, \boldsymbol{h}_{t+1}]$ we can write:

$$\begin{aligned}\mathbb{E}\left[exp(\boldsymbol{u}^T \boldsymbol{x}_{t+1})\right] &= \mathbb{E}\left[exp(\boldsymbol{u}_g^T \boldsymbol{g}_{t+1}) exp(\boldsymbol{u}_h^T \boldsymbol{h}_{t+1})\right] \\ &\mathbb{E}_h\left[\mathbb{E}_{g|h}\left[exp(\boldsymbol{u}_g^T \boldsymbol{g}_{t+1}) exp(\boldsymbol{u}_h^T \boldsymbol{h}_{t+1})\right]\right]\end{aligned} \tag{E.13}$$

with the conditional expectation $\mathbb{E}_{g|h}\left[\boldsymbol{u}_g^T \boldsymbol{g}_{t+1}\right]$ from E.11 with $\bar{\boldsymbol{u}}_g = \boldsymbol{u}_g$ we get:

$$\begin{aligned}\mathbb{E}\left[exp(\boldsymbol{u}^T \boldsymbol{x}_{t+1}\right] &= \mathbb{E}_h[exp((\boldsymbol{\mu}_g - \boldsymbol{\lambda}_g + (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g)\boldsymbol{g}_t + (\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh})\boldsymbol{h}_t \\ &+ \boldsymbol{\Sigma}_{gh}\boldsymbol{h}_{t+1} - \boldsymbol{\Sigma}_{gh}\left[(\mathbf{I}_{H\times H} + \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h(\boldsymbol{v}_h - \boldsymbol{\lambda}_h) + (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{h}_t]\right)^T \bar{\boldsymbol{u}}_g \\ &+ \frac{1}{2}\boldsymbol{u}_g^T \boldsymbol{\Sigma}_{gt}\boldsymbol{\Sigma}_{gt}^T \boldsymbol{u}_g) \exp(\boldsymbol{u}_h^T \boldsymbol{h}_{t+1})] \\ &= \mathbb{E}_h[exp(\boldsymbol{u}_g^T(\boldsymbol{\mu}_g - \boldsymbol{\lambda}_g + (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g)\boldsymbol{g}_t + (\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh})\boldsymbol{h}_t \\ &- \boldsymbol{\Sigma}_{gh}\left[(\mathbf{I}_{H\times H} + \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h(\boldsymbol{v}_h - \boldsymbol{\lambda}_h) + (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{h}_t]\right) \\ &+ \frac{1}{2}\boldsymbol{u}_g^T \boldsymbol{\Sigma}_{gt}\boldsymbol{\Sigma}_{gt}^T \boldsymbol{u}_g + \boldsymbol{u}_g^T \boldsymbol{\Sigma}_{gh}\boldsymbol{h}_{t+1}) \exp(\boldsymbol{u}_h^T \boldsymbol{h}_{t+1})] \\ &exp(\boldsymbol{u}_g^T(\boldsymbol{\mu}_g - \boldsymbol{\lambda}_g + (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g)\boldsymbol{g}_t + (\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh})\boldsymbol{h}_t \\ &- \boldsymbol{\Sigma}_{gh}\left[(\mathbf{I}_{H\times H} + \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h(\boldsymbol{v}_h - \boldsymbol{\lambda}_h) + (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{h}_t]\right) \\ &+ \frac{1}{2}\boldsymbol{u}_g^T \boldsymbol{\Sigma}_{gt}\boldsymbol{\Sigma}_{gt}^T) \, \mathbb{E}_h\left[exp((\boldsymbol{u}_g^T \boldsymbol{\Sigma}_{gh} + \boldsymbol{u}_h^T)\boldsymbol{h}_{t+1})\right])\end{aligned} \tag{E.14}$$

Using the Laplace transform of $\boldsymbol{h}_{t+1}$ in E.12 with $\bar{\boldsymbol{u}}_h = \boldsymbol{\Sigma}_{gh}^T \boldsymbol{u}_g + \boldsymbol{u}_h$ leads to:

$$
\begin{aligned}
\mathbb{E}\left[exp(\boldsymbol{u}^T \boldsymbol{x}_{t+1})\right] &= exp(\boldsymbol{u}_g^T(\boldsymbol{\mu}_g - \boldsymbol{\lambda}_g + (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g)\boldsymbol{g}_t + (\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh})\boldsymbol{h}_t \\
&- \boldsymbol{\Sigma}_{gh}\left[(\mathbf{I}_{H\times H} + \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h(\boldsymbol{v}_h - \boldsymbol{\lambda}_h) + (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{h}_t\right]) \\
&+ \frac{1}{2}\mu_g^T \boldsymbol{\Sigma}_{gt}\boldsymbol{\Sigma}_{gt}^T \boldsymbol{\mu}_g) \\
&\times exp((\boldsymbol{u}_g^T \boldsymbol{\Sigma}_{gh} + \boldsymbol{u}_h^T)\boldsymbol{\mu}_h \\
&+ \sum_{i=1}^{H} \frac{\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T(\boldsymbol{\Sigma}_{gh}^T \boldsymbol{u}_g + \boldsymbol{u}_h)}{(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T(\boldsymbol{\Sigma}_{gh}^T \boldsymbol{u}_g + \boldsymbol{u}_h))} \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1}(\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)(\boldsymbol{h}_t - \boldsymbol{\mu}_h) \\
&- \sum_{i=1}^{H}(v_{hi} - \lambda_{hi}) ln(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T(\boldsymbol{\Sigma}_{gh}^T \boldsymbol{u}_g + \boldsymbol{u}_h)))
\end{aligned}
\tag{E.15}
$$

Keeping E.15 in mind bond prices are described similar to the ATSM with constant volatility by the exponential affine expression:

$$
\begin{aligned}
P(t,T) &= exp(A_\tau + \mathbf{B}_\tau^T \boldsymbol{F}_t) \\
&= exp(A_\tau + \mathbf{B}_{g\tau}^T \boldsymbol{g}_t + \boldsymbol{B}_{h\tau}^T \boldsymbol{h}_t)
\end{aligned}
\tag{E.16}
$$

with maturity dependent coefficients $A_\tau, \boldsymbol{B}_{g\tau}$ and $\boldsymbol{B}_{h\tau}$. For a one-period bond with time to maturity $\tau = 1$ the price $P(t, t+1)$ is:

$$
P(t, t+1) = \mathbb{E}_t\left[exp(-r_t)P(t+1, t+1)\right] \tag{E.17}
$$

where the $\tau = 0$ bond has a price of $P(t+1, t+1) = 1$, such that we can write with respect to 6.47 and 6.48:

$$
P(t, t+1) = exp(-\delta_0 - \boldsymbol{\delta}_{1g}^T \boldsymbol{g}_t - \boldsymbol{\delta}_{1h}^T \boldsymbol{h}_t) \tag{E.18}
$$

