**Appendices S1 to S4**

**Hill−Chao numbers allow decomposing gamma multifunctionality into alpha and beta components**

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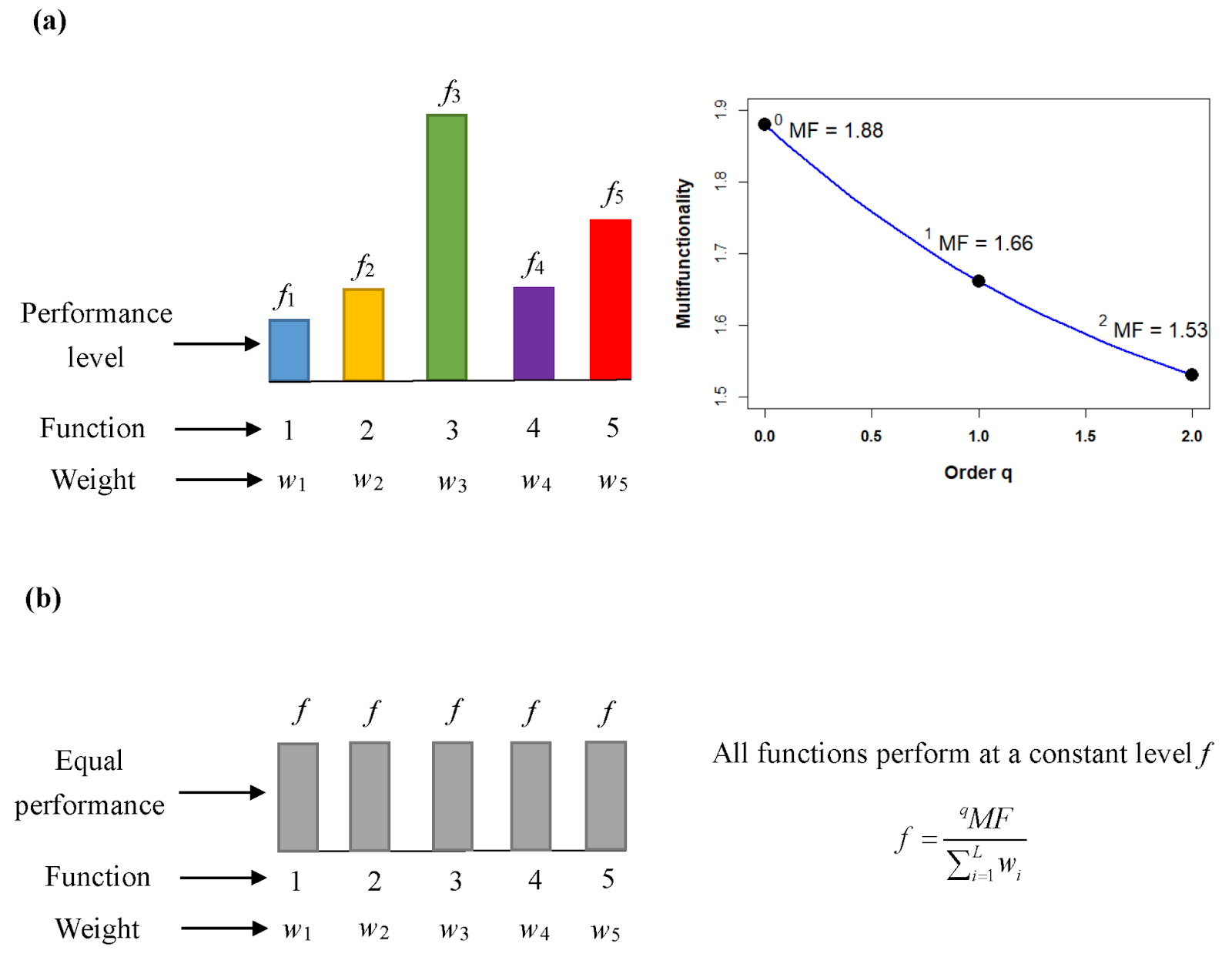
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**Appendix S1. Interpreting the proposed multifunctionality measure and comparing it with Byrnes et al.’s measure**

*An intuitive interpretation of the proposed multifunctionality measure*

Consider the illustrative ecosystem in Figure S1.1(a) below: there are *L* = 5 functions with performance levels = (0.2, 0.3, 0.9, 0.3, 0.5) and function weights = (1, 1, 1, 0.6, 0.6). Our proposed multifunctionality measure (in Equations 3a and 3b of the main text) quantifies the effective weighted sum of function performance levels for any order *q* ≥ 0. In Figure S1.1(a), we also present the corresponding multifunctionality profile which depicts  with respect to the order *q*, 0 ≤ *q* ≤ 2. Figure S1.1(b) shows a *simple reference* ecosystem in which there are *L* = 5 functions with the same set of function weights (1, 1, 1, 0.6, 0.6), but all functions perform at a constant level of .

The multifunctionality value of any order *q* ≥ 0 for the illustrative ecosystem in Figure S1.1(a) is identical to that of the same order for the simple reference ecosystem in Figure S1.1(b). For example, for *q* = 0, we have , i.e., the actual weighted sum of function performance levels. Then the illustrative ecosystem has the same weighted sum as the simple reference ecosystem with all functions performing at a constant level of 1.88/4.2 = 0.45. For *q* = 1, Equation (3b) for the illustrative ecosystem gives a numerical value: , which can be interpreted as the effective weighted sum of levels based on moderate-performing functions. The illustrative ecosystem has the same multifunctionality value of order *q* = 1 as the simple reference ecosystem with all 5 functions performing at a constant level of 1.66/4.2 = 0.40. Analogously, for *q* = 2, Equation (3a) for the illustrative ecosystem gives a numerical value: , which can be interpreted as the effective weighted sum of levels based on high-performing functions. In the corresponding simple reference ecosystem, all 5 functions perform at a constant level of 1.53/4.2 = 0.36.



**Figure S1.1. (a)** Multiple functions in an illustrative ecosystem (left) and the corresponding multifunctionality profile (right). In the illustrative ecosystem, there are 5 functions with performance levels = (0.2, 0.3, 0.9, 0.3, 0.5) and weights = (1, 1, 1, 0.6, 0.6). The multifunctionality profile depicts the value of  with respect to order *q*. The multifunctionality numerical values specifically for *q* = 0, 1 and 2 are shown along the profile: ,, and . **(b)** A simple reference ecosystem with equal-performance of functions. For example, for *q* = 0, 1, and 2, the above illustrative ecosystem has the same value as the simple reference ecosystem in which all functions perform at a constant level of 0.45, 0.40 and 0.36, respectively.

