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IF NUMBERS ARE RIGHT: ON THE USE OF RECKONING IN THE ISLAMIC MIDDLE AGE

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In the year 1615 the grape-harvest in Southern Austria had broken all records. John Kepler, the famous astronomer, who was passing that autumn in Linz, took advantage of the lucky situation. At the local wine-market he made a strange observation. In order to determine the content of their barrels the wine-merchants did not employ - as was usual everywhere - a standard jug, but pushed a kind of measure-stick through the upper hole of the barrel until it hit the bottom. From the wet mark on the stick they calculated directly the amount of wine contained in the barrel and calculated the price.

Kepler is said to have locked himself up for three days in his appartment in Linz to ponder on the cubic measurement of the wine-barrels. Shortly after, in the same year, he published his 'Nova Stereometrica Doliorum Vinariorium' the 'New Stereometry of the Wine Barrels', where 92 algorithms for the cubation of rotating barrels are explained.

John Kepler was not the first scholar to whom drinking presented a mathematical riddle. Almost exactly 600 years earlier the Islamic mathematician ᶜAbd al-qāhir al-Baghdādi dealt - albeit for different reasons - with a problem that had to do with wine. Al-Baghdadi was an eminent all-round scholar who is said to have taught 17 sciences later in his life in his east-persian homeland. Besides his works on the dogmatic movements in Islam and on Islamic law, especially the law of inheritance, only his 'Takmila fī l-ḥisāb' was to exert a lasting influence on the following generations. One chapter of the '*Takmila*' is entitled al-muᶜāmalāt, the inter-relationships of men.[1] There he touches upon a problem that had arisen from the quranic prohibition against drinking Khamr.

It had been unveiled to the Prophet in one of the last suras and the Islamic jurists not only wrote this prohibition into the sharīᶜa, but concluded pars pro toto the prohibition against all fermented drinks, as those, made form dates, honey, wheat or barley. In addition, the production, the trade and even the medicinal application as ointment or clyster was forbidden.

The juridical arguments were supplied through analogy to the religiously legitimated facts of the Qur'ān and the traditions on the life of the Prophet. Analogies of this kind were constructed so as to legalize the prohibition and make its observance possible. But the answer to a central question could not be found on religious grounds. If asked, what actually was to be regarded as wine and which kind of preparation turned drinks made from fruits or cereals into the forbidden khamr, the systematists of the sharīᶜa has to look for help from a science which was largely

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1- ᶜAbd al-qāhir al-Baghdādī : *at-Takmila fī l-ḥisāb*. Maᶜa risāla lahu fī l-misāḥa. Taḥqīq wa-dirāsa muqarina: Ahmed Salim Saᶜidan. Kuwait 1406/1986, pp. 247-296.

independant from God Almighty. The most general answer, that all fermented drinks are forbidden, was insufficient as long as they could not be compared with the permitted ones. The list of beverages in the Ancient Orient contained a variety of fruity, sourish and lightly fermented drinks which had to be allocated to one class or the other. Was not the Prophet being served in the morning a grape-juice which his wifes had prepared the evening before in a skin bag in order for delaying fermentation? In Syria the Arabs became acquainted with a sort of sirup which had the consistence of a camel's saliva and which was called ṭila^c accordingly. The peculiar method which produced this ṭila^c, became the dominant criterion of Islamic law for declaring juices from potted or squeezed-out fruits permitted or forbidden. In order to make ṭila^c, the juice had to be cooked and condensed until the initial volume was reduced to one third. Therefore this type of sirup was also called - besides from ṭila^c and ^{c}aṣīr - al-muthallath or 'tripled'. This production method offered the best guarantee of forestalling post-fermentation. And - above all - it offered the possibility of controlling production. However, one had to know how to reckon.