According to E.18 it can be written $A_1 = \delta_0, \boldsymbol{B}_{g1} = \boldsymbol{\delta}_{1g}$ and $\boldsymbol{B}_{h1} = \boldsymbol{\delta}_{1h}$. For $\tau > 1$ with $T > t+1$ the following recursive pricing scheme can be applied:

$$
\begin{aligned}
P(t,T) &= \mathbb{E}_t\left[exp(-r_t)P(t+1, T-1)\right] \\
&= \mathbb{E}_t\left[exp(-\delta_0 - \boldsymbol{\delta}_{1g}^T \boldsymbol{g}_t - \boldsymbol{\delta}_{1h}\boldsymbol{h}_t) exp(A_{\tau-1} + \boldsymbol{B}_{g\tau-1}\boldsymbol{g}_{t+1} + \boldsymbol{B}_{h\tau-1}\boldsymbol{h}_{t+1})\right] \\
&= exp(-\delta_0 - \boldsymbol{\delta}_{1g}^T \boldsymbol{g}_t - \boldsymbol{\delta}_{1h}^T \boldsymbol{h}_t + A_{\tau-1})\, \mathbb{E}_t\left[exp(\boldsymbol{B}_{g\tau-1}^T \boldsymbol{g}_{t+1} + \boldsymbol{B}_{h\tau-1}\boldsymbol{h}_{t+1})\right]
\end{aligned}
\tag{E.19}
$$

defining $\boldsymbol{u}^T = [\boldsymbol{B}_{g\tau-1}, \boldsymbol{B}_{h\tau-1}]$ in E.19 we get with E.15:

$$
\begin{aligned}
P(t,T) = & \; exp(-\delta_0 - \boldsymbol{\delta}_{1g}^T \boldsymbol{g}_t - \boldsymbol{\delta}_{1h}^T \boldsymbol{h}_t + A_{\tau-1}) \\
& \times exp((\boldsymbol{\mu}_g - \boldsymbol{\lambda}_g + (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g)\boldsymbol{g}_t + (\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh})\boldsymbol{h}_t \\
& - \boldsymbol{\Sigma}_{gh} \left[(\mathbf{I}_{H\times H} + \boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h + \boldsymbol{\Sigma}_h(\boldsymbol{v}_h - \boldsymbol{\lambda}_h) + (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{h}_t \right])^T \boldsymbol{B}_{g\tau-1} \\
& + \frac{1}{2}\boldsymbol{B}_{g\tau-1}^T \boldsymbol{\Sigma}_{gt} \boldsymbol{\Sigma}_{gt}^T \boldsymbol{B}_{g\tau-1}) \\
& \times exp((\boldsymbol{B}_{g\tau-1}^T \boldsymbol{\Sigma}_{gh} + \boldsymbol{B}_{h\tau-1}^T)\boldsymbol{\mu}_h \\
& + \sum_{i=1}^{H} \frac{\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1})}{(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1}))} \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1} (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)(\boldsymbol{h}_t - \boldsymbol{\mu}_h) \\
& - \sum_{i=1}^{H} (v_{hi} - \lambda_{hi}) ln(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1}))) \\
= & \; exp(-\delta_0 + A_{\tau-1} + \boldsymbol{\mu}_g^T \boldsymbol{B}_{t\tau-1} + \frac{1}{2}\boldsymbol{B}_{g\tau-1}^T \boldsymbol{\Sigma}_{gt} \boldsymbol{\Sigma}_{gt}^T \boldsymbol{B}_{g\tau} \\
& + \boldsymbol{\mu}_h^T \left[\boldsymbol{B}_{h\tau-1} + (\boldsymbol{\Psi}_h^T - \boldsymbol{\Lambda}_h^T)\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} \right] + (\boldsymbol{\mu}_g^T - \boldsymbol{\lambda}_g^T)\boldsymbol{B}_{g\tau-1} - (\boldsymbol{v}_h - \boldsymbol{\lambda}_h)\boldsymbol{\Sigma}_h^T \boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} \\
& - \sum_{i=1}^{H} (v_{hi} - \lambda_{hi}) ln(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1})) \\
& - \sum_{i=1}^{H} \frac{\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1})}{(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1}))} \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1} (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h \\
& + \left[\boldsymbol{B}_{g\tau-1}^T (\boldsymbol{\Psi}_g - \boldsymbol{\Lambda}_g) - \boldsymbol{\delta}_{1g}^T \right] \boldsymbol{g}_t \\
& + [\sum_{i=1}^{H} \frac{\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1})}{(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1}))} \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1} (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h) \\
& \quad + \boldsymbol{B}_{g\tau-1}^T \left[(\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh}) - \boldsymbol{\Sigma}_{gh}(\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h) \right] - \boldsymbol{\delta}_{1h}^T \\
& \quad + \frac{1}{2}(\mathbf{I}_{H\times H} \otimes \boldsymbol{B}_{g\tau-1})^T \boldsymbol{\Sigma}_g \boldsymbol{\Sigma}_g^T (\boldsymbol{e}_H \otimes \boldsymbol{B}_{g\tau-1})]\boldsymbol{h}_t)
\end{aligned}
\tag{E.20}
$$

such that the constant A_τ and the bond loadings with respect to the latent yield and heteroscedasticity factors $\boldsymbol{B}_{g\tau}$ and $\boldsymbol{B}_{h\tau}$ can be written as:

$$\begin{aligned}
A_\tau = & -\delta_0 + A_{\tau-1} + \boldsymbol{\mu}_g^T \boldsymbol{B}_{t\tau-1} + \frac{1}{2}\boldsymbol{B}_{g\tau-1}^T \boldsymbol{\Sigma}_{gt} \boldsymbol{\Sigma}_{gt}^T \boldsymbol{B}_{g\tau} \\
& + \boldsymbol{\mu}_h^T \left[\boldsymbol{B}_{h\tau-1} + (\boldsymbol{\Psi}_h^T - \boldsymbol{\Lambda}_h^T)\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1}\right] + (\boldsymbol{\mu}_g^T - \boldsymbol{\lambda}_g^T)\boldsymbol{B}_{g\tau-1} - (\boldsymbol{v}_h - \boldsymbol{\lambda}_h)\boldsymbol{\Sigma}_h^T \boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} \\
& - \sum_{i=1}^{H} (v_{hi} - \lambda_{hi}) ln(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1})) \\
& - \sum_{i=1}^{H} \frac{\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1})}{(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1}))} \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1} (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\boldsymbol{\mu}_h
\end{aligned} \tag{E.21}$$