*Interpretation of the multifunctionality profile when all weights = 1*

As indicated in the main text, the multifunctionality profile conveys all the information in the function performance levels and thus completely characterizes multifunctionality in an ecosystem. More interpretations regarding the profile when all weights = 1 are given as follows.

(1) For *q* = 0, our measure reduces to the actual sum of function performance levels in the data. Thus, the ratio between the measure and the number of functions (*L*) represents the average of function performance levels, a measure which has been used in BEF research (e.g., Maestre et al., 2012). The difference between the measure value and the number of functions represents the difference in the total function level between the actual ecosystem and an ideal ecosystem with all functions performing maximally.

(2) When all functions perform at the same level (i.e., functional levels are even), the multifunctionality profile curve is a horizontal line. Otherwise, the profile curve is a decreasing function of the order *q* ≥ 0. The magnitude of the slope generally reflects the unevenness of function performance levels, but for *q* > 0 it is heavily sensitive to high-performing functions, as shown in later numerical examples.

*Byrnes et al.’s (2023) measure*

Byrnes *et al*. (2023) proposed the following multifunctionality measure of order *q* ≥ 0 as a product of *A* (the simple average level of functions) and the effective number of functions:

, (S1.1)

where . Thus, if all weights are the same with  and correlations between functions are not considered, our measure for the special case of *q* = 0 (i.e.,) is identical to their measure ; both reduce to the actual sum of function performance levels. However, for *q* > 0, our measure generally differs from  to some extent; see later comparisons.

*A theoretical difference between our measure and Byrnes et al.’s (2023) measure*

As indicated in the main text, our proposed multifunctionality measure () and the measure introduced by Byrnes *et al*. (2023) (, as shown in Equation S1.1) are both based on the framework of Hill numbers. Nevertheless, there exists a theoretical difference between the two approaches, which is reiterated below. Consider the simplest uncorrelated case with all weights = 1. Then both measures quantify the effective sum of function performance levels. When one additional function with a very low performing level is added, our multifunctionality measure can be proved (see later text) to obey the weak-monotonicity property, i.e., adding an additional very low-performing function should result in an increase in the multifunctionality value. However, Brynes *et al*.’s measure does not satisfy this property for *q* > 0. (For *q* = 0, their measure is identical to ours.)

We use the example presented in the main text to explain why the above-described difference arises. Suppose we have 4 uncorrelated functions with performance levels (0.3, 0.5, 0.1 and 0.2) with all weights = 1; by adding one additional low-performing function (say, 0.01), we then have five functions with levels (0.3, 0.5, 0.1, 0.2 and 0.01). For *q* = 1, Byrnes *et al*.’s measure *decreases* from 0.951 (for 4 functions) to 0.799 (for five functions). Our measure increases from 1.0047 to 1.0059, correctly signifying that the sum of performance levels slightly increases due to adding an additional function. For *q* = 2, our measure increases from 0.9447 to 0.9452, whereas Byrnes *et al*.’s measure *decreases* from 0.853 to 0.701.

The reason for the possible decrease in the value of Byrnes *et al*.’s measure is that their measure is expressed as a product of two terms; see Equation (S1.1). The weak-monotonicity property is fulfilled for the second term which represents a Hill number (i.e., the effective number of functions). However, the property may fail for the first term (i.e., the simple average of performance levels), leading to a failure of the property for the product of the two terms. For example, in the special case of *q* = 2, the effective number of functions in the above example increases from 3.103 to 3.158, but the simple average of performance levels reduces substantially from 0.275 to 0.222. Consequently, the product decreases from 0.853 (= 0.275×3.103) to 0.701 (= 0.222×3.158).

*More comparisons of our multifunctionality measure with Byrnes et al.’s (2023) measure*

Consider the following four ecosystems. In each ecosystem, there are *L* = 5 uncorrelated functions; their performance levels are given below. We consider the simplest case with all weights = 1. In all ecosystems, the total performance levels are fixed to be 2, and the corresponding simple averages are fixed to be 0.4.

Ecosystem 1: = (0.4, 0.4, 0.4, 0.4, 0.4).

Ecosystem 2: = (0.3, 0.3, 0.4, 0.4, 0.6).

Ecosystem 3: = (0.1, 0.3, 0.4, 0.4, 0.8).

Ecosystem 4: = (0.1, 0.1, 0.1, 0.8, 0.9).

We first illustrate the Byrnes et al.’s (2023) approach in terms of ordinary Hill numbers in communities; see Equations (1a) and (1b) in the main text. Note that there exists a constant *K* such that *K*× *fi* is an integer for any *i =* 1, 2, …, *L*. In the above example, without losing generality, we take *K* = 10. In Byrnes et al.’s (2023) measure, ordinary Hill numbers are obtained by considering 4 “communities” (referred to as Communities 1−4 below); each community has 5 species with species absolute abundances = . For example, for Ecosystem 1, we consider Community 1 in which there are 5 species each with raw abundance 4. For simplicity, this community structure is denoted as {5×4}. Similarly, for Ecosystem 4, we consider Community 4 in which there are three species each with raw abundance 1, one species with raw abundance 8, and one species with raw abundance 9. This community structure is denoted as {3×1, 1×8, 1×9}. The four corresponding “communities” are listed below:

Community 1 with structure {5×4}: 5 species with raw abundances = (4, 4, 4, 4, 4).

Community 2 with structure {2×3, 2×4, 1×6}: 5 species with raw abundances 

= (3, 3, 4, 4, 6).

Community 3 with structure {1×1, 1×3, 2×4, 1×8}: 5 species with raw abundances: 

= (1, 3, 4, 4, 8).

Community 4 with structure {3×1, 1×8, 1×9}: 5 species with raw abundances 

= (1, 1, 1, 8, 9).

Based on Equations (1a) and (1b) in the main text, we obtain Hill numbers (TD, taxonomic diversity) of *q* = 0, 1 and 2 for the above 4 communities as follows. Then each Hill number is multiplied by the average performance level (0.4) to obtain Byrnes et al.’s (2023) multifunctionality value; the profiles for the four ecosystems are shown in Figure S1.2(b).

Community 1: = (5, 5, 5), = 0.4 × (5, 5, 5) = (2, 2, 2).

Community 2: = (5, 4.83, 4.65), = 0.4 × (5, 4.83, 4.65)

= (2, 1.93, 1.86).