Until the 10th century regulations were inserted into the canonical law books which simply tended to protect the believer against the diabolic effects of alcohol. The correct interpretations of the traditions and the One-Third-Clause seemed to be sufficient. But the first to analyse the khamr problem more thoroughly and to develop a control procedure was our mathematician from Baghdad, ^{c}Abd al-Qāhir.[2]

Since one part of the juice evaporated and anoher part - to keep the sirup pure - was skimmed off during the cooking process, one had to deal with four different quantities. If we translate the verbal explanations of ^{c}Abd al-Qāhir into a modern notation, we can call the initial quantity M_0, the evaporated quantity M_1, the qauntity skimmed off M_2 and the remainig third M. The condition of reduction to one third can then be expressed by the following proportion :

$$(M_0 - M_1) : {}^1/_3\, M_0 = (M_0 - M_1 - M_2) : M$$

With this proportion any fourth unknown quantity can be found out if the three others are given. Al-Baghdādī demonstrates all variants among which we also find the one that can be used to examine the declaration of a sirup producer to have condensed his khamr to one third. For if he declares to have produced for example $8\,{}^1/_3$ buckets of ^{c}aṣīr after 6 buckets had evaporated and 4 had been skimmed off, then it can be proven that he had started with at least with 30 buckets of wine-juice. If we insert these quantities into our proportion given above we get

$$(M_0 - 6 - 4) : 3\,{}^1/_3 = (M_0 - 6) : {}^1/_3\, M_0$$

and after transformation

$$M_0^2 - 35\,M + 150 = 0$$

$$M_0 = 17\,{}^1/_2 + (-)\sqrt{625\ /_4} = 30\ (4)$$

A few decades after these still playful considerations by al-Baghdādī, the law had already taken

2- *at- Takmila*, pp. 283-284.

posession of this control procedure. In the first extensive and systematic commentary on hanafit law as-Sarakhsī, a famous law-scholar from Central Asia, followed exactly al-Baghdādī's train of thought. After having exercised all four variant proportions he states verbally "that some of our collegues have solved such problems with the algebraic method from the science of hisab", [3] To make clear his.

But the considerations did not end there: The practice of the producers to dilute their juice with water - in order, of course, to sell a higher quantity of ᶜashir - tore holes into the juridical walls of the experts who had in the meantiome generally accepted al-Bagdadi's method.

A later collegue of as-Sarakhsi, the Egyptian Ibn al-Humām, tried to define the ratio of the evaporation of water to that of wine-juice. He incorporated this relation into our proportion which now grew more complicated but not more exact.[4]

For Ibn al-Humām could not decide if the diluted juice evaporated before the added and - as he called it - 'lighter' water or if it escaped simultaneously.

Unfortunately the historical sources do not tell us enough about the degree to which this method affected the administration of justice. One of the reasons for this silence may have been the deeply rooted moral condamnation of everything that had to do with alcohol. Certainly the scruples to leave such a delicate field to the logic of numbers and proportions did not loosen the historians tongue. This supposition is both grave and near at hand; near at hand because the acceptance of Islamic juridical sources was so strongly dependant on the godly wording of the Qur'ān and the ideal biographies of the Prophet and his companions that even the classic analogy had a hard time being accepted as a means of interpreting the unveiled principles of law; grave it is because it operates with an unspoken image of Islamic legal thought in which there is no room for other than religiously achieved methods.

I do not wish to trace back this supposition, but rather would like to take the opposite direction and to look for further evidence from the science of reckoning that the Islamic society not only was open to, but sometimes even forced to accept the rational power of numbers.

It would certainly be fitting here to discuss the enormously fertile development of practical astronomy which was stimulated by the various qur'ānis descriptions to orientate the religious performances to Mecca, to determine exactly the times of prayer ant to bring the course of day and year into line with the orbit of the sun and the moon. Recent research, however, has shown that Islamic astronomy not only delivered more and more precise solutions to these problems. The scientific process also favoured the tendency to tackle problems which no longer belonged to a Muslim's every day life.

Let us stick then to our search in the lowlands of reckoning and remain in close touch with social concern.

The Qur'ān contains but a few numerical instructions. The most detailed ones are to be found in verses relevant to the law of inheritance.[5] Not without reason. The Islamic rules came to replace a common feature of all Oriental laws of inheritance which regulated the fluctuation of private

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3- as-Sarakhsī, Kitāb al-Mabsuṭ. Bairut 1406/1986, xxiv, p. 12f.

4- Ibn al-Humām: Sharḥ Fatḥ al-Kabir. Bulaq 1318h, viii, p. 168/10f.

5- For verses relevant to farā' id see Sura 4: 7, 11-12, 176; wasaya: 2: 180-182, 240; 5: 106.

property within a patrilinear family with full testamentary freedom. The Prophet Muhammad's exemplary but unsystematic regulations, however, which were elaborated by the exegetes and designers of the Islamic law, shifted the crucial point of property fluctuation from the family towards collective (umma-) beliefs.[6]

By virtue of his or her rank of relation each male or female member of the deceased's family was awarded an absolutely or relatively fixed share and the bequest was limited to one third of the estate.