$$\boldsymbol{B}_{g\tau} = -\boldsymbol{\delta}_{1g} + (\boldsymbol{\Psi}_g^T - \boldsymbol{\Lambda}_g^T)\boldsymbol{B}_{g\tau-1} \tag{E.22}$$

$$\begin{aligned}
\boldsymbol{B}_{h\tau} = & [\sum_{i=1}^{H} \frac{\boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1})}{(1 - \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^T (\boldsymbol{\Sigma}_{gh}^T \boldsymbol{B}_{g\tau-1} + \boldsymbol{B}_{h\tau-1}))} \boldsymbol{\delta}_i^T \boldsymbol{\Sigma}_h^{-1} (\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h) \\
& + \boldsymbol{B}_{g\tau-1}^T \left[(\boldsymbol{\Psi}_{gh} - \boldsymbol{\Lambda}_{gh}) - \boldsymbol{\Sigma}_{gh}(\boldsymbol{\Psi}_h - \boldsymbol{\Lambda}_h)\right] - \boldsymbol{\delta}_{1h}^T \\
& + \frac{1}{2}(\mathbf{I}_{H\times H} \otimes \boldsymbol{B}_{g\tau-1})^T \boldsymbol{\Sigma}_g \boldsymbol{\Sigma}_g^T (\boldsymbol{e}_H \otimes \boldsymbol{B}_{g\tau-1})]^T
\end{aligned} \tag{E.23}$$

where $\mathbf{I}_{H\times H}$ is the $H \times H$ identity matrix and $\boldsymbol{e}_H$ is the $H \times 1$ column vector of ones. Bond yields are determined by:

$$\begin{aligned}
y(t,T) &= -\frac{1}{\tau} ln(P(t,T)) \\
&= a_\tau + \boldsymbol{b}_{g\tau}^T \boldsymbol{g}_t + \boldsymbol{b}_{h\tau}^T \boldsymbol{h}_t
\end{aligned} \tag{E.24}$$

with $a_\tau = -A_\tau/\tau, \boldsymbol{b}_{g,\tau} = -\boldsymbol{B}_{g,\tau}/\tau$ and $\boldsymbol{b}_{h\tau} = -\boldsymbol{B}_{h\tau}/\tau$.

E.4 Comparison of term-structure models' in-sample fit

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
	In-Sample-Fitting RMSYE Netherlands 03/2005 - 02/2014									**In-Sample-Fitting MAD Netherlands 03/2005 - 02/2014**								
12	0.072	0.255	0.300	0.194	0.058	0.553	0.021	0.155	0.245	0.056	0.198	0.228	0.155	0.042	0.496	0.013	0.122	0.237
24	0.110	0.193	0.166	0.064	0.045	0.407	0.000	0.001	0.000	0.098	0.165	0.121	0.052	0.035	0.376	0.000	0.001	0.000
36	0.090	0.157	0.064	0.010	0.023	0.246	0.026	0.172	0.179	0.078	0.129	0.045	0.008	0.019	0.229	0.020	0.136	0.170
60	0.014	0.075	0.029	0.020	0.013	0.123	0.040	0.173	0.337	0.011	0.064	0.020	0.016	0.010	0.117	0.033	0.144	0.330
120	0.091	0.108	0.125	0.124	0.015	0.124	0.015	0.058	0.504	0.070	0.092	0.100	0.097	0.012	0.095	0.012	0.047	0.476
	In-Sample-Fitting RMSYE Portugal 03/2005 - 02/2014									**In-Sample-Fitting MAD Portugal 03/2005 - 02/2014**								
12	0.372	0.789	1.010	1.069	1.055	1.004	0.099	0.338	1.863	0.219	0.538	0.627	0.703	0.653	0.627	0.061	0.191	1.791
24	0.141	0.555	0.488	0.477	0.431	1.190	0.000	0.002	0.000	0.079	0.438	0.331	0.353	0.285	0.794	0.000	0.001	0.000
36	0.104	0.446	0.323	0.234	0.172	1.220	0.088	0.456	2.092	0.069	0.349	0.208	0.174	0.118	0.782	0.063	0.250	2.016
60	0.067	0.178	0.170	0.050	0.041	1.170	0.124	0.676	2.116	0.046	0.136	0.113	0.034	0.027	0.709	0.086	0.371	2.029
120	0.360	0.489	0.155	0.258	0.258	0.960	0.037	0.240	0.431	0.291	0.347	0.101	0.194	0.185	0.660	0.027	0.133	0.411

Table E.1: In-sample RMSYEs and MADs for the Netherlands and Portugal - Estimation period 03/2005 - 02/2014.