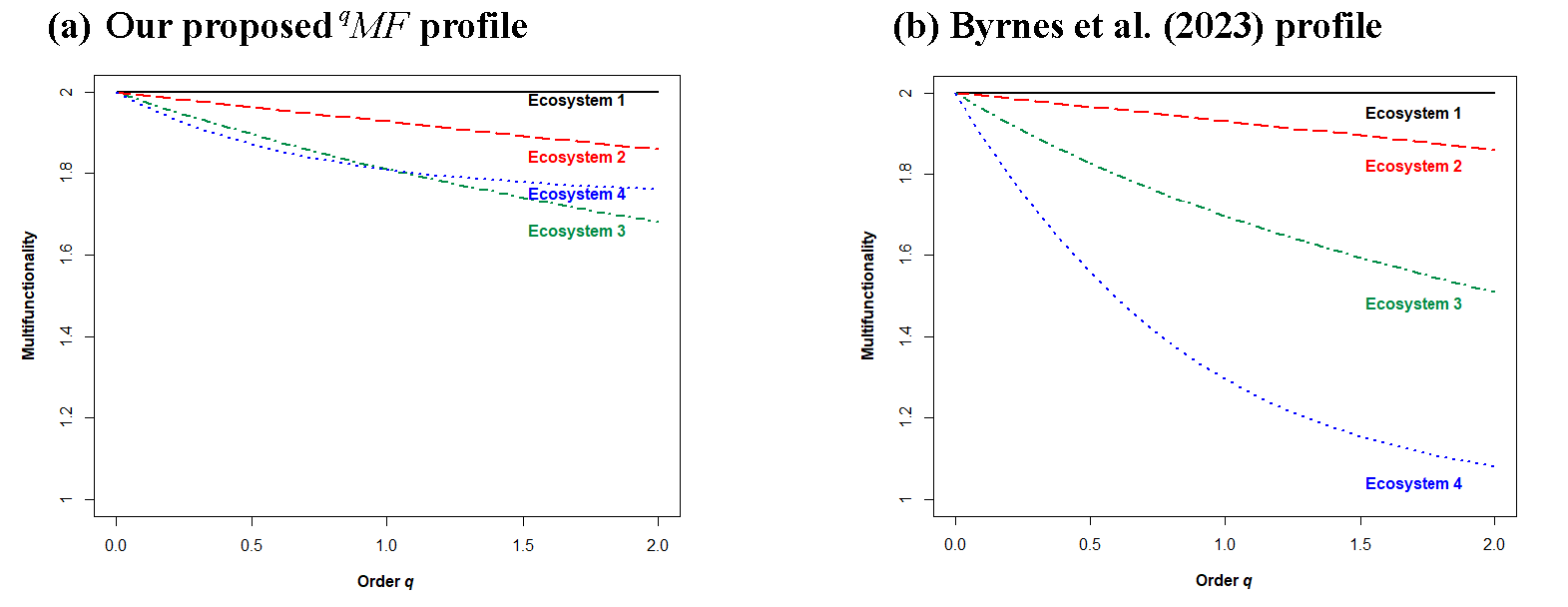
Community 3: = (5, 4.24, 3.77), = 0.4 × (5, 4.24, 3.77)

= (2, 1.70, 1.51).

Community 4: = (5, 3.24, 2.70), = 0.4 × (5, 3.24, 2.70)

= (2, 1.30, 1.08).

The magnitude of the slope for each curve in Figure S1.2(b) reflects the unevenness of 5 species abundances. For example, the species abundances (1, 1, 1, 8, 9) for Community 4 are more uneven than the species abundances (1, 3, 4, 4, 8) for Community 3. Thus, the profile curve for Community 4 lies below the curve for Community 3 for all orders *q* between 0 and 2; the two curves do not cross.



**Figure S1.2.** The multifunctionality profile which depicts multifunctionality value with respect to the order *q*, 0 ≤ *q* ≤ 2, for the four Ecosystems 1−4 (described in the text) based on **(a)** our proposed measure (Equations 3a and 3b in the main text), and **(b)** Byrnes et al.’s (2023) measure (Equation S1.1). The scales of the Y-axes in the two plots are the same.

For our proposed measure based on the Hill−Chao numbers, we can also adopt an ordinary Hill-numbers approach to intuitively understand the measure. Note that when each performance level is multiplied by a constant *K* (here *K* = 10), then the resulting value is *K* times the original value. Thus, for an ecosystem with performance levels , it is equivalent to considering a community in which there are *Kfi* species each with raw abundance *Kfi*, *i* = 1, 2, …, 5. For example, for Ecosystem 3 with = (0.1, 0.3, 0.4, 0.4, 0.8), we consider a community in which there is 1 species (with raw abundance 1), 3 species (each with raw abundance 3), 4 species (each with raw abundance 4), another 4 species (each with raw abundance 4), and 8 species (each with raw abundance 8). For simplicity, this community structure is denoted as {1×1, 3×3, 8×4, 8×8}. Corresponding to the four ecosystems described above, we thus consider the following 4 communities (referred to as Community 1\* to Community 4\* below, each has 20 species):

Community 1\*: {20×4}, i.e., there are 20 species each with raw abundance 4.

Community 2\*: {6×3, 8×4, 6×6}, i.e., there are 6 species (each with raw abundance 3), 8 species (each with raw abundance 4), and 6 species (each with raw abundance 6).

Community 3\*: {1×1, 3×3, 8×4, 8×8}, i.e., there is 1 species (with raw abundance 1), 3 species (each with raw abundance 3), 8 species (each with raw abundance 4), 8 species (each with raw abundance 8).

Community 4\*: {3×1, 8×8, 9×9}, i.e., there are 3 species (each with raw abundance 1), 8 species (each with raw abundance 8), and 9 species (each with raw abundance 9).

Based on Equations (1a) and (1b) in the main text, we obtain ordinary Hill numbers (TD) of *q* = 0, 1 and 2 for the above 4 communities as follows. Then each value is divided by *K* = 10 to obtain our  values; the profiles for the four ecosystems are shown in Figure S1.2(a).

Community 1\*: = (20, 20, 20), = (20, 20, 20)/10 = (2, 2, 2).

Community 2\*: = (20, 19.27, 18.58), = (20, 19.27, 18.58)/10

= (2, 1.93, 1.86).

Community 3\*: = (20, 18.10, 16.82), = (20, 18.10, 16.82)/10

= (2, 1.81, 1.68).

Community 4\*: = (20, 18.09, 17.61), = (20, 18.09, 17.61)/10

= (2, 1.81, 1.76).