Nobody, not even the Prophet, could have anticipated that the ten quranic verses which talk about shares and bequests, would give rise to an independant science which combined law and reckoning in a unique manner. The historical and methodical points of departure were some seriously disputed inheritance cases whose solutions caused a collision between revelation and arithmetic.

In the so-called 'pulpit-case', al-minbariya, a Muslim had died and left a widow, a father, a mother and two daughters. According to the judgements of sūras 2,4 and 5 the widow - in absence of children - was entitled to one fourth, the mother to one third and the father to the rest of the inheritance. Surviving children halved the qoute of widow and mother and reduced the share of the father to a fixed sixth. But for two or more daughters the Prophet had fixed in the 4, sūra two third of the inheritance.[7]

Let us look at this constellation in a more comfortable way

W	M	F	[2 D] -		
$\frac{1}{4}$	$\frac{1}{3}$	R	–		-- without children
$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$		-2 daughters→halves
$\Rightarrow\frac{3}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{16}{27}$	$\rightarrow \frac{27}{24}$	- sum of shares surmounts estate (=1)
$\Rightarrow\frac{3}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{16}{27}$	$\rightarrow \frac{27}{27}$	- reduction by way of in-creasing denominator

Thus the divine proscriptions proved to be unrealizable. But a solution had to be found. God's omnipotence did not tolerate gaps in his revelation. So, again, the jurists were forced to leave the wording of the Qur' ān and to put themselves into the hands of arithmetic. Since the sum of all

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6- A condensed presentation of this processes is given in Aaron Skaist: Inheritance Laws and their Social Background, in: JHOS, 95, 1 (Jan. -Mar. 1975), pp. 242-7.

7- For a detailed presentation of this case, see N.J. Coulson: Succession in the Muslim Family. Cambridge 1971, pp. 47-49; David S. Powers: Studies in Qur'an and Ḥadith. The formation of the Islamic Law of Inheritance. Berkeley 1986, pp. 66-67.

shares exceeded the whole, the size of the portions was adjusted to the number of the shares. One simply had to increase the denominator in order to satisfy the different claims respectively.

With this arithmetical trick, called al-^{c}aul, the caliph ^{c}Umar is said to have solved the minbariya and set a precedent for the adjustment of quranic prescriptions. We can easily imagine that al- ^{c}aul did not remain the only tricky method. If we insert, for example, a bequest into the minbariya-case and further assume that the deceased - which happened fairly often - had to pay off debts then we have to deal with a new factor in the equation since the shari^{c}a prescribed that debts take priority of shares.

Being part of the quranic revelation and - at the same time of considerable social relevance? the numerical instructions of the inheritance verses raised the interests of experts. A new and purely Islamic science, ^{c}ilm al-fara' id, was borne. The oldest known work on the calculation of shares is ascribed to M. as-Shaibani, the founder of the Hanafiya law-school in the 9thcentury.[8] And it is significant - on the other hand - that the most eminent text of early Islamic mathematics written some few decades later, the so-called 'Algebra' of M. b. Musa al-Khuwarizmi, was composed for the greater part of artistical inheritance cases. Al-Khuwarizmi transformed these cases into linear and square equations and solved then algebraically. The complementary knowledge the ^{c}ilm al-fara' id required, made it an interdisciplinary science which developed into a student's nightmare. All basic works were written by scholars who were well acquainted with both: fiqh and hisab.[9]

The above mentioned jurist as-Sarakhsi placed an introduction to arithmetic before his chapter on the law of inheritance. He was well aware of the conflict that the different methods of solution had provoked.[10] To make clear his point of view he unfolds with acrobatic skill a most peculiar but extremely suitable case: the case of two inheriting hermaphrodites, khuntha, people who would inherit neither the full male nor the half female but an intermediate share which was not foreseen by the primary legal sources. After several pages filled with sexological and legal arguments on the one hand and with arithmetical proportions on the other he ends up with a categorical decision: legal prescriptions take precedence over numerical laws.[11]

This decision is - in itself - rather insignificant. But the decision was made upon a background in which the classification of sciences acceptable for the Islamic community or not took shape. Mathematics too were subject to the dispute. In the course of the adaption of the classical sciences, mathematics came to be called a foreign,^{c}ajami science. Being represented by translations of the Greek works like those of Euclid, Archimedes, Pappos and Diophant, this science - just like philosophy and metaphysics - was not only regarded as not Arabic but also as not Islamic. The first Islamic encyclopedias of sciences clearly remove these disciplines from the the traditional

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8- an-Nadim, *Kitab al-Fihrist*. (Not dated), p. 257/-2.