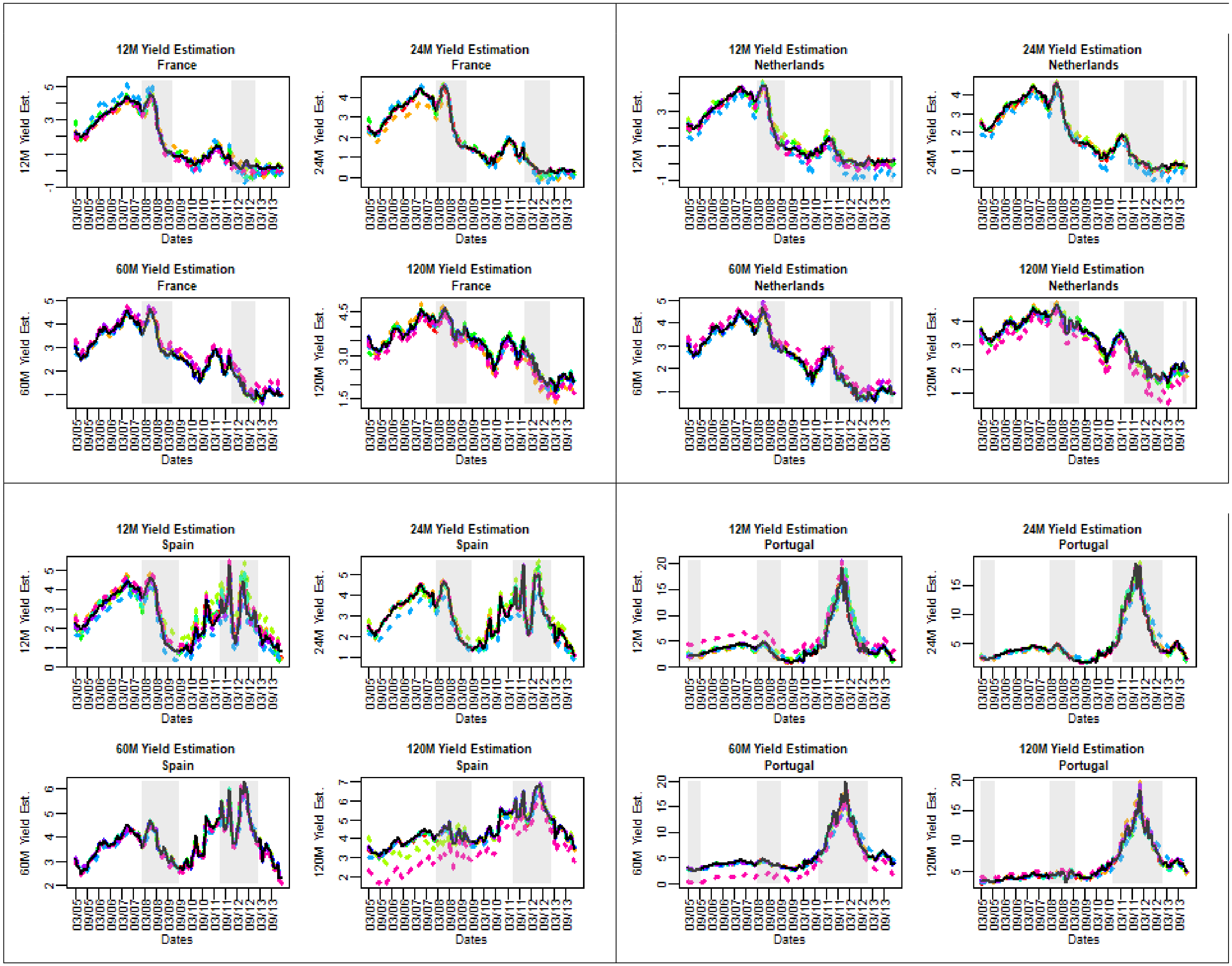


Table E.2: Observed and estimated yields with $\tau = 12, 24, 60$ and 120 month maturities for France, Netherlands, Spain and Portugal between 03/2005 and 02/2014

E.5 Comparison of term-structure models' out-of-sample point forecasts

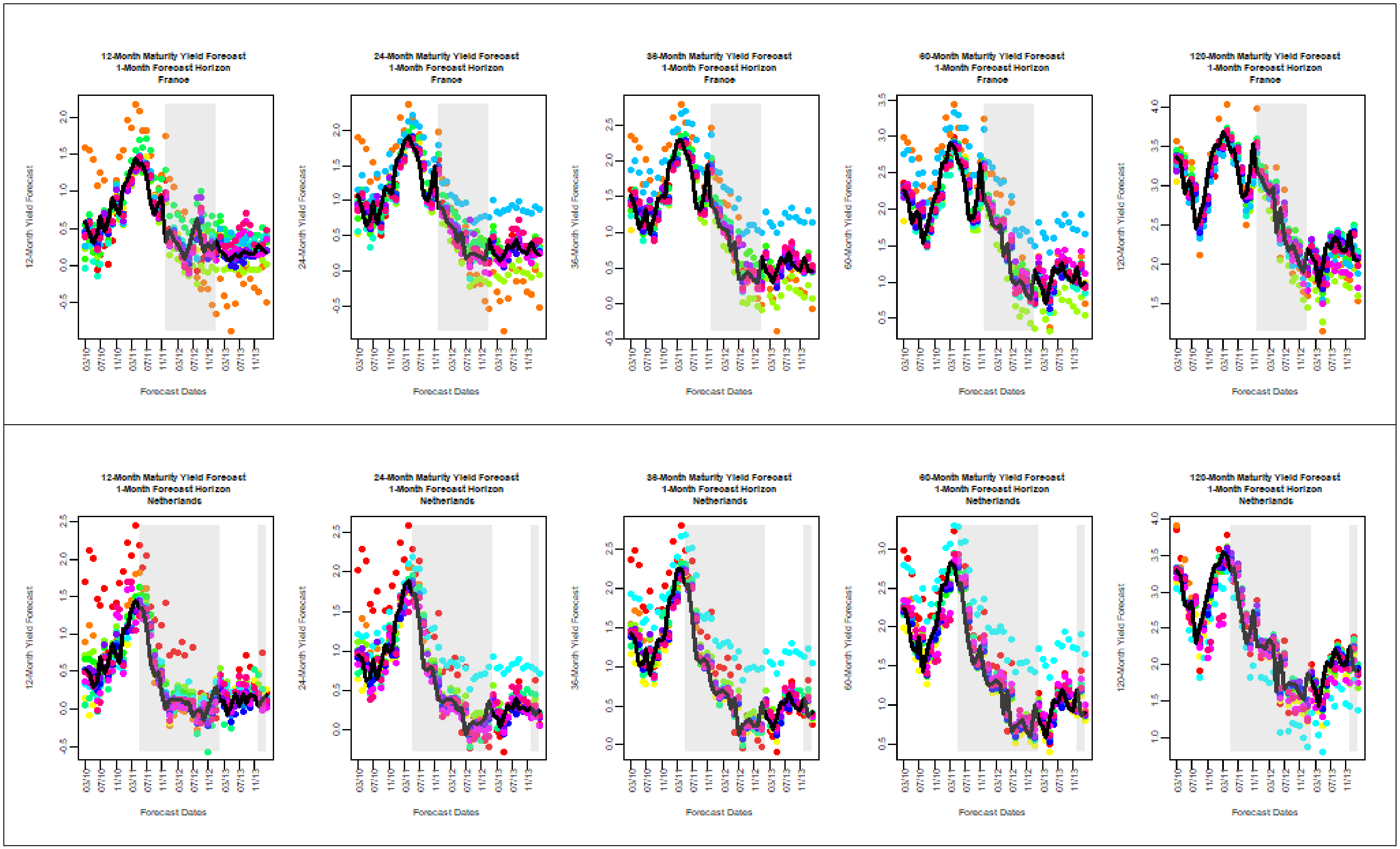


Table E.3: Observed and 1-month ahead predicted yields with $\tau = 12, 24, 36, 60$ and 120 month maturities for France and Netherlands between 03/2010 and 02/2014