The above examples clearly show that our measure of order *q* (times a constant *K*) is identical to a Hill number of the same order in a community in which there are *Kfi* species each with raw abundance *Kfi*, *i* = 1, 2, …, *L*, i.e., the community structure is . Thus, our measure satisfies the weak-monotonicity for all *q* ≥ 0 because it is known that ordinary Hill numbers fulfill the weak-monotonicity property. Moreover, we rigorously prove a stronger form of weak-monotonicity property at the end of this appendix. Note that in Byrnes et al.’s approach, the corresponding community structure is . Our measure is more heavily sensitive to high-performing functions, as also shown from comparing the structures of Communities1− 4 and Communities 1\* − 4\*. Consequently, our measure is generally higher than Byrnes et al.’s; see Figure S1.2(a)−(b).

The magnitude of the slope for each curve in Figure S1.2(a) reflects the unevenness of 20 species abundances, rather than 5 species abundances. For example, the degree of unevenness among the 20 species abundances for Community 3\* with structure {1×1, 3×3, 8×4, 8×8} is slightly higher than that for Community 4\* with structure {3×1, 8×8, 9×9} for *q* < 1. Thus, the profile curve for Community 4 lies below the curve for Community 3 when *q* < 1. However, for *q* > 1, the ordering is *reversed*. The two curves cross at *q* = 1. This example also implies that high multifunctionality values for our measure can be attained by increasing the evenness among function performance levels (as in Ecosystems 1 and 2) or by maintaining several functions at very high levels (as in Ecosystem 4) because of heavy sensitivity to high-performing functions. By contrast, high multifunctionality values for Byrnes et al.’s measure can be attained only by maintaining functions at equal performance levels.

*A stronger form of the weak-monotonicity property*

Consider an ecosystem with *L* functions with normalized performance levels  and function weights . When one additional function with very low performing level *a* > 0 (and weight *wa*) is added to the ecosystem, the multifunctionality measure is an increasing function of *a*, as long as *a* ≤ *fi* for all *i*.

Proof. We first prove the above property for the case of *q* ≠ 1. When an additional very low-performing function with level *a* > 0 (and weight *wa*) is added, the multifunctionality measure (Equation 3a in the main text) based on the new set of performance levels and weights can be expressed as the following function in terms of *a*:

Taking the derivative of  with respect to *a*, we obtain



.

When *q* < 1, the following inequality holds if *a* ≤ *fi* for all *i*:

> 0.

When *q >* 1, a reverse inequality holds under the same conditions:

.

Therefore, we obtain  for the case of *q* ≠ 1, implying that is an increasing function of *a*. For *q* = 1, Equation (3b) of the main text leads to



.

Thus, we have



.

The above inequality follows from  ≥ 0 under the condition *a* ≤ *fi* for all *i*. Thus, for the case of *q* = 1, the property is proved.

*References used in Appendix S1*

Byrnes, J. E. K., Roger, F. and Bagchi, R. (2023). Understandable multifunctionality measures using Hill numbers. *Oikos*, e09402.

Maestre, F.T., Quero, J.L., Gotelli, N.J., Escudero, A., Ochoa, V., Delgado-Baquerizo, M. et al. (2012). Plant species richness and ecosystem multifunctionality in global drylands. *Science*, 335, 214–218.

**Appendix S2: Supplemental theory in the decomposition of multifunctionality measures**

We prove that the proposed alpha and gamma multifunctionality measures for *N* ecosystems satisfy the following properties.

**(a)** The gamma multifunctionality measure is always greater than or equal to the alpha multifunctionality measure: , for all *q* ≥ 0. When all *N* ecosystems have identical performance levels in each function, we have , i.e., , for all *q* ≥ 0.

**(b)** The gamma multifunctionality measure is always less than or equal to *N* times the alpha multifunctionality measure, i.e., . When there are no shared functions among the *N* ecosystems, we have , i.e., .

Combining (a) and (b), we have

, for all *q* ≥ 0.

The above inequalities ensure that the beta multifunctionality measure is always between 1 and *N*.

**Proof:** To simplify the presentation, we let . From Equations (5a) and (5b) of the main text, the gamma and alpha multifunctionality measures for all *q* ≥ 0, *q* ≠ 1 respectively are

,

and

.

**(a)** To prove , note that for *q* >1,  is a convex function. The Jensen inequality implies that for any species *i*,

,

and thus



which is equivalent to

.

This yields  for *q* >1. Note that the Jensen inequality becomes an equality if and only if  for each fixed *i* = 1, 2, …, *S*. Thus,  if and only if all *N* ecosystems have identical performance levels in each function.

For ,  is a concave function. The Jensen inequality leads to the following reverse inequality for any function *i*:



Similar steps imply that  for , and similar arguments also imply  if and only if all *N* ecosystems have identical functions. Taking the limit of Equations (5a) and (5b), as *q* tends to 1, we have

,



When, we thus obtain the following two formulas:

=,

and



Because is a concave function, the Jensen inequality leads to

.

Then we obtain

,

which is equivalent to

.

This implies  because

.

It is readily seen that  when all *N* ecosystems have identical functions.

**(b)** The conclusion  for *q* > 1 follows from the following inequality:

.

The above becomes an equality if and only if  for some *m*, i.e., if and only if there are no shared functions among the *N* ecosystems. Similarly, for, the same conclusion follows from the following inequality:

.

The above becomes an equality if and only if  for some *m*, the same condition as in the case of *q* > 1. For *q* = 1, we have

=

and

.

Then the conclusion of part (b) follows from  for any *m* = 1, 2, …, *N*.

*Decomposition of multifunctionality measures for correlated functions*

As indicated in the main text, the level of function *i* is  in the pooled ecosystem, where  denotes the performance level for function *i* in ecosystem *k*, ,  Applying similar arguments as we obtained Equation (4a) in the main text leads to that the performance level of function *i* is increased from  to  due to correlations between functions:

. (S2.1)

Note that the performance level for function *i* in ecosystem *k* is increased from to , where

. (S2.2)

We can also express  as a simple average of :

.

In the pooled ecosystem, the corresponding function contribution becomes

.

Based on Equation (3a) in the main text, gamma multifunctionality for correlated functions is expressed as



 , *q* ≠ 1. (S2.3)

Likewise, the performance level of function *i* in ecosystem *k* is increased from *fik* to . Parallel derivations lead to alpha multifunctionality for correlated functions:



 (S2.4)

Then beta multifunctionality for the correlated case is defined as

. (S2.5)

As within a single ecosystem, we can consider all plausible thresholds in the interval [0, 1] and depict as a function of *τ*. Then we compute the area under the *τ*-curve (AUC) in [0, 1] to obtain an integrated measure, which is used in our real data analysis. All the properties for the uncorrelated case can be derived for the beta measure for correlated functions in a parallel way.