9- A detailed list is to be found in Ulrich Rebstock: *Rechnen im islamischen Orient*. Wiesbaden 1991 (forthcoming), Appendice I.

10- as-Sarakhsi, *Kitab al-Mabsut*, xxx, p. 94/5f.

11- *Kitab al-Mabsut*, xxx, p. 99f. For a closer discussion, see Ulrich Rebstock: Mathematische Quellen zur Rechtsgeschichte: Das Problem des Hermaphroditen, in: *Die Welt des Orients*, xx/xxi (1989/90), pp. 100-114.

Islamic sources of knowledge, such as the study of qur'ān and ḥadith. But by the end of the 9th century a distinction was becoming clear though we cannot trace back its pattern. al-Fārābi, for example, removed mathematics from their cognitive context and split them into disciplines which contained theoretical or practical parts.[12] Later authors of classifications, such as Ibn Fariᶜūn, al-Gazzāli, Ibn Khaldūn or Ibn al-Akfānī, emphasized the utility of natural sciences for a pious communal life. Among the physical disciplines which were raised to the rank of useful and independant sciences, are the amazing techniques for moving beaving beavy loads, the transport of water and the fabrication of burning mirrors and pneumatic instruments. Certain mathematical procedures were set off as especially helpful and were recommended to be taught and studied, such as reckoning with Indian ciphers, Ḥisāb al-hindī, which came to replace the Arabic Ḥisāb al-yad, reckoning with unknowns by 'adjusting and 'balancing', al-jabr wa l-muqabala, and the 'reckoning with mistakes', the 'regulafalsorum'[13]. Thus Greek, Mesopotamian and Indian heritage were joined to a very specific Islamic esteem for mathematics which thereby - at least partly - got rid of the brand-mark of heresy.[14]

Had not both, mathematics and physics, supplied the atheistic Greek philosphers with elementary models for the recognition of motion, matter and cohesion of the cosmos? And had they not remained to be the sources of such models during the powerful contoversy between rationalism and Islamic religious orthodoxy, the Muᶜtazila and the ahl as-sunna wa l-jamāᶜa? Although the mathematicians and astronomers were highly respected at the caliphs' courts in the 9th and 10th century, they were surrounded at the same time by an aura of heresy, of suspicion of doubting or limiting God's omnitpotence. With the division of mathematics as reflected by the encyclopedias this aura disappeared and made room for a secular respect and growing appreciation of socially useful techniques of reckoning. How this 'domestication' of mathematics helped to avoid a debacle like the trial of Galileo Galilei is made clear by the words of al-Gazzāli, the most effective religious philosopher of the 11th century. In his opus magnum Ihyā al-ᶜulūm the introduction mentions several times the necessary contribution of reckoning to the communal life of the believers. Al-Gazzāli says:

"Philosphy, however, is no science in itself but consists of four parts; the first part is Geometry, al-handasa, together with Arithmetic, al-ḥisāb, which are both permitted. None is to be kept away from them except those who are in danger of tresspassing the border-line of the blameworthy sciences. For most of those who practise them were led to innovations, bidaᶜ. The weak should therefore be protected against them, just like a boy must be with-held from the river-bank for fear he could fall in, or like a new convert to Islam should be kept away from

12- Matthias Schramm: Theoretische und praktische Disziplin bei al-Farabi, in: *Zeitschrift fur die Geschichte der islamisch-arabischen Wissenschaften*, 3 (1986), pp. 1-55.

13- See for example M. b. Ibrahim al-Ansari al-Akfani: Irshad al-qasid ila asna al-maqasid. (Ed.: ᶜAbdallatif M. al-ᶜAbd), al-Qahira 1398/1978, pp. 134/5ff (Geometry), 149/-4ff(Arithmetic).