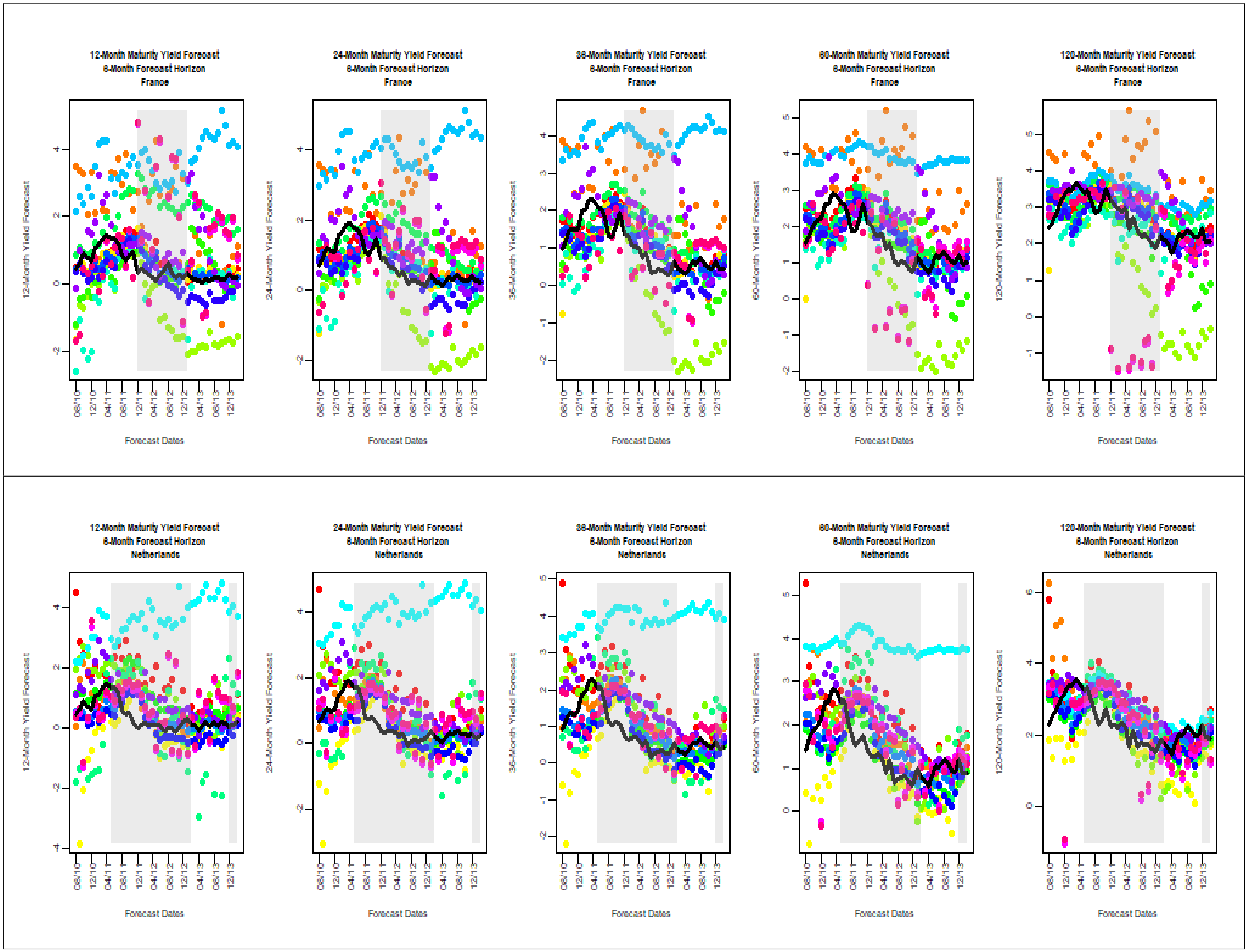


Table E.4: Observed and 6-month ahead predicted yields with $\tau = 12, 24, 36, 60$ and 120 month maturities for France and Netherlands between 03/2010 and 02/2014