**Appendix S3: Supplemental data and analysis for the worked example**

**Table S3.1.** The distribution of 209 plots among the six countries; each of the 209 plots consisted of one to five tree species.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| # of species in plot → | 1 | 2 | 3 | 4 | 5 | Row  total |
| FIN | 11 | 14 | 3 | 0 | 0 | 28 |
| GER | 6 | 14 | 14 | 4 | 0 | 38 |
| ITA | 9 | 10 | 9 | 7 | 1 | 36 |
| POL | 6 | 11 | 13 | 11 | 2 | 43 |
| ROM | 8 | 10 | 8 | 2 | 0 | 28 |
| SPA | 11 | 18 | 4 | 3 | 0 | 36 |

The basal area of each tree species within each plot was used as a proxy for species abundance to compute tree species diversity of orders *q* > 0. The basal area data are available from the Dryad Digital Repository (Scherer-Lorenzen et al., 2023; see Data Availability Statement in the main text).

*A list of the 26 functions used in the worked example along with brief descriptions*

**Table S3.2.** The 26 ecosystem functions and their brief descriptions (taken from Table 1 of Ratcliffe et al., 2017). All ecosystem function data are available from the Dryad Digital Repository (Scherer-Lorenzen et al., 2023; see Data Availability Statement in the main text). Smaller values are desirable for functions (7) and (19) in the table; higher values are desirable for the other 24 functions. Their pairwise correlations are shown in Figure S3.1.

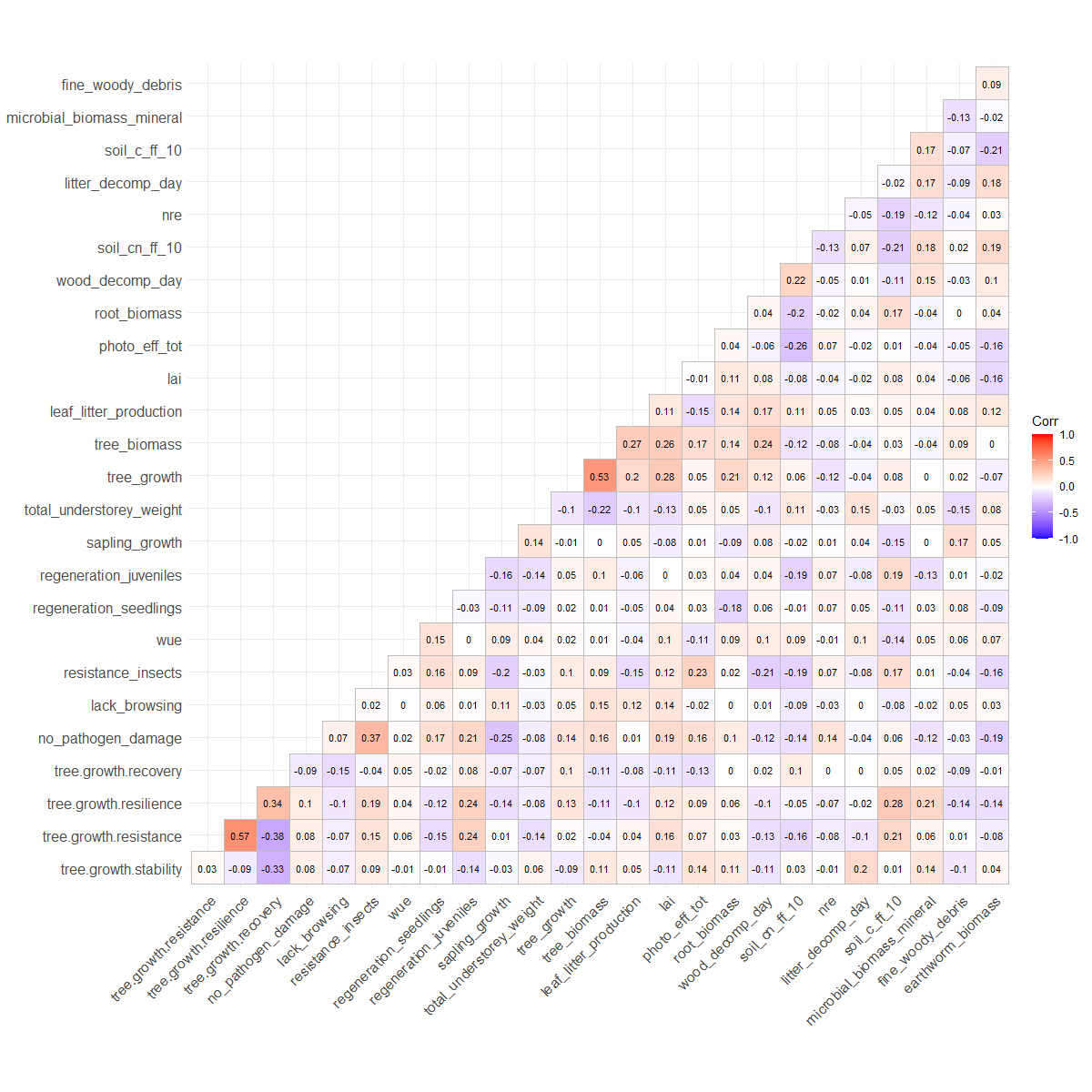
|  |  |
| --- | --- |
| **Ecosystem function** | **Description** |
| 1. Earthworm biomass | Biomass of all earthworms (g m-2 ) |
| 1. Fine woody debris | Snags and standing dead trees shorter than 1.3 m and thinner than 5 cm DBH, and all stumps and other dead wood pieces lying on the forest floor |
| 1. Microbial biomass | Mineral soil (0–5 cm layer) microbial biomass carbon |
| 1. Soil carbon stock (soil\_c\_ff\_10) | Total soil carbon stock (Mg ha-1 ) in forest floor and 0–10 cm mineral soil layer combined |
| 1. Litter decomposition | Decomposition of leaf litter using the litterbag methodology (% daily rate) |
| 1. Nitrogen resorption efficiency   (nre) | Difference in N content between green and senescent leaves divided by N content of green leaves (%) |
| 1. Soil C/N ratio (soil\_cn\_ff\_10) | Soil C/N ration in forest floor and 0–10 cm mineral soil layer combined (smaller values are desirable) |
| 1. Wood decomposition | Decomposition of flat wooden sticks placed on forest floor (% daily rate) |
| 1. Fine root biomass | Total biomass of living fine roots in forest floor and 0–10 mineral soil layer combined (g m-2 ) |
| 1. Photosynthetic efficiency | Chlorophyll fluorescence methodology (ChlF) |
| 1. Leaf mass (lai) | Leaf area index |
| 1. Litter production | Annual production of foliar litter dry mass (g) |
| 1. Tree biomass | Aboveground biomass of all trees (Mg C ha-1 ) |
| 1. Tree productivity | Annual aboveground wood production (Mg C ha-1 year-1 ) |
| 1. Understory biomass | Dry weight of all understory vegetation in a quadrant (g) |
| 1. Sapling growth | Growth of saplings up to 1.60 m tall (cm) |
| 1. Tree juvenile regeneration | Number of saplings up to 1.60 m tall |
| 1. Tree seedling regeneration | Number of tree seedlings less than a year old |
| 1. Resistance to drought (wue) | Difference in carbon isotope composition in wood cores between dry and wet years (smaller values are desirable) |
| 1. Resistance to insect damage | Foliage not damaged by insects (%) |
| 1. Resistance to mammal browsing | Twigs not damaged by browsers (%) |
| 1. Resistance to pathogen damage | Foliage not damaged by pathogens (%) |
| 1. Tree growth recovery | Ratio between post-drought growth and growth during the respective drought period |
| 1. Tree growth resilience | Ratio between growth after and before the drought period |
| 1. Tree growth resistance | Ratio of tree growth during a drought period and growth during the previous 5-year high-growth period |
| 1. Tree growth stability | Mean annual tree growth divided by standard deviation in annual tree growth between 1992 and 2011 |