14- U. Rebstock : Rechnen, chapter 1.

unbelievers who could divert him from the right believe".[15]

This is not an anathema of mathematics but instead a warning against its ideological dangers. Why the teaching and acceptance of mathematics could not be dispensed with, al-Gazzāli explains elsewhere: Most of the economical interactions, 'buying and selling', al-bai^{c} wa sh-shirā', for example, the calculation of profits, rent, leasehold and loans are regulated by legal prescriptions. There, legal and illegal practises go hand in hand and it is up to the responsible authority to make sure the law is respected. But there again this requires the knowledge of reckoning. Therefore, al-Gazzāli expressly calls hisab a collective obligation. It is a technique whose mastery must be guaranteed everywhere.[16] Al-Gazzāli classifies the fields affected by this obligation as al-mu^{c}āmalat, the 'mutual relations'.

This term, the same that al-Baghdadi had used for his wine-problem, contains a significant double-meening. In the oldest collections of legal traditions, certain fields of study where Muslims enter economic relations among themselves or to God, are already called al-mu^{c}āmalat. There, islamic law intervenes this relation by means of procedural and sometimes quantifying prescriptions, especially when it comes to the fixing of taxes, rents or the profit-rates in capital companies. These fixations appear in the shape of fractions. Hence it must not surprise that a proper literary genre came to deal with it. This genre appropriated the term al-mu^{c}amalat and spread the methods to solve mathematical problems of every-day life under the title hisāb al-mu^{c}amalāt. At the same time when the legal mu^{c}āmalāt were fixed, around the middle of the 9th century, a mathematician of Baghdād, ^{c}Abdalḥamid b. Turk al-Khuttali, composed a Kitab al-mu^{c}amalat, the first represntative of this genre we know about.[17] The oldest preserved text, however, comes from one of the most brilliant Arabic scientists: Ibn al-Haitham. His treatise, written about the turn of the millenium, deals mainly with the role and practise of proportions in al-mu^{c}amalat. He introduces the reader with the assertion, that

"one who is not well versed in this art resembles a man who has lost one of his senses".[18]

The treatise of Ibn al-Haitham is not only worth mentioning because he - like many others - addressed himself to the public with the fruits of his scientific studies, but also in that he offered the useful art of reckoning to the whole society as an elementary part of education and culture. Two centuries before al-Jahiz had still ridiculed a certain katib who was versed in hisab.

The books on the art of administration which appeared around the end of the 9th century, emphasized that the clerks need this knowledge in order to understand the monetary transactions, to execute devaluations and revaluations, to control the laws of the market, to satisfy the financial

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15- al-Gazzāli: *Ihyā' ^{c}ulum ad-din.* al-Qahira 1316h, i, p. 19/7f.

16- Ihya, i, p. 13/-8f.

17- G.P. Matvievskaja/B.A. Rozenfeld: Matematiki i Astronomi Musulmanskogo Srednevekovja i ix Trudi (VIII-XVII wek.). Moskwa 1983, ii, p. 54.

18- Ibn al-Haitham: *al-Qawl al-ma^{c}ruf fi hisab al-mu^{c}amalat.* Berlin, MS Staatsbibliothek-Ost, MO 2970, fol. 178b.

needs of the sultan by way of manipulating the tax-rates and to solve logistic problems on military compaigns. The hand-books classify this measures as *mu^c^amalat sultaniya*, relations between state and citizen.[19] On the clerks it was made incumbent to be trained in the six basic mathematical operations, addition, subtraction, multiplication, division, root extraction and proportions as well as in rudimentary algebra. But quite a few scholars did not put up with the one-sided perspective that hisab should be a privileged instrument of state officials. Like Ibn al-Haitham, though in a less anthroplogical but more realistic manner, they extended their vote for ḥisab. Towards the end of the 10^{th} century the Persian mathematician Abu l-Wafa al-Buzjani makes the following appeal to his collegues in his book on ḥisab:

"The clerks very often apply methods of calculation to the disadvantage of the state or other parties. They neglect their work and do not penetrate the procedures because of their ignorance".[20]

After detecting their failures to measure triangles and circles - once benefit of the buyer, once to the benefit of the seller of an estate- he goes on to a profound introduction to arithmetical, planimetrical and stereometrical operations. In order to demonstrate the necessity of such knowledge he gives a detailed example of how the calculation of the Islamic legal taxes can be manipulated.[21] Another anonymous author of that time explicitly offered his mathematical skill to a taxpayer who had been cheated by the official tax-agent.[22]

This perspective includes the basic Islamic demand for justic, ^c^adl. In the treatises on ḥisab the authors time and again denounce the dubious methods applied by certain professionals, like tax-agents or money-changers. They urge them to use the clear, demonstrable, and just methods of Geometry, Arithmetic and Algebra. They do not, however, stop at the moral call for justice.