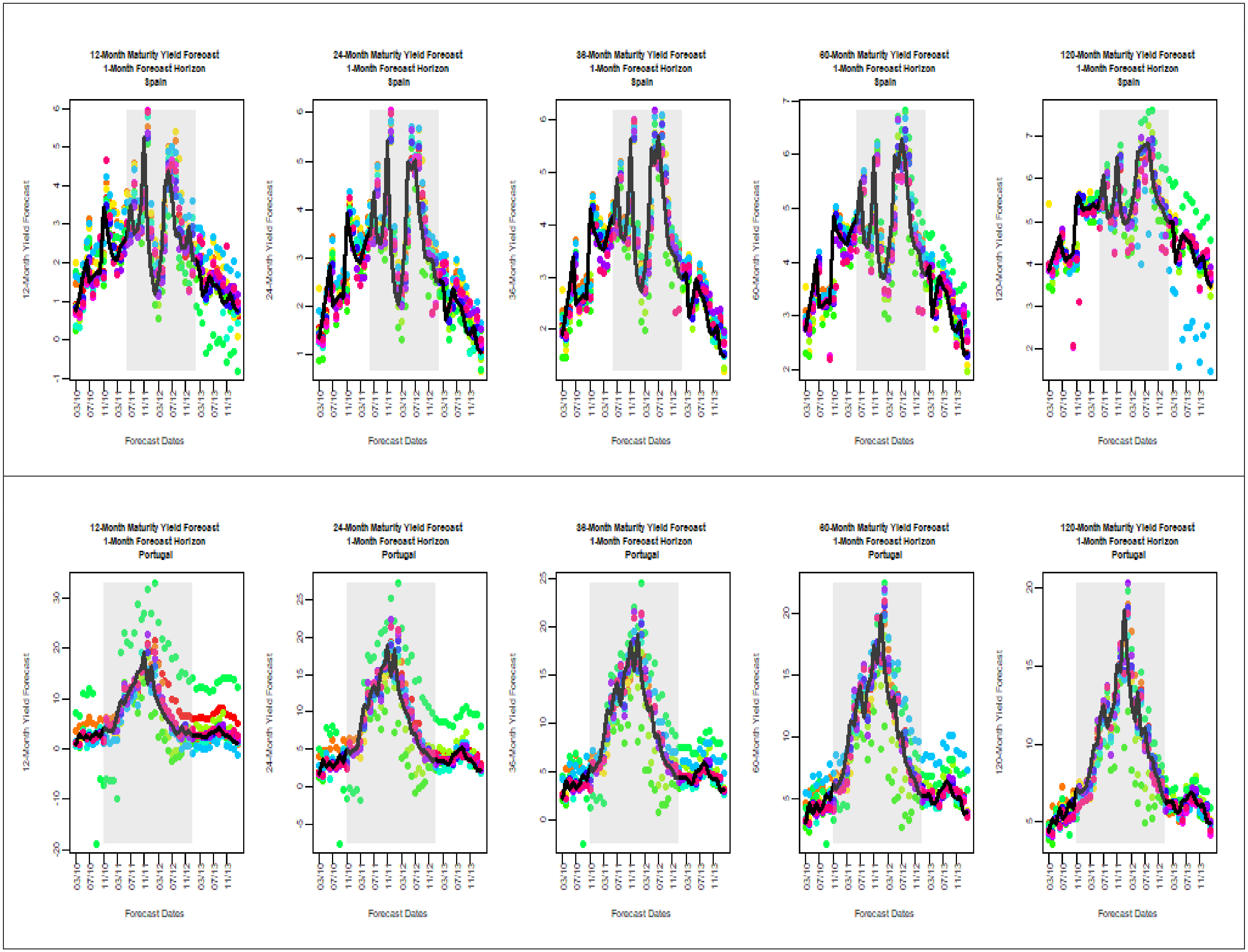


Table E.5: Observed and 1-month ahead predicted yields with $\tau = 12, 24, 36, 60$ and 120 month maturities for Spain and Portugal between 03/2010 and 02/2014

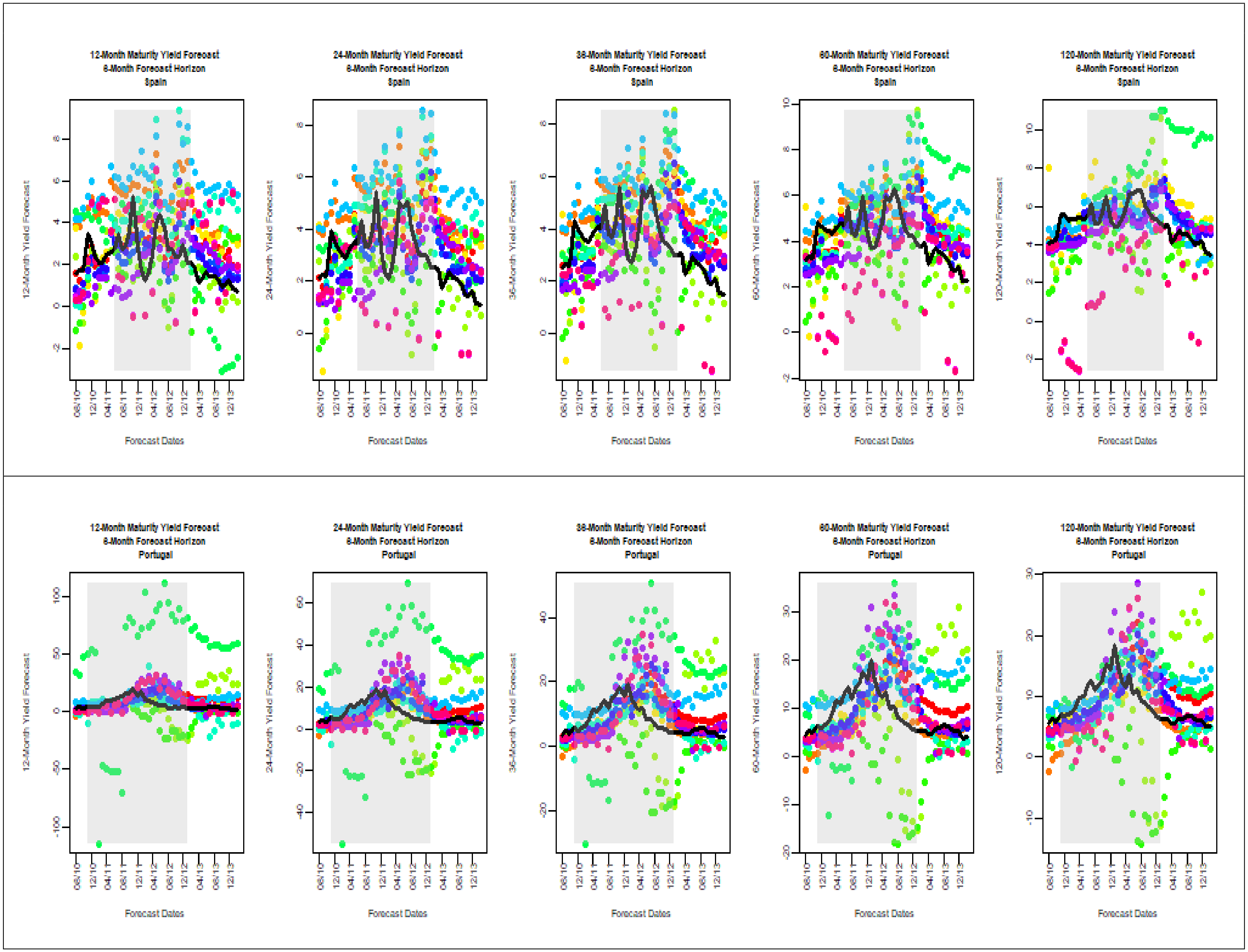


Table E.6: Observed and 6-month ahead predicted yields with $\tau = 12, 24, 36, 60$ and 120 month maturities for Spain and Portugal between 03/2010 and 02/2014