*Imputing NA values and normalizing each function*

In our analysis based on 26 functions, measurements are missing for a few functions in some plots; the total percentage of missing values was about 5%. For species basal area data, there are only two missing values. For each missing value, the mean of the non-missing values of the given variable within the country was imputed. The resulting BEF patterns are generally consistent with those by imputing missing values from a machine learning approach (random forests).

After all NA values in the functions were imputed, each function was normalized to the range of [0, 1], separately within a country. For positive functionality, the transformation is , that is, ecosystems with the highest value in the raw function data are transformed to the maximal value of 1, and those with the lowest raw value are transformed to the minimum value of 0. Note that the value “0” always implies an absent function; if the lowest raw value is not 0, the transformed 0 should be replaced by a very small number, e.g., 10−5. For negative functionality, the transformation is . In a similar manner, if the highest raw value is not 0, the transformed 0 should also be replaced by a very small number, e.g., 10−5. These replacements will not affect any numerical computations, but will help indicate that the transformed values represent functions that should be regarded as “present” ones. Therefore, in the transformed data, present or absent functions can be clearly distinguished; the information on presence/absence of functions is required in the decomposition of multifunctionality among ecosystems.

*Correlations between any two normalized functions*

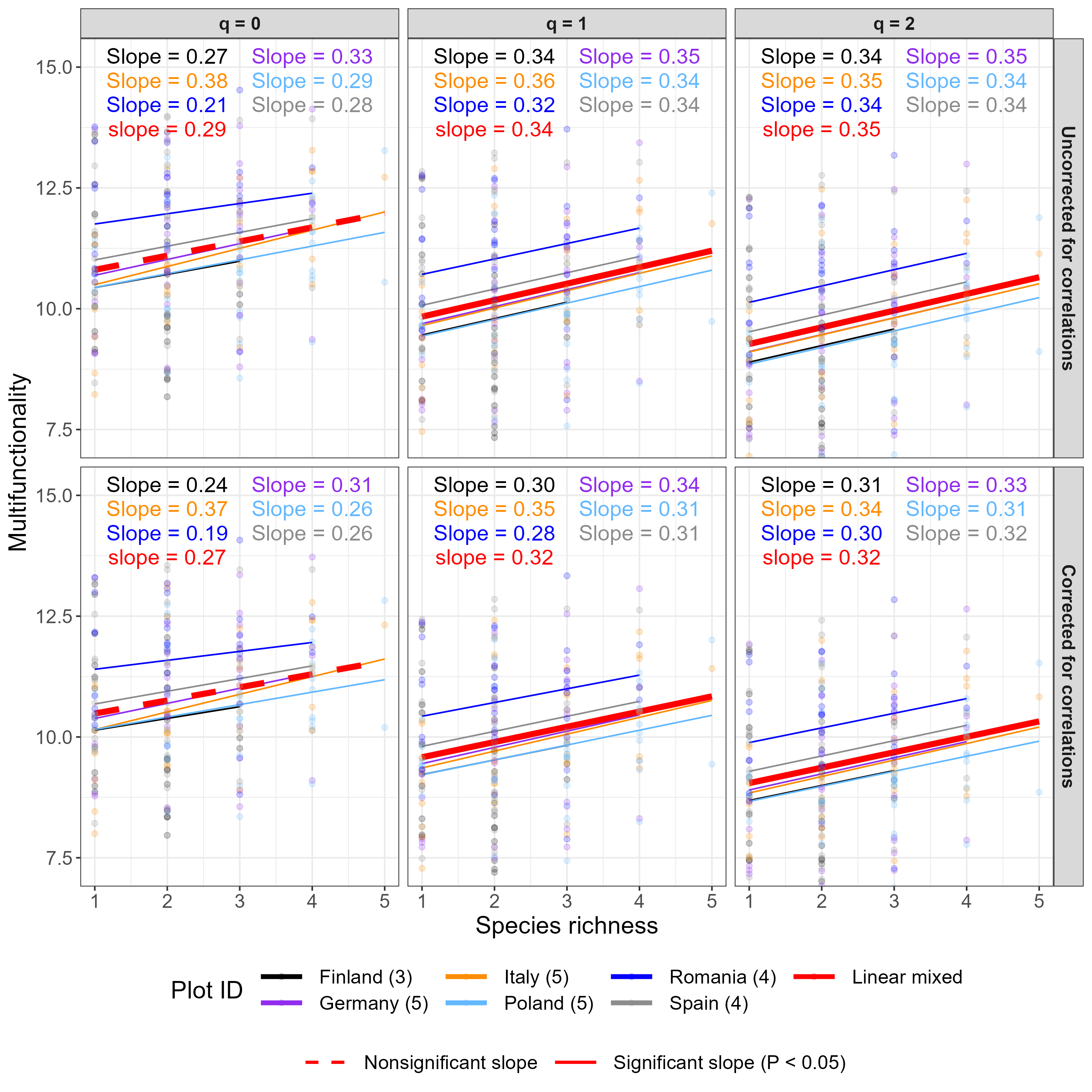


**Figure S3.1**: Correlations between the performance levels of any two normalized functions; all pairwise correlations are generally weak.