Since the Islamic expansion had vastly changed the economical structures of the early days, which had set the frame for the quranic prescriptions, new methods were necessary to make them applicable to the transcontinental economical space that had developed in the meantime. The contribution of mathematics of mathematics to this development cannot be underestimated.

Let me just pick out one central and apparantly simple example: the measuring units. As the basis of every kind of calculation they were equally essential for trade, administration and legal order. In order to faciliate their handling, the ḥisab authors spared no effort to compose lists of measures and of their conversion into others. Abu l-Wafa, however, does not content himself with lists. He isolates factors to convert quantities of one measuring system into another and shows new ways to perform his easily and swiftly.

19- See U. Rebstock: *Rechnen*, chapter 2.

20- Abu l-Wafa al-Buzjani: *Kitab fi ma yahtaj ilaihi al-kuttab wa l-^c^ummal wa-ghairuhum min ^c^ilm al-ḥisab.* (Ed. Ahmad Sa^c^idan), ^c^Amman 1971, pp. 202-204.

21- al-Buzjani: *Kuttab*, p. 287f, 316f.

22- Abu ^c^Abdallah Ahmad ash-Shaqqaq: *Kitab al-Hawi li-a^c^mal* assultaniya wa-rusum al-hisab ad-diwaniya. Paris, MS Bibl. Nat. ar. 2462, fol. 40b.

A whole chapter of his hisab-book deals with measurements.[23] From the babylonian chaos five units are joined to a coherent system. The biggest unit, 1 aṣl (rope), has 10 bāb (rod); 10 bab have 60 dhirā^c (yard), 60 yards have 360 qabḍa (fist's breadth) and 360 fists have 1440 iṣba^c (finger's breadth).

So 1 iṣba^c = $^1/_{1440}$ aṣl = $^1/_{144}$ bāb = $^1/_{24}$ dirā^c = $^1/_4$ qabda.

For planimetrical calculation further units are used.

One square asl equals 1 gharīb; 1 gharib equals 10 qafiz, 10 qafiz 100 ^{c}ashir, 100 ^{c}ashir are 3600 dhirā^c (i.e. square dhirā^c), these equal 21 600 square qabḍa which are composed of 345 600 square iṣba^c. So $1d^2$ can be expressed as $^1/_{4.9}$ ^{c}ashir. 1d multiplied with 1 $q = {}^1/_{3.8.9}\, d^2$ ^{c}ashīr; and 1 d multiplied with 1 iṣba^c 1 $q = {}^1/_{4.8.8.8.9}$ ^{c}ashir or $^1/_{8.8.9}\, d^2$.

The stereometrical unit azala replaces the planimetrical unit asl. One aẓala contains $100d^3$ (cubic yard) which equal 172 800 $qf^c b^3$ or 11 059 200 iṣba^{c3}. Now, Abū l-Wafā recommends spliting such big numbers into factors of powers of ten and powers of the unit :

172 800 $qf^c b^3$ can thus be expressed as $10^2 . (12\, qd^c b)^3$, and 11059200 is^3 as $10^2 . (48\ is)^3$.

This is especially useful for the calculation of volumes. If 'yards' and 'fists' and 'fingers' are multiplied with each other 10 combinations are possible.

The multiplication of the three sides of a regular solid measured in yards, f.e., yield a product which can be decomposed into $a . 10^0 + b . 10^2$ where the 'a' stands for the number of d^3 and the 'b' for the number of aẓala ($= a . d^3 + b . az$).

We could draw a deep breath now - if the units mentioned above had not differed from place to place. There existed 11 different dhirā^c lenghts. One was called the 'black-ell'. It was introduced by the caliph al-Mansur who is told to have taken measure of the ell of his negro-slave. This 'black ell' corresponds to $^{89}/_{72}$ of the so-called 'iron ell' which was used by the blacksmiths. Consequently, before being convertible and comparable, the volumes had first to be adjusted to one another.