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	RW	Slope Regression	VAR[1]	PC-3-AR	PC-6-AR	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
Panel A: 1-month ahead RMSFYEs for France														
12	0.267	0.638	0.217	0.282	0.245	1.474	0.179	0.188	0.204	0.218	0.241	0.348	0.263	0.223
24	0.241	0.593	0.217	0.291	0.235	1.145	0.191	0.197	0.214	0.218	0.234	0.302	0.255	0.390
36	0.237	0.566	0.225	0.303	0.236	0.947	0.203	0.208	0.225	0.224	0.235	0.285	0.252	0.483
60	0.238	0.548	0.231	0.314	0.236	0.835	0.212	0.216	0.232	0.236	0.238	0.277	0.253	0.536
120	0.235	0.517	0.233	0.325	0.237	0.782	0.216	0.219	0.231	0.243	0.236	0.272	0.251	0.493
Panel B: 2-month ahead RMSFYEs for France														
12	0.308	0.955	0.279	0.492	0.356	1.437	0.245	0.264	0.331	0.584	0.608	0.647	0.424	0.600
24	0.295	0.901	0.290	0.511	0.341	1.104	0.259	0.273	0.342	0.474	0.499	0.538	0.399	0.841
36	0.301	0.881	0.307	0.530	0.338	0.923	0.277	0.288	0.356	0.429	0.448	0.487	0.387	0.931
60	0.311	0.867	0.320	0.549	0.337	0.832	0.292	0.301	0.365	0.430	0.446	0.458	0.380	0.952
120	0.309	0.820	0.323	0.570	0.336	0.777	0.297	0.304	0.357	0.466	0.472	0.442	0.372	0.862
Panel C: 6-month ahead RMSFYEs for France														
12	0.502	1.917	0.531	1.469	0.779	0.952	0.418	0.562	1.036	1.748	1.781	1.783	1.145	3.186
24	0.513	1.874	0.566	1.501	0.739	0.796	0.453	0.571	1.047	1.422	1.460	1.452	1.038	3.271
36	0.536	1.887	0.597	1.533	0.725	0.808	0.486	0.584	1.053	1.239	1.270	1.271	0.965	3.187
60	0.556	1.895	0.613	1.574	0.724	0.844	0.511	0.590	1.039	1.188	1.214	1.155	0.906	3.000
120	0.549	1.824	0.600	1.632	0.734	0.790	0.515	0.578	0.990	1.295	1.306	1.088	0.859	2.712
Panel C: 12-month ahead RMSFYEs for France														
12	0.748	2.880	0.955	3.087	1.677	0.849	0.597	1.451	0.974	1.392	1.396	3.282	2.319	7.064
24	0.809	2.840	1.044	3.166	1.414	1.122	0.670	1.348	0.987	1.237	1.245	2.707	2.050	6.789
36	0.853	2.856	1.101	3.236	1.280	1.310	0.715	1.261	0.997	1.150	1.157	2.364	1.863	6.405
60	0.878	2.858	1.121	3.323	1.207	1.385	0.734	1.169	0.992	1.084	1.088	2.128	1.708	5.916
120	0.853	2.765	1.085	3.445	1.186	1.276	0.717	1.081	0.952	1.029	1.033	1.982	1.596	5.350

Table E.7: Out-of-sample RMSFYEs for France - Forecasting period 03/2010 - 02/2014.

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	RW	Slope Regression	VAR[1]	PC-3-AR	PC-6-AR	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
Panel A: 1-month ahead RMSFYEs for Netherlands														
12	0.205	0.738	0.240	0.262	0.251	0.363	0.176	0.184	0.184	0.245	0.287	0.214	0.271	0.210
24	0.207	0.668	0.225	0.240	0.239	0.357	0.178	0.184	0.188	0.222	0.258	0.199	0.248	0.395
36	0.213	0.618	0.223	0.235	0.235	0.340	0.185	0.190	0.196	0.217	0.242	0.197	0.239	0.494
60	0.221	0.573	0.229	0.238	0.236	0.324	0.198	0.201	0.208	0.239	0.255	0.205	0.242	0.562
120	0.228	0.532	0.242	0.252	0.245	0.315	0.210	0.212	0.218	0.275	0.278	0.215	0.247	0.555
Panel B: 2-month ahead RMSFYEs for Netherlands														
12	0.319	0.919	0.312	0.403	0.412	0.512	0.251	0.270	0.272	0.378	0.420	0.348	0.494	0.650
24	0.321	0.849	0.311	0.385	0.392	0.511	0.260	0.275	0.281	0.338	0.379	0.315	0.442	0.891
36	0.325	0.798	0.316	0.378	0.379	0.494	0.270	0.282	0.293	0.321	0.351	0.306	0.414	0.981
60	0.331	0.752	0.327	0.377	0.367	0.474	0.282	0.290	0.307	0.330	0.349	0.306	0.406	1.009
120	0.332	0.702	0.350	0.388	0.368	0.458	0.291	0.297	0.316	0.347	0.355	0.309	0.397	0.922
Panel C: 6-month ahead RMSFYEs for Netherlands														
12	0.756	1.328	0.590	1.139	0.980	1.145	0.479	0.571	0.758	1.018	1.059	0.820	1.541	3.384
24	0.759	1.273	0.606	1.120	0.933	1.128	0.512	0.582	0.781	0.903	0.944	0.706	1.374	3.471
36	0.760	1.235	0.624	1.102	0.894	1.096	0.536	0.595	0.794	0.841	0.872	0.664	1.280	3.382
60	0.757	1.201	0.649	1.083	0.856	1.053	0.556	0.604	0.797	0.826	0.847	0.652	1.225	3.191
120	0.738	1.153	0.727	1.079	0.846	1.005	0.563	0.602	0.779	0.851	0.862	0.640	1.147	2.872
Panel C: 12-month ahead RMSFYEs for Netherlands														
12	1.408	1.690	1.049	3.051	3.574	1.933	0.662	1.066	1.154	2.054	2.102	1.298	3.368	7.616
24	1.396	1.629	1.069	3.016	3.716	1.893	0.709	1.016	1.150	1.687	1.733	1.068	3.027	7.311
36	1.377	1.588	1.091	2.959	3.814	1.836	0.739	0.975	1.140	1.445	1.481	0.970	2.831	6.895
60	1.344	1.548	1.121	2.863	3.899	1.761	0.756	0.932	1.118	1.351	1.387	0.927	2.694	6.374
120	1.282	1.478	1.219	2.744	3.983	1.666	0.749	0.886	1.075	1.432	1.450	0.885	2.480	5.746

Table E.8: Out-of-sample RMSFYEs for Netherlands - Forecasting period 03/2010 - 02/2014.