*Assessment of the effect of species richness on multifunctionality*

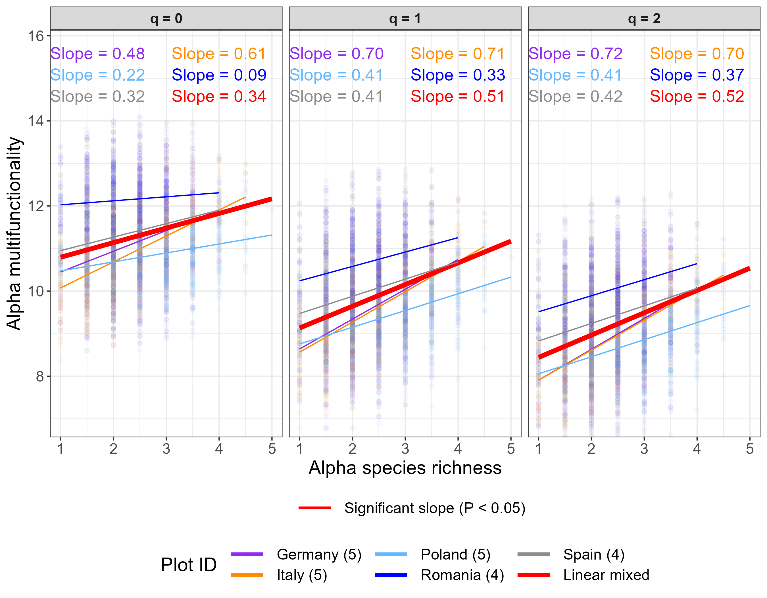
In Figure 2 of the main text, the BEF relationships were plotted between *species diversity* measured with a specific *q* value and the corresponding multifunctionality measure using the same *q* value. We can use different *q* values for species diversity and multifunctionality. For example, to assess how species richness (*q* = 0) influences multifunctionality of *q* = 0, 1 and 2, Figure S3.2 below shows the patterns of within-plot multifunctionality measure of order *q* = 0 (left panels), *q* = 1 (middle panels), and *q* = 2 (right panels). These patterns are in relation to tree *species richness*, rather than species diversity. In the upper panels, all functions are uncorrected for correlations, while in the lower panels, correlations between any two functions are accounted for.

All statistical modeling and fitting remained consistent with those presented in the main text. In the left panels (*q* = 0) of Figure S3.2, the fittings are exactly the same as those in Figure 2 of the main text. For *q* = 1 and *q* = 2, the BEF patterns in Figure 2 and Figure S3.2 are generally similar, except that each fitted slope in Figure S3.2 is lower than the corresponding slope in Figure 2. This observation leads to the conclusion that the effect of species richness on multifunctionality is weaker than species diversity. A similar conclusion is also valid for the BEF relationships across spatial scales when correlations between any two functions are uncorrected for (Figure S3.3) or corrected for (Figure S3.4).

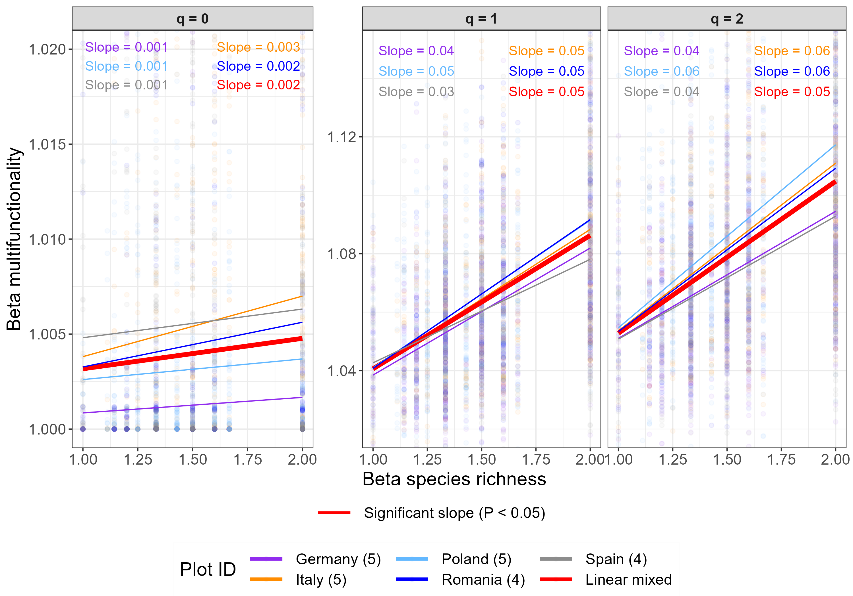


**Figure S3.2**. The legend is the same as in Figure 2 of the main text, except that the X-axis in each panel represents species richness, instead of species diversity of order *q*.

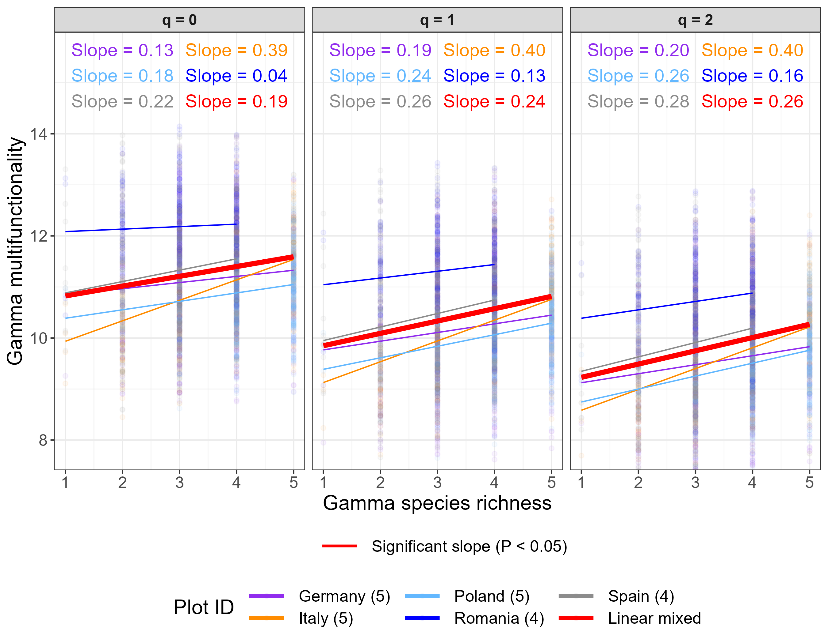
**(a)**



**(b)**

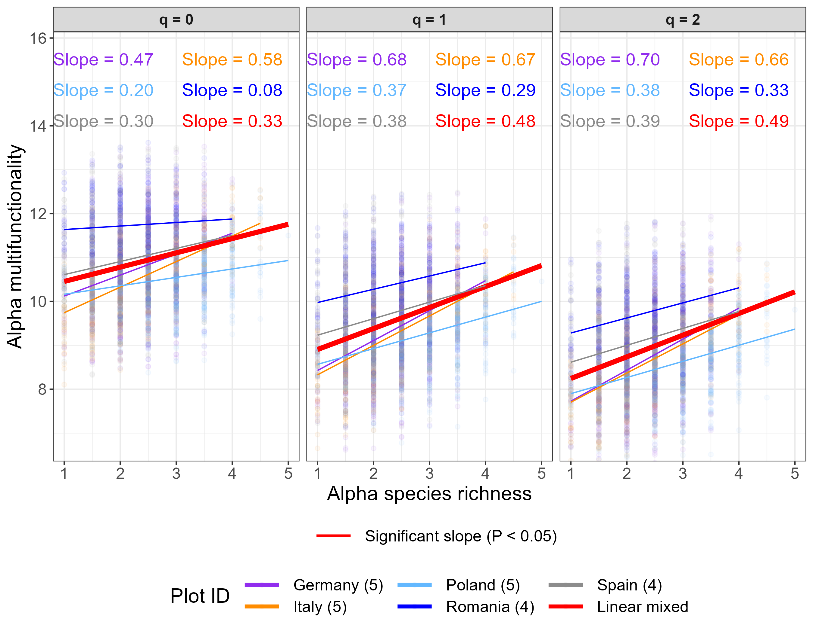


**(c)**

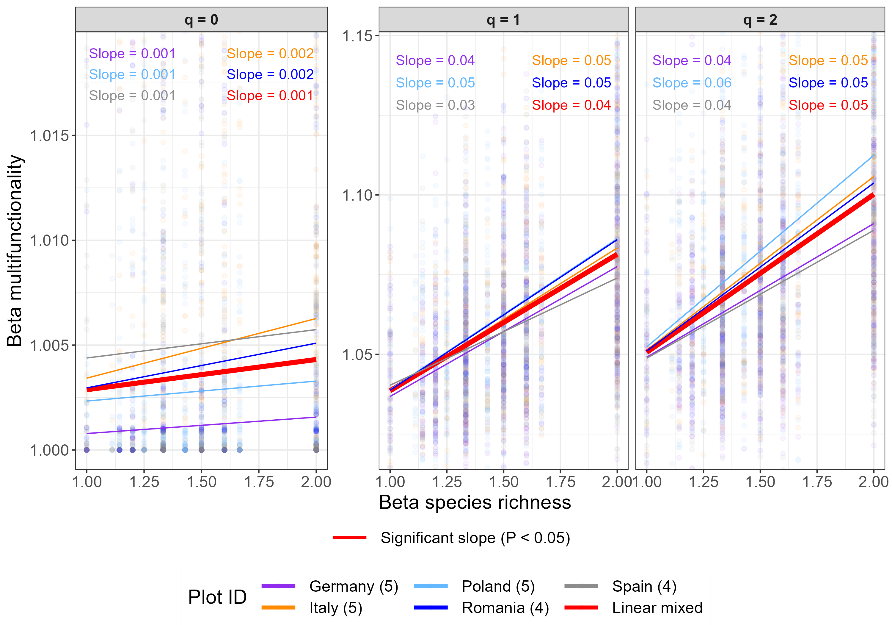


**Figure S3.3.** The legend is the same as in Figure 3 of the main text, except that the X-axis in each panel represents alpha/beta/gamma species richness, instead of alpha/beta/gamma species diversity of order *q*.

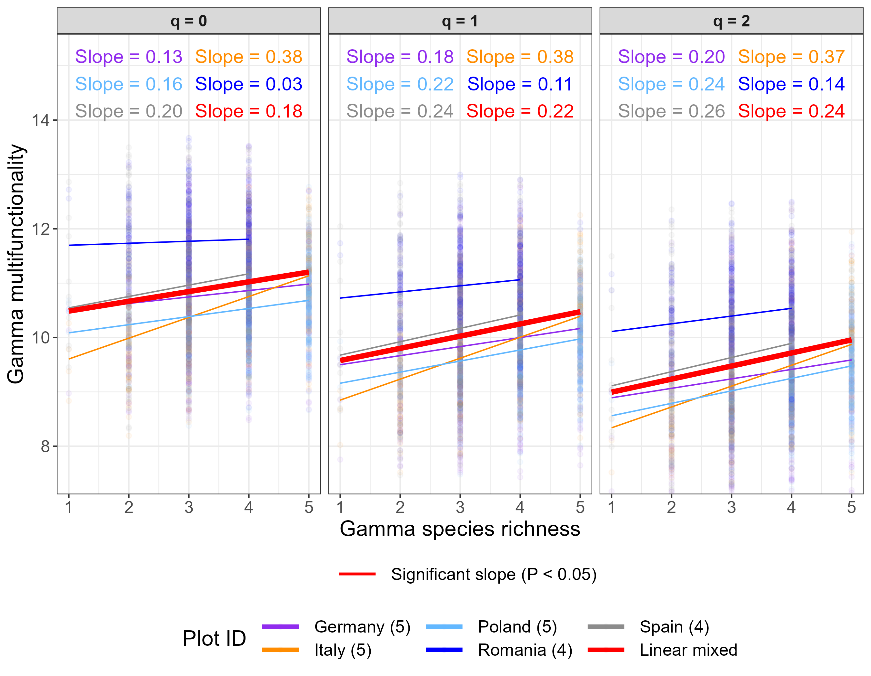
**(a)**



**(b)**



**(c)**



**Figure S3.4.** The legend is the same as in Figure 4 of the main text, except that the X-axis in each panel represents alpha/beta/gamma species richness, instead of alpha/beta/gamma species diversity of order *q*.

*References used in Appendix S3*

Ratcliffe, S. Wirth, C., Jucker, T. van der Plas, F., Scherer-Lorenzen, M. Verheyen, K. *et al*. (2017). Biodiversity and ecosystem functioning relations in European forests depend on environmental context. *Ecology Letters,* 20, 1414–1426.

Scherer-Lorenzen, M. *et al*. (2023). The functional significance of tree species diversity in European forests - the FunDivEUROPE dataset [Dataset]. Dryad. <https://doi.org/10.5061/dryad.9ghx3ffpz>