Other texts[24] contain similar conversion lists and factors of the different coin weights and product weights then current in the Islamic world.

Services of this kind were incorporated into the non-mathematical professional literature from the 11th century onwards and brought about the acceptance of practical mathematics, not only as a science but also as a respectable part of Islamic education.

The rising reputation of this science is best mirrored by the 10th century encyclopedist Ibn

23- The following extracts are chosen from al-Buzjani: Kuttab, pp. 205-212,

24- See for example Ibn al-Mīlī: *Munqidh al-halik wa-^{c}umdat as-salik*, MS Leiden Cod. Or. 1511, p. 50/4ff.

Fari^{c}un. He divides hisāb into 'legal reckoning', hisab fiqhī, into 'administrative reckoning', hisāb diwāni, and into 'geometrical reckoning', hisāb handasī, which surveyors' and architects needed for their job.[25] This distinction roughly corresponds to ^{c}ilm al-farā' id, hisāb al-mu^{c}āmalāt and ^{c}ilm al-misaha, the three major disciplines and literatures on applied mathematics unfolding at that time.

The acceptance is based on the understanding that these methods developed and offered by experts, facilited the verification and correction of unjust arbitrariness.

The select demontration of the development and acceptance of practical mathematics should not mislead us. A much more important proof waits to be given. For the point in question is whether these methods were actually applied, whether mathematics had become a generally accepted tool. Unfortunately the sources of the Early Middle Ages seldom report on the daily social life, let alone on the real process of calculating debts or measuring tax areas. Who should have registered it and noted it down? For which reason should it have been transmitted? Only by accident are we informed about the division of a heritage that had taken place somewhere or about the calculation of customs elsewhere.

On the other hand, the search in the vicinity of mathematical literature has yielded a substantial harvest. The particular Islamic division of mathematics in theoretical and practical disciplines, the growing acceptance of the skilful services for economy and administration can be interpreted as signs of a modern and rational way to think. A way of thinking which had - if we recall the warning of Ibn al-Haitham - even gained the rank of a sixth sense of the pious Muslim. One of the keys of this development may have been the readiness of the Islamic mathematicians to leave their ivory-tower and make their knowledge available to the public. A readiness which may have been supported by the religious pressure on philosophical freethinkers like ^{c}Umar al-Khayyām fro example.

One particular fact may also have supported this readiness or may even partly explain this readiness: The diffusion of the Indian numbers. The bibliographical findings make clear that their progation was successfully persued only 200 years after their first import to Syria. But from now on countless treatise on hisāb al-hind are written betwenn Central Asia and al-Andalus. Must of them have not yet been studied or even catalogued Being simply introductions to the reckoning with Indian numerals, to basic geometry and to hisab al-mu^{c}amalat they were ignored by both, the historians of mathematics and the historians of civilization. The temporal co-occurence of the universal diffusion of the Indian numerals with the growing appreciation and teaching of practical mathematics is not accidental. The Indian type of notation was superior to the classical Greek notation and to the Arabic finger-reckoning. Its decimal system allowed elegant and quick operations with large numbers. It was easy to communicate and - most important - it was applicable everywhere. A wooden stick and some fine sand, at-takht wa l-makl, was all one needed and everywhere at hand. With this method at hand and the rising need for basic mathematical knowledge the circulation was set in motion.

Even in Christian Toledo interest was raised. Around 1140 an anonymous monk, presumably Johannes Hispalensis, took time to translate one of those hisāb-treatises as 'Liber Mahamelet' into

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25- Ibn Fari^{c}un: Gawami' al- ulūm. (Facs. Ed. Fuat Sezgin), Frankfurt 1985, p. 69.

Latin. And one of the last problems - and here we are back at al-Baghdādī - dealt with the watering down of wine and beer.

The French historian Le Goff has called the 12th century the century of the arithmetizaton of the Occident. That both, Christian and Islamic reckoners bothered themselves with wine-problems does not mean that they shared common grounds. But we have grown curious enough to look more closely at the amazing achievements in the field of Islamic economy, administration and architecture.

To define the contribution of applied mathematics will be difficult. But we will see a mobile and rational spirit at work who has hitherto, until today, been taken as a privilege of the Occident.