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	RW	Slope Regression	VAR[1]	PC-3-AR	PC-6-AR	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
Panel A: 1-month ahead RMSFYEs for Spain														
12	0.740	0.930	1.055	0.713	0.834	0.996	0.700	0.708	0.733	0.942	0.927	1.038	0.814	1.110
24	0.717	0.861	0.942	0.705	0.817	1.017	0.686	0.698	0.726	0.865	0.855	0.874	0.767	0.988
36	0.698	0.810	0.871	0.696	0.802	0.977	0.674	0.687	0.714	0.811	0.805	0.813	0.733	0.904
60	0.672	0.764	0.814	0.679	0.778	0.909	0.653	0.665	0.691	0.776	0.769	0.802	0.702	0.843
120	0.637	0.715	0.764	0.649	0.743	0.848	0.619	0.631	0.655	0.766	0.763	0.805	0.661	0.937
Panel B: 2-month ahead RMSFYEs for Spain														
12	1.063	1.407	1.313	1.114	1.279	1.470	1.034	1.068	1.078	1.454	1.448	1.585	1.349	1.794
24	1.029	1.308	1.214	1.096	1.259	1.433	1.002	1.039	1.061	1.350	1.347	1.299	1.235	1.608
36	0.998	1.228	1.147	1.077	1.238	1.357	0.973	1.012	1.038	1.298	1.297	1.211	1.151	1.472
60	0.957	1.151	1.087	1.050	1.205	1.260	0.936	0.973	1.001	1.294	1.292	1.226	1.082	1.367
120	0.905	1.071	1.030	1.007	1.160	1.171	0.886	0.922	0.949	1.353	1.353	1.278	1.012	1.330
Panel C: 6-month ahead RMSFYEs for Spain														
12	1.319	2.762	1.958	1.690	1.729	2.367	1.127	1.312	1.562	2.326	2.307	2.141	2.918	3.894
24	1.323	2.556	1.902	1.801	1.777	2.243	1.152	1.325	1.562	2.122	2.110	1.872	2.614	3.518
36	1.317	2.377	1.862	1.882	1.806	2.111	1.163	1.322	1.542	2.086	2.079	2.006	2.356	3.239
60	1.293	2.195	1.823	1.939	1.820	1.959	1.154	1.298	1.495	2.240	2.233	2.305	2.147	3.004
120	1.237	2.018	1.786	1.945	1.820	1.818	1.117	1.249	1.426	2.579	2.577	2.610	1.973	2.715
Panel C: 12-month ahead RMSFYEs for Spain														
12	1.991	4.647	3.458	3.129	1.970	3.360	1.396	2.491	1.979	3.311	3.299	3.288	6.200	6.592
24	1.965	4.280	3.442	3.416	2.045	3.169	1.433	2.337	1.989	2.956	2.949	3.534	5.414	5.990
36	1.932	3.958	3.432	3.631	2.100	2.983	1.445	2.225	1.975	2.812	2.808	3.841	4.737	5.539
60	1.876	3.621	3.421	3.823	2.155	2.773	1.437	2.125	1.937	2.887	2.881	4.179	4.190	5.137
120	1.785	3.302	3.428	3.952	2.219	2.566	1.405	2.029	1.881	3.174	3.174	4.492	3.793	4.635

Table E.9: Out-of-sample RMSFYEs for Spain - Forecasting period 03/2010 - 02/2014.

τ	Indep. AF-DNS	Corr. AF-DNS	Indep. DNS	Corr. DNS	MF-DNS	Vasicek-3-Factor	RW	Slope Regression	VAR[1]	PC-3-AR	PC-6-AR	Latent ATSM	MF-ATSM	Stoch. Volatility ATSM
Panel A: 1-month ahead RMSFYEs for Portugal														
12	4.082	2.767	1.961	2.388	3.491	4.283	1.848	1.933	2.305	2.153	2.156	11.910	2.095	2.611
24	3.223	2.468	1.912	2.328	3.597	3.994	1.756	1.832	2.213	2.042	2.049	9.596	1.921	2.226
36	2.786	2.299	1.885	2.287	3.633	3.713	1.707	1.774	2.149	1.972	1.979	8.203	1.822	2.252
60	2.531	2.152	1.847	2.217	3.580	3.444	1.661	1.717	2.074	1.900	1.903	7.220	1.752	2.511
120	2.335	2.024	1.769	2.104	3.408	3.236	1.592	1.636	1.968	1.798	1.800	6.499	1.673	2.319
Panel B: 2-month ahead RMSFYEs for Portugal														
12	5.194	3.488	2.502	4.367	6.862	5.138	2.237	2.508	3.183	3.110	3.110	23.355	2.848	2.741
24	4.238	3.085	2.483	4.227	6.906	4.785	2.164	2.402	3.062	2.923	2.931	18.786	2.538	2.750
36	3.698	2.841	2.472	4.097	6.869	4.461	2.122	2.321	2.956	2.793	2.800	16.027	2.363	3.025
60	3.393	2.642	2.438	3.921	6.711	4.144	2.077	2.225	2.828	2.660	2.662	14.071	2.247	3.397
120	3.149	2.470	2.346	3.696	6.394	3.891	2.000	2.106	2.676	2.493	2.498	12.641	2.152	3.131
Panel C: 6-month ahead RMSFYEs for Portugal														
12	9.356	6.211	5.364	15.766	12.656	7.566	5.180	7.169	10.962	9.290	9.294	66.986	10.008	8.739
24	8.362	5.883	5.208	15.223	12.744	7.141	5.053	7.210	10.851	9.240	9.250	53.934	8.548	9.160
36	7.731	5.651	5.070	14.755	12.759	6.760	4.939	7.138	10.617	9.104	9.113	46.027	7.653	9.412
60	7.331	5.426	4.886	14.192	12.658	6.353	4.770	6.866	10.170	8.823	8.827	40.388	7.023	9.519
120	6.923	5.182	4.629	13.481	12.332	5.977	4.510	6.384	9.495	8.362	8.367	36.258	6.518	8.876
Panel C: 12-month ahead RMSFYEs for Portugal														
12	14.045	10.647	7.354	25.910	9.770	9.542	8.058	13.471	24.508	13.899	13.950	127.804	21.882	19.419
24	13.183	10.562	7.338	25.980	9.774	9.190	8.129	13.909	24.772	13.729	13.778	103.003	17.941	19.466
36	12.601	10.435	7.261	25.943	9.735	8.849	8.092	13.823	24.624	13.444	13.480	87.963	15.657	19.313
60	12.187	10.234	7.078	25.701	9.616	8.441	7.905	13.363	23.942	12.956	12.977	77.218	14.137	18.897
120	11.661	9.912	6.757	25.032	9.348	8.003	7.509	12.506	22.597	12.223	12.245	69.335	12.979	17.663

Table E.10: Out-of-sample RMSFYEs for Portugal - Forecasting period 03/2010 - 02/2014.